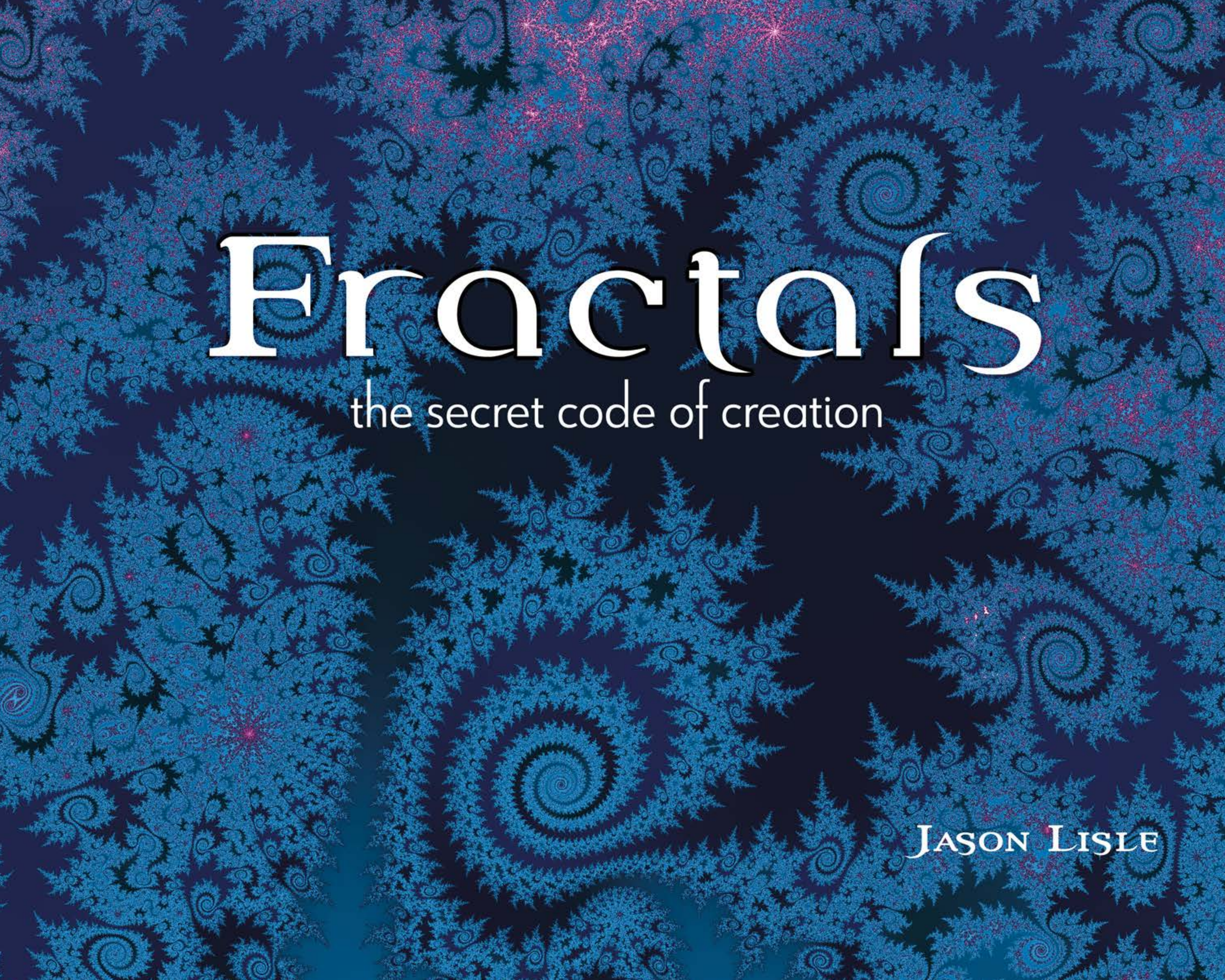

Fractals

the secret code of creation

Jason Lisle

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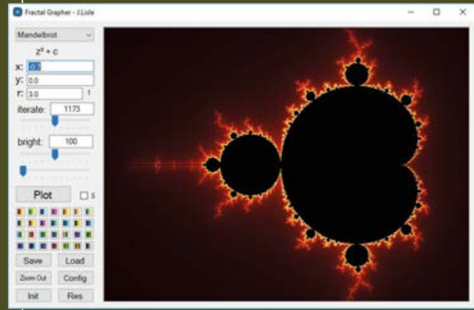

Dr. Jason Lisle takes the reader on a unique journey through fractals and how complex, beautiful, and amazing they are in *Fractals: The Secret Code of Creation*. Now you can explore fractals further and print your own images with Fractal Grapher©, an included program developed by Dr. Lisle of Biblical Science Institute (available for Windows®). It allows you to

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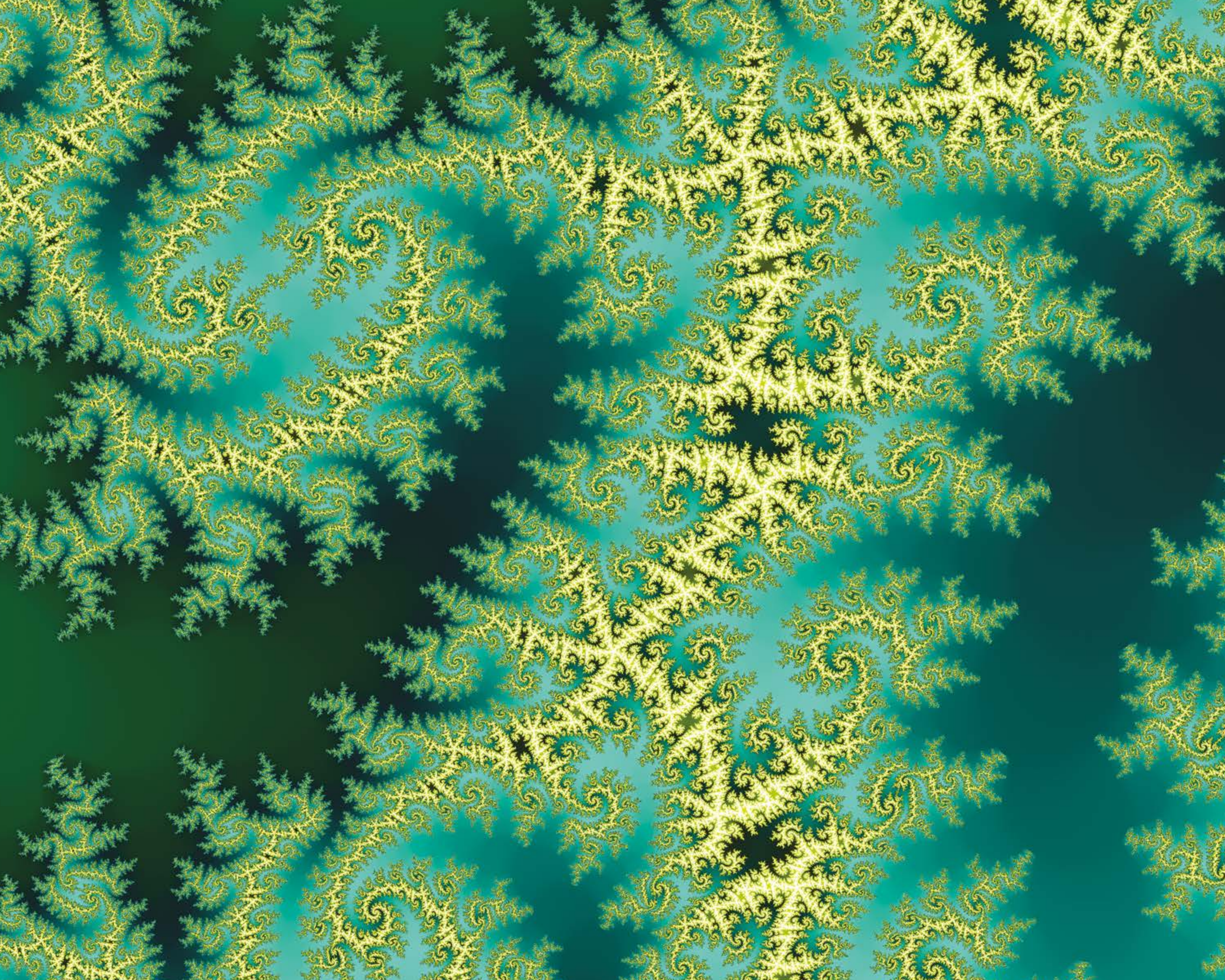
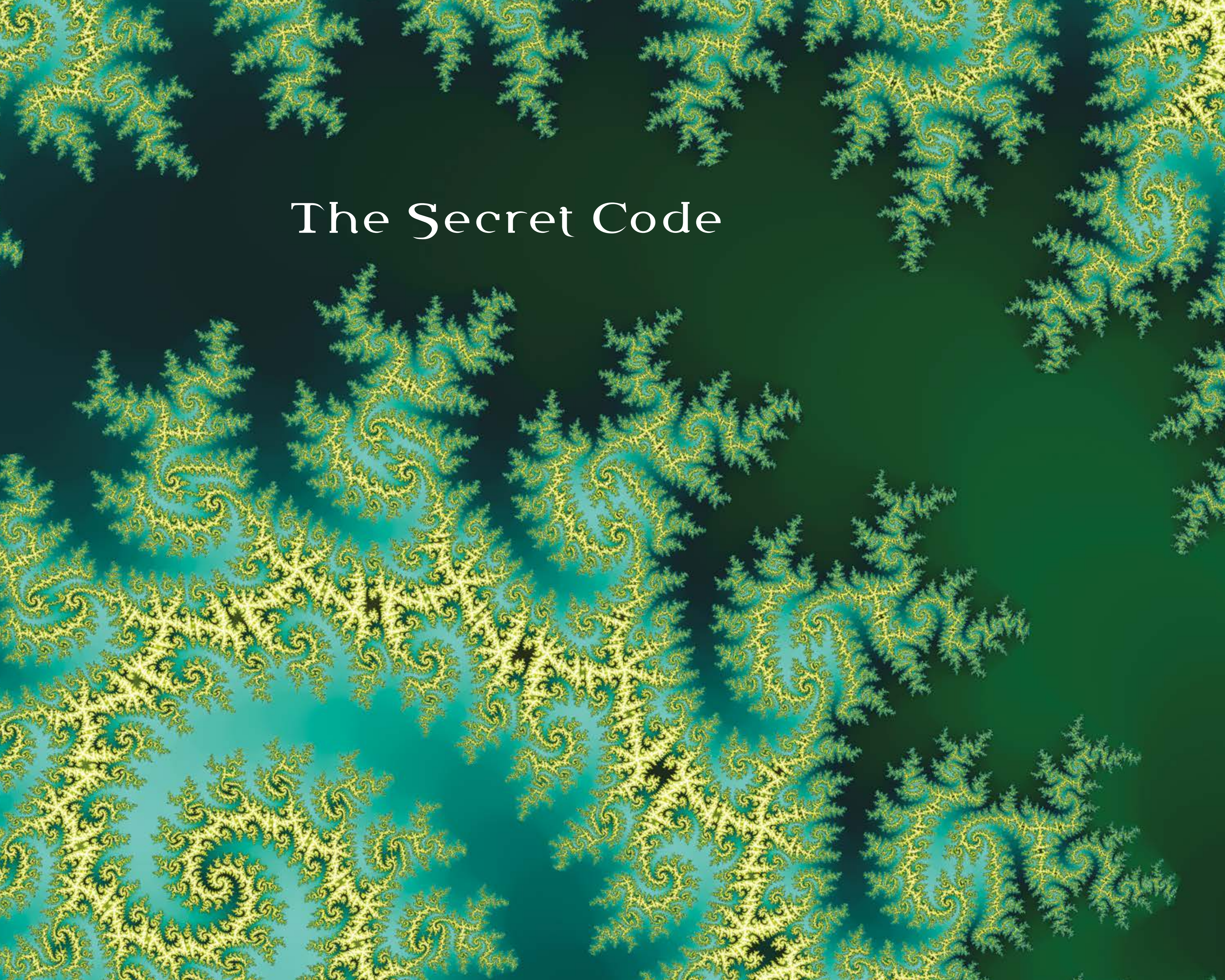
Table of Contents

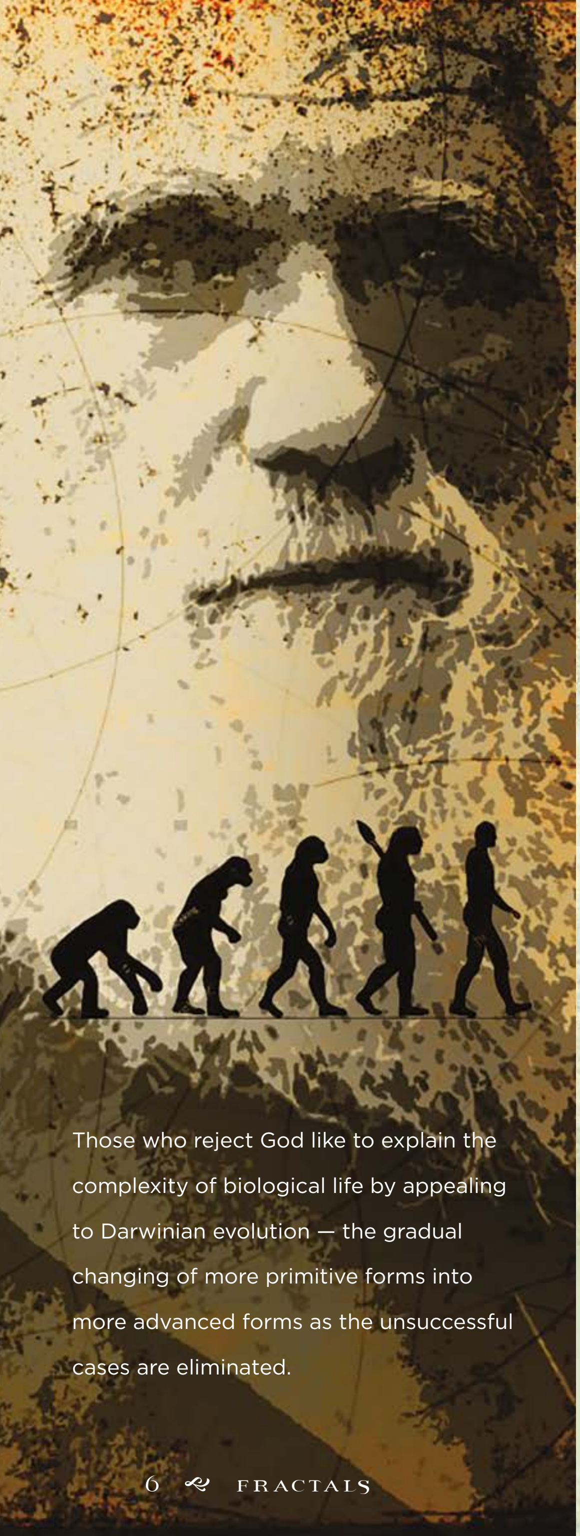
The Images in this book are not the result of human creativity. The only human element is the selected color scheme, but the shapes stem from the mind of God.

Dr. Jason Lisle is a Christian astrophysicist who writes and speaks on various topics relating to science and the defense of the Christian faith. He graduated from Ohio Wesleyan University where he majored in physics and astronomy and minored in mathematics. He then earned a master's degree and a Ph.D. in astrophysics at the University of Colorado in Boulder. Dr. Lisle began working in full-time apologetics ministry, specializing in the defense of Genesis. His most well-known book, *The Ultimate Proof of Creation,* demonstrates that biblical creation is the only logical possibility for origins.



The Secret Code





Those who reject God like to explain the complexity of biological life by appealing to Darwinian evolution — the gradual changing of more primitive forms into more advanced forms as the unsuccessful cases are eliminated.

What if mathematicians discovered a secret code embedded in math itself? What would that mean? Suppose that when analyzing certain sets of numbers, we found an amazing work of art embedded in them, far more intricate and complex than any work of man. How would we make sense of such complex beauty in something as simple as numbers? Who put it there?

In fact, just such a code of astounding beauty was discovered in the 1980s. Artwork of tremendous beauty and infinite complexity had been hidden in numbers from the beginning of time. Yet it lay undiscovered until computer technology had advanced to the point that otherwise tedious computations could be performed with rapid efficiency. The beautiful images in this book are not the work of man. They are the very images that were discovered in sets of numbers, hidden in plain sight. How can we make sense of this? Who or what is responsible for these amazing shapes?

I suggest that secular thinking has no answer. Those who reject God like to explain the complexity of biological life by appealing to Darwinian evolution — the gradual changing of more primitive forms into more advanced forms as the unsuccessful cases are eliminated. This view has its problems, of course, but my point is that such an explanation is not even plausible for numbers because numbers do not evolve. It is not as though the number 7 gradually evolved from the number 3. Numbers have always been what they are. Therefore, the artwork displayed in this book did not evolve. It has always existed, being built into numbers.

I suggest that the Christian worldview alone can make sense of this secret code built into numbers. As such, the images in this book are a demonstration of the truth of the Christian worldview. The same God who built beauty into the physical world has also built beauty into the abstract world of numbers. We don't often think about God creating numbers. We tend to think of God creating physical things.

But even abstract conceptions like numbers could not exist apart from God.

Numbers are a concept of quantity. As concepts, they exist in the mind. We can represent numbers with a written numeral like the Arabic numeral "2" or the Roman numeral "II." But these are merely physical representations of an idea. After all, erasing the physical symbol "2" will not cause the number 2 to cease to exist! The number itself is abstract; it cannot be touched or seen, but it exists as a concept of the mind.

We can think about numbers in our mind, but we did not create them or the rules pertaining to them. It is not as though some ancient human simply decided to invent the number 2 and arbitrarily decreed that 2 + 2 should equal 4. No. Humans *discovered* numbers and the relationships between them. That means that numbers and the relationships between them existed before humans. This makes sense in the Christian worldview because numbers existed in the mind of God from the beginning of time. The mind of God is responsible for the existence of numbers and the rules governing their relationships. It has been the privilege of human beings to discover these rules by the gift of logical reasoning that the Lord has so graciously given. When we discover a mathematical truth, we have discovered something about the way God thinks.

When we discover a mathematical truth, we have discovered something about the way God thinks.

The images in this book therefore represent an infinitesimal glimpse into the mind of God. God's thinking is not only flawlessly rational, but supremely beautiful as well. But exactly how were these images discovered? Where do they come from? A little background information will be helpful.

Sets

The images in the pages of this book are maps of sets of numbers. A set of numbers is just what you think it is: a group of numbers that have something in common. There are all kinds of sets. Most sets include some numbers and exclude others. Consider the set of even numbers. This set includes numbers like 2, 4, 6, 8, 10, but excludes numbers like 1, 3, 5, 7, 9. The set of negative numbers includes numbers like -3, -4, -5, -1/2, but excludes numbers like 2, 5, 7, π, and so on. You can even have the set of all numbers, which includes everything and excludes nothing. You can also have an empty set, which includes no numbers at all.

In sets like those mentioned above, you can tell if a number belongs or not just by looking at it. You know the number 24,389 does not belong in the set of even numbers because even numbers always end in 0, 2, 4, 6, or 8. You know that 57 does not belong in the set of negative numbers because there is no negative sign in front of it. But with some sets, you cannot tell just by looking at the number if it belongs or not. You have to do some work.

Consider the set of prime numbers: those natural numbers that cannot be formed by the product of two natural numbers other than themselves and 1. Does the number 14,351 belong to this set? You probably cannot tell just by looking at it. You have to do some work to see if some product of natural numbers will generate this number. In fact, this number is the product of 113 and 127. So, it does not belong to the set of prime numbers.

The Mandelbrot Set

In the late 1970s and early 1980s, mathematicians began using computers to analyze solutions in a branch of mathematics called *complex dynamics*. This field involves sets of numbers that are defined by functions that involve *iteration* — that is, doing a calculation repeatedly. For example, take the number 1 and double it. Now double the result. Then double that result, and so on forever. This iteration will generate the sequence of numbers 1, 2, 4, 8, 16, 32, 64, 128 . . . and so on. We might represent this expression as $2z \rightarrow z$, meaning that we multiply the number (z) by 2, and this becomes the next value of z. This particular sequence is *unbound*, meaning the numbers just get larger and larger without limit.

Now, let's try a different iteration. Take the number 1 and divide it by 2. Then *divide* the result by 2, and that result by 2 and so on. This iteration produces the numbers 1, ½, ¼, ⅛, 1/16, 1/32, and so on, getting closer to but never actually reaching zero. We represent this iteration as $z/2 \rightarrow z$. This iteration is *bound*, meaning the numbers never exceed a certain value (in this case, 1). So *unbound* sequences get larger without limit, but *bound* sequences have a largest number that none of the members will exceed.

With the iterations mentioned above, it is easy to see whether they are bound or unbound. But with other iterations, it is not so obvious. Some iterations must be done many times before we know whether the sequence is bound or unbound. In the late 1970s and early 1980s, computers were finally fast enough and affordable enough to be useful in this kind of analysis. This allowed mathematicians to explore sets that are defined by iterative functions. One set in particular that caught their interest came to be called the Mandelbrot set, after Benoit Mandelbrot, who explored and popularized this particular number set.

The Mandelbrot set involves the iteration $z^2 + c \rightarrow z$, where z is initially zero. This means that the value of the number z is squared and then added to a different number (c) to become the new value of z, which is plugged back into the formula and so on.

The images in this book therefore represent an infinitesimal glimpse into the mind of God.

The Mandelbrot set is defined as the set of all numbers c for which the sequence z remains *bound* according to this iteration. A more conventional way to write this is:

$$z_n^2 + c = z_{n+1}$$

Mathematicians use subscripts to indicate that the next value of z (which is z_{n+1}) depends on the *current* value of z (which is z_n). The number "c" is the number we are testing to see if it belongs to the Mandelbrot set. So if you want to see if the number 1 belongs to the set, then c = 1. The symbol z_n represents a sequence of numbers. So the sequence z_n might look like this: 0, 1, 2, 5, 26, 677. . . . That's an unbound sequence because it gets larger without limit. The subscript n represents the position in the sequence starting with n = zero. So $z_0 = 0$, and $z_4 = 26$. By construction, the first element in the sequence is always zero. So $z_0 = 0$ always. After that, each value of z depends on the previous value of z (and the value of c), according to the formula above. So, the symbol z_{n+1} means that the *next* number in the sequence will be the square of the previous number (z_n^2) plus c.

The easiest way to get a feel for this is to simply do a few examples. We ask, "Is the number 1 part of the Mandelbrot set?" since we are evaluating the number 1, c = 1. So our iterative formula will be:

$$z_n^2 + 1 = z_{n+1}$$

By definition, the first value in the sequence of z_n is always 0, so $z_0 = 0$. Starting with z = 0, we take this number (zero), square it (it's still zero), and add it to c (which is one) to get the next z in the sequence. Namely, $0^2 + 1 = 1$. So the next value in our sequence of z_n is 1. Now we do this again. Take 1, square it (it's still 1), add it to 1, and this results in the number 2. Do this again: 2 squared (is 4) plus one is 5. Do this again: 5 squared (is 25) plus one is 26. So our sequence of z_n looks like this: 0, 1, 2, 5, 26, 677. . . . We can see that this sequence is unbound. Since the number 1 (c = 1) generated a sequence of z_n that is *un*bound, the number one is *not* part of the Mandelbrot set. We might construct a table to record this:

Number (c)	Part of Mandelbrot set?
1	No

What about the number zero? Does it belong? To test this, we set c = 0, start with $z_n = 0$, and plug it into the formula: $0^2 + 0 = 0$. So the next value of z is also 0. Doing this again, we see the next value of z is also 0 and so on. Our sequence of z_n is: 0, 0, 0, 0, 0, 0, 0. . . . Now this sequence is clearly bound because the value of z will never exceed zero. Since the sequence is bound, the number we were checking, namely zero (z = 0), is indeed part of the Mandelbrot set. So we can add it to our table:

Number (c)	Part of Mandelbrot set?
1	No
0	Yes

One more example involves a number that generates a very interesting sequence. Does the number *negative one* belong to the Mandelbrot set? In this case, c = -1 and substituting this into the formula we have:

$$z_n^2 - 1 = z_{n+1}$$

As before, our first value (z_0) will be zero by definition. We square this number (still zero) and subtract 1 to get -1. We then take this new value of z (negative one) and plug it back into the formula. Negative one squared (which is positive one) minus one is zero. Plugging this back in, we then get the next value of z as -1. So our sequence of z_n is: 0, -1, 0, -1, 0, -1, 0, -1. . . . This sequence cycles between two values forever! But clearly the sequence is bound. Its absolute magnitude never exceeds 1. Therefore, the number -1 is indeed part of the Mandelbrot set, and we can add it to the table:

Number (c)	Part of Mandelbrot set?
1	No
0	Yes
-1	Yes

You can see why this branch of mathematics flourished after the development of computers. It is tedious to do these computations by hand. But computers can do such tasks quickly and test many different numbers to see if they belong to the Mandelbrot set.

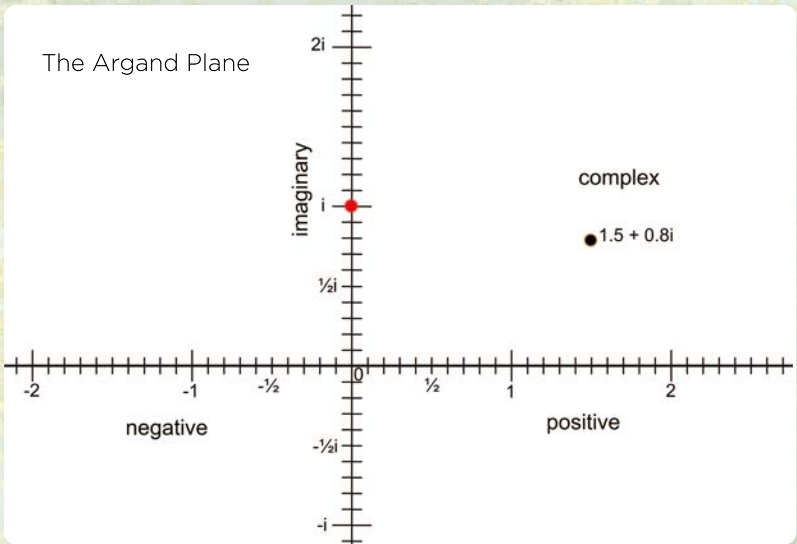
Complex Numbers

There is one more nuance to the Mandelbrot set before we get to the really interesting stuff. The Mandelbrot is not limited to the so-called "real numbers," but also includes complex numbers and "imaginary numbers." I hate the terminology because it is misleading. The name suggests that imaginary numbers are made-up or somehow less valid than the so-called "real" numbers. But in fact, both real numbers and imaginary numbers do exist. They are equally legitimate and are useful. And the terminology has become standard.

An imaginary number is a number that when squared produces a *negative* number. So imaginary numbers are not positive (because a positive number squared is a positive number), and imaginary numbers are not negative (because a negative number squared is a positive number), and imaginary numbers are not zero (because zero squared is zero). So how can you have a number that is not positive, not negative, and not zero?

To answer this, consider a number line. Those numbers to the right of zero are positive. Those numbers to the left are negative. We can think of the imaginary numbers as being on a different axis, directly above or below zero (see figure 1.1). Such numbers are not to the right (not positive) and not to the left (not negative) and yet are not zero. This satisfies the definition of an imaginary number.

Figure 1.1

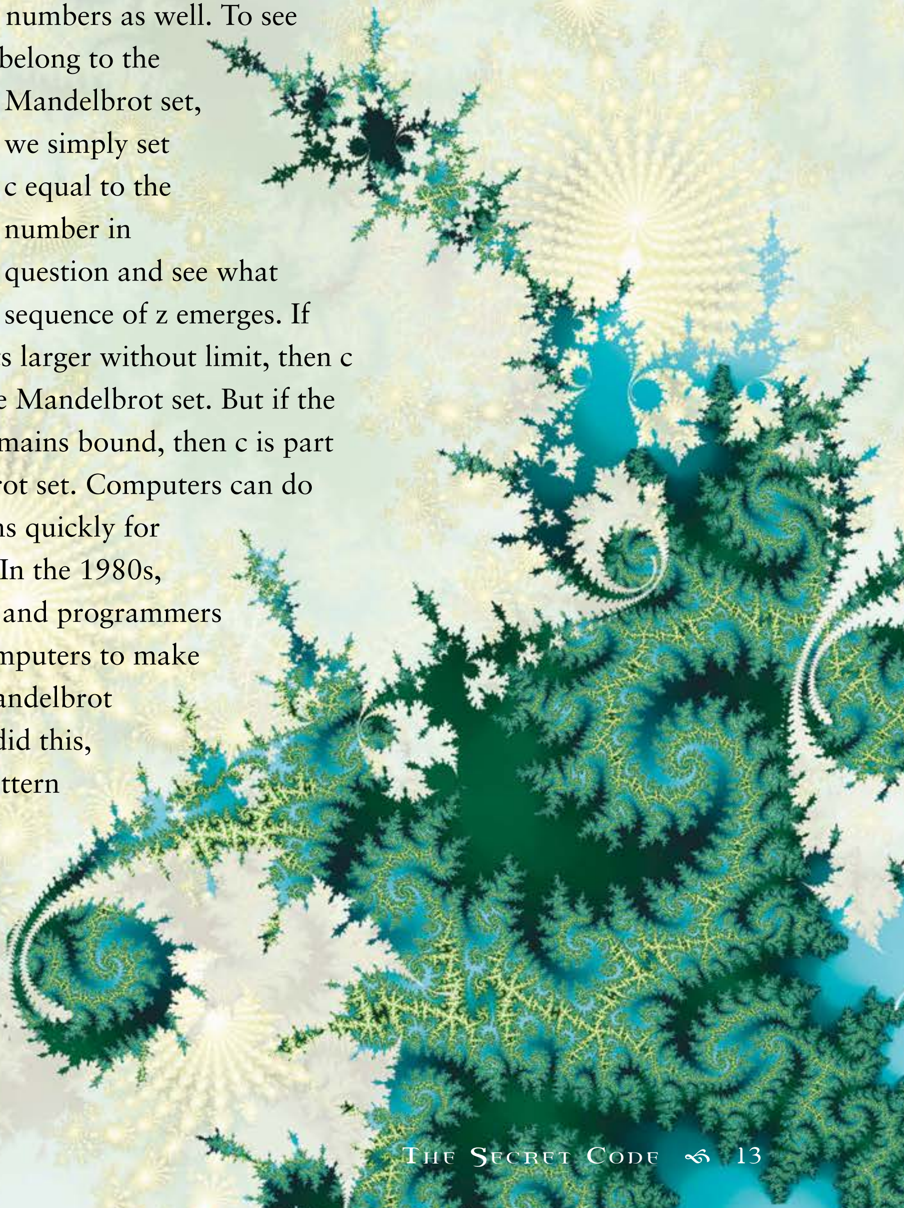


The imaginary number equivalent of the number one is symbolized by a lowercase letter i. This number squared is equal to negative one. By convention, we place i directly above zero. It is placed at the same distance as the number one is from zero. All the other imaginary numbers are generated by multiplying i by any real number. So, 2i, 3i, and so on. These numbers obey the ordinary rules of mathematics; it's just that whenever you encounter an i^2, this is equal to negative one. For example, $(3i)^2 = -9$.

> An imaginary number is a number that when squared produces a *negative* number.

We might also consider numbers that are not on either axis. These are called *complex numbers*. They get their name because they have two parts: a real part and an imaginary part. For example, the number 3 + 2i is complex. It has two parts but is one number. We can plot it using the real component as the x-coordinate and the imaginary component as the y-coordinate. This way of depicting complex numbers by coordinates on a surface is called an *Argand plane*.

The Mandelbrot set includes some complex (and imaginary) numbers as well. To see which numbers belong to the Mandelbrot set, we simply set c equal to the number in question and see what sequence of z emerges. If the sequence gets larger without limit, then c is not part of the Mandelbrot set. But if the sequence of z remains bound, then c is part of the Mandelbrot set. Computers can do these calculations quickly for many numbers. In the 1980s, mathematicians and programmers began to use computers to make a map of the Mandelbrot set. When they did this, a remarkable pattern emerged.



Mapping the Mandelbrot Set

Using computers, we can make a map of the Mandelbrot set in the Argand plane. Recall that each point on the plane represents one complex number, with the x-coordinate representing the real component and the y-coordinate representing the imaginary component of that number. The computer quickly checks each number by running it through the formula, generating a sequence of z_n, which either remains bound (for numbers belonging to the Mandelbrot set) or becomes larger without limit (for numbers *not* in the Mandelbrot set).

But how does the computer decide if the sequence grows larger *forever* or remains small forever? The computer can do a lot of iterations quickly, but it cannot do them forever! We humans do not need to run the iteration forever to see what will happen because we intuitively understand patterns. We know that the sequence 1, 2, 4, 8, 16, 32 . . . will get larger without limit because we can see that each number is twice the previous number. We understand that the pattern 0, -1, 0, -1, 0, -1 will be bound because it cycles. But computers have no such comprehension or intuition.

Using computers, we can make a map of the Mandelbrot set in the Argand plane.

Since computers have no real understanding, in practice, the way they decide if a sequence is bound is for their programmer to set an "escape value" and an "iteration limit." In other words, if after 1,000 iterations the value of z is still smaller than, say, 10, we can be pretty confident (though not absolutely sure) that the sequence is bound. If, however, the value of z is larger than 10 after only 4 iterations, we can be confident that the sequence will grow without limit. For the Mandelbrot set, mathematicians have shown that the escape value can be as small as the number 2. That is, for a given c, if any number in the sequence z_n is larger than 2, then the sequence will grow large without limit and is unbound. So most programmers set the escape value to the number 2. The iteration limit is harder to guess, but suffice it to say that larger values produce a more accurate map.

So the computer systematically checks

each point in the Argand plane to see if the sequence of z remains bound (and is part of the Mandelbrot set) or exceeds the escape value (and is therefore not part of the set) after the prescribed number of iterations. Then the computer assigns a color to the point depending on whether it does or does not belong to the set. By convention, the computer colors the point *black* if the number does belong to the Mandelbrot set and uses some other color (such as red or yellow) if it does not. What will the map look like when the computer has checked every point?

Now the Really Cool Part

Naïvely, we might think the map would be a circle or some basic shape based on the simplicity of the definition of the set. After all, $z^2 + c$ is a pretty simple expression. But instead, the map of the Mandelbrot set turns out to be remarkably interesting and complex, as shown in figure 1.2. This basic shape was first discovered and plotted (in black and white) in 1978, but at much lower resolution than we can do today.

In the 1980s, Benoit Mandelbrot developed software to improve the plotting of the Mandelbrot set and its exterior, eventually in shades of color that represent how quickly the sequence z_n grows — how many iterations it took to exceed the escape value. In other words, for numbers that do not belong to the Mandelbrot set, the sequence of z_n can grow large rapidly (such as z_n = 0, 10, 1000, 100000, 1000000000), in which case the second iteration has already exceeded the escape value of 2. Or, the sequence can grow slowly (such as z_n = 0, 1, 1.1, 1.15, 1.19), in which case it might take a hundred iterations or more for the sequence to exceed the escape value. By convention, we normally use bright colors (like bright yellow) for numbers where the sequence of z_n grows slowly, indicating numbers that are *very close to* (yet not part of) the Mandelbrot set. And we use darker colors (like deep red) for numbers where the sequence of z_n grows quickly, indicating numbers that are far away from being part of the Mandelbrot set. And again, numbers that are part of the Mandelbrot set are colored black.

So, in figure 1.2, the black regions represent all complex numbers that belong to the Mandelbrot set. The yellow regions represent numbers that are very close to the set, but do not belong to it. And the dark red regions represent numbers that are not even close to being part of the Mandelbrot set. Using this map, we can easily check whether any given number belongs to the set simply by checking the color at its coordinates because the computer has already done the calculation. We can see that -1.5 does belong to the Mandelbrot set, but +1.5 does not. We can see that zero belongs (as we proved earlier), but 2i does not. And so on.

Now the amazing thing here is not so much that we have a convenient map, but rather the shape of the map itself. No one had imagined that this map of the Mandelbrot set would have such an amazing and complicated shape. And as we will see later, when we zoom in, some sections of the Mandelbrot set are immensely beautiful. The shape itself has wonderful mathematical properties. I suppose that is not too surprising given that it is a mathematical graph. But the particular geometric and mathematical properties it exhibits were a surprise to everyone. Who knew that such properties had been hidden in the little formula $z^2 + c$?

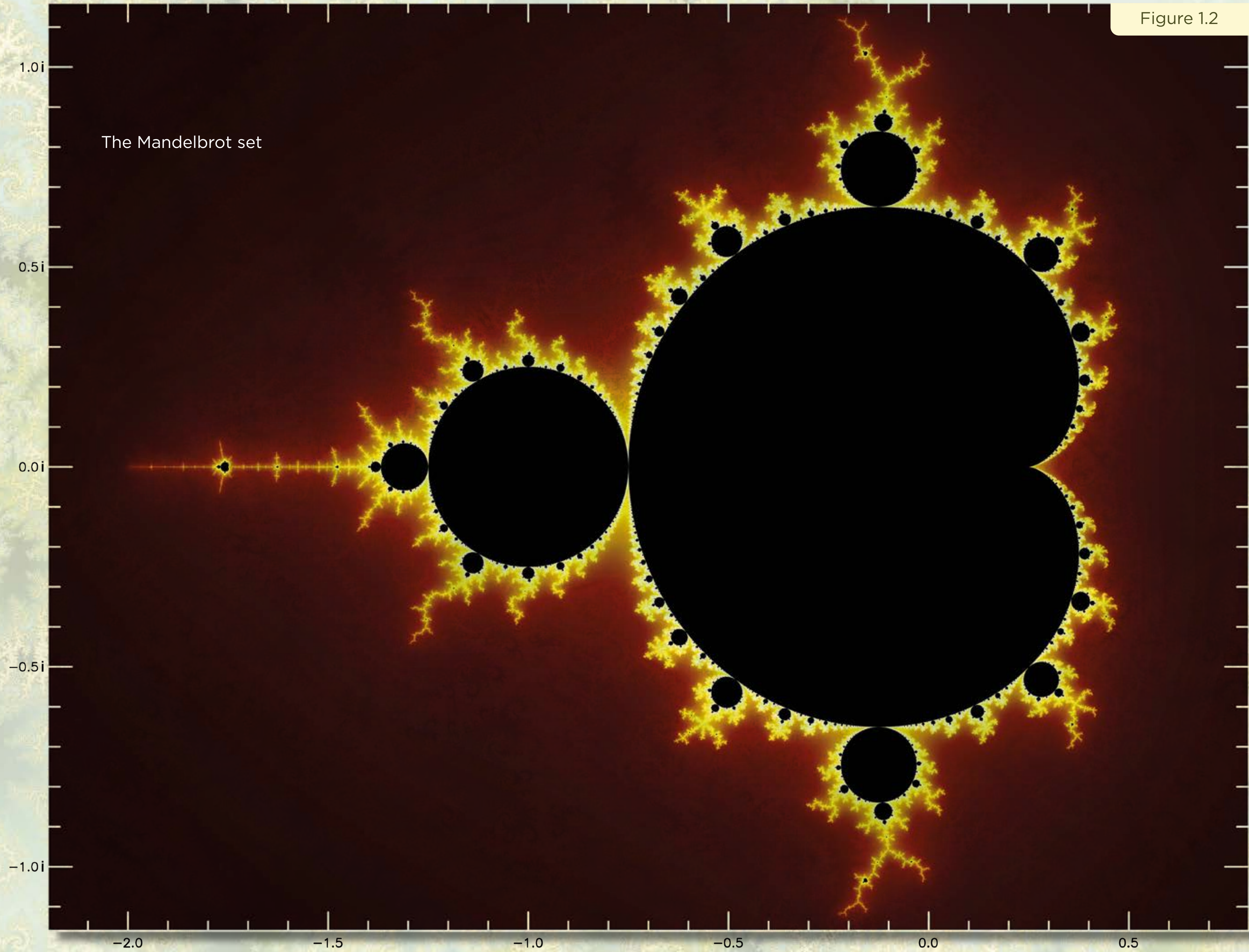
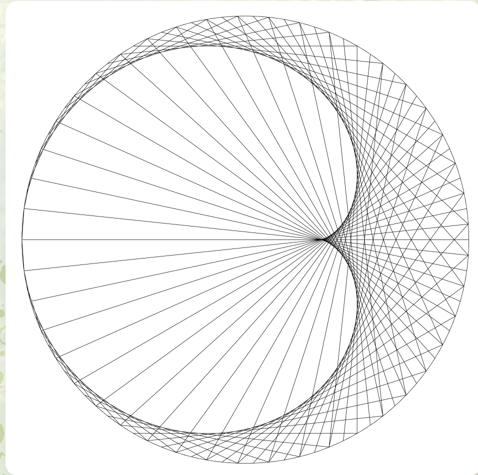


Figure 1.2

The Mandelbrot set

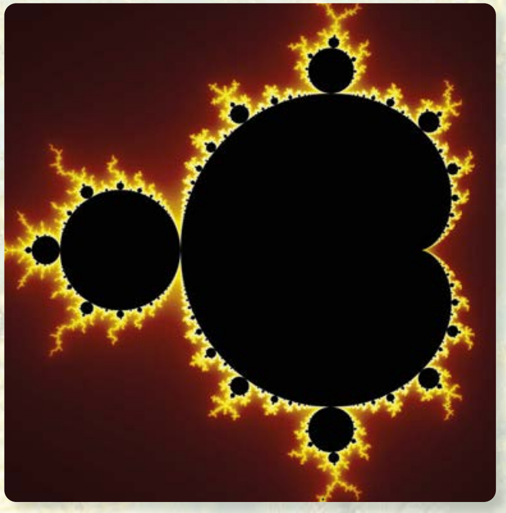
Geometry

At first glance, we notice that the Mandelbrot set has three types of geometric shapes. The largest and most prominent is the heart-shaped structure. This shape is called a *cardioid*. It is the shape generated when you roll one circle around another of equal size, keeping your pencil affixed to a point on the rolling circle. The cusp of the cardioid is located exactly at ¼, and its opposite side ends at exactly -3/4. The cardioid has an area of $3\pi/8$.



Cardioid - sinusoidal spiral - mathematical plane curve.

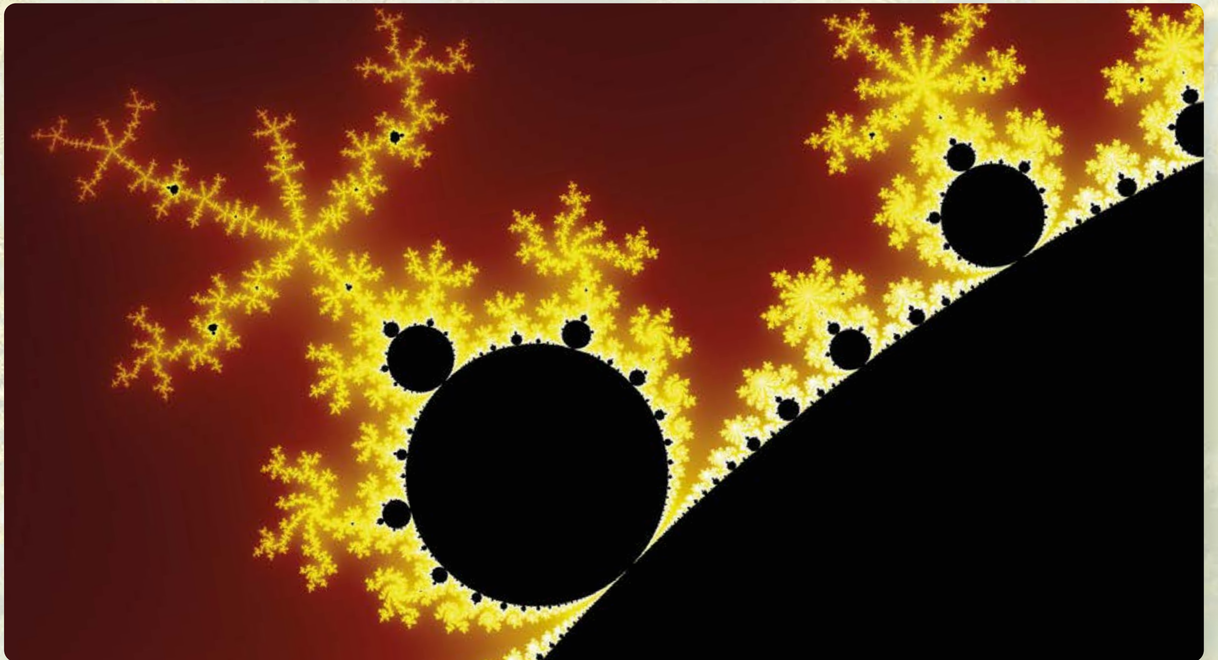
Next, we notice lots of perfect circles budding off the cardioid. The largest of these circles is affixed to the left side of the cardioid, is centered exactly on the number -1, and has a radius of exactly ¼. Another smaller circle grows off of its left side, with another growing off of it, and so on, as far as the eye can detect. The second largest circles in this map are affixed to the top and bottom of the cardioid.



Next, we notice lots of perfect circles budding off the cardioid.

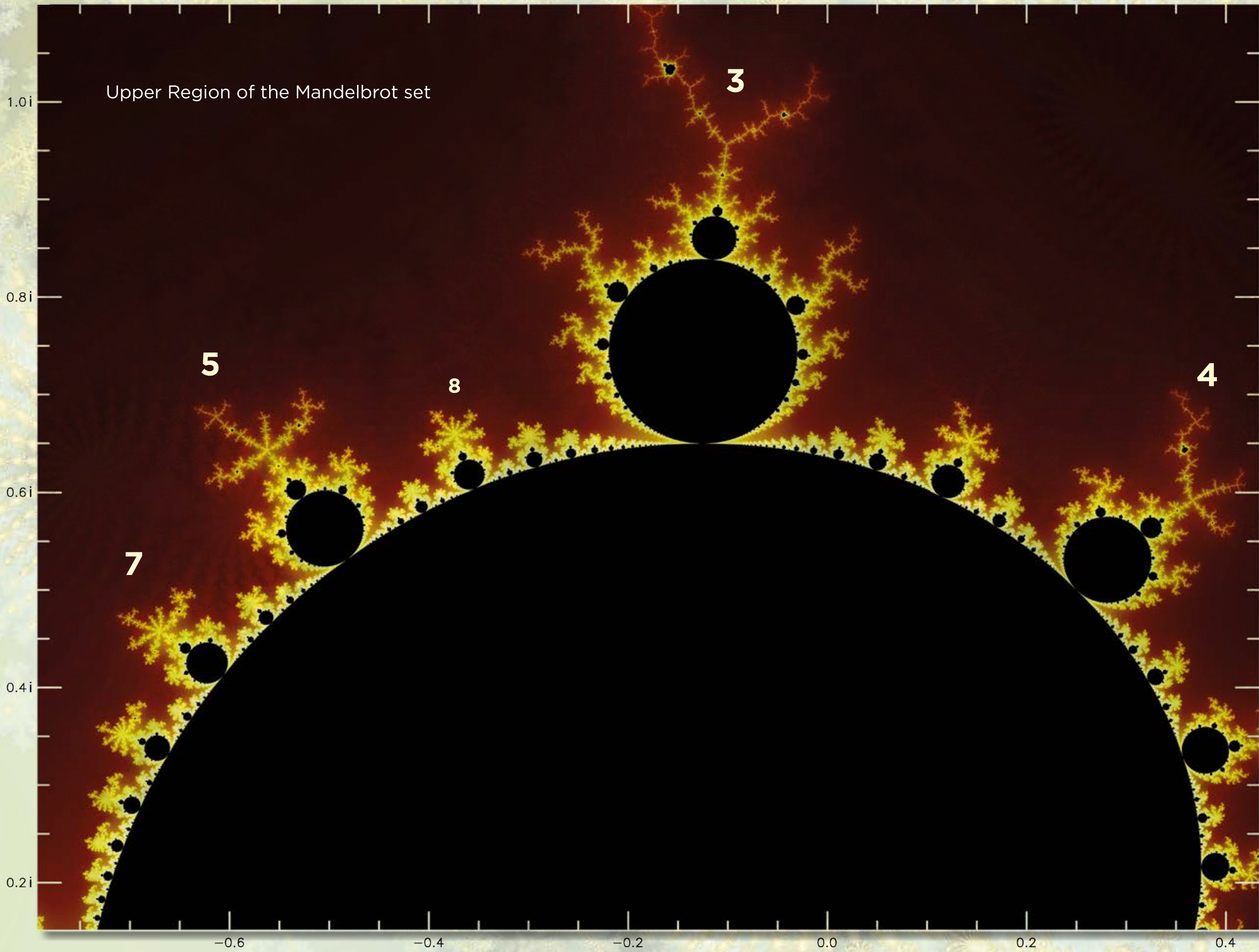
The third shape we notice are thousands of tiny "branches" or "dendrites" that are rooted in the circles that bud off of the cardioid. All of these dendrites are very "wiggly" with one exception: the antenna extending directly to the left on the real number line is perfectly straight and ends at exactly c = -2. It may seem at first that these branches are not part of the Mandelbrot set because most of them are not colored black. But since we see bright yellow in these branches, we must conclude that such points are extremely close to the Mandelbrot set. In other words, the actual (black) threads are too thin to be visible but are surrounded by yellow. They branch into dendrites, which then branch into more dendrites. We will see that this type of feature is common in the Mandelbrot set.

The Mandelbrot set has an infinite number of smaller versions of itself built into itself!



Thousands of tiny "branches" or "dendrites" that are rooted in the circles that bud off of the cardioid

Figure 1.3


Upper Region of the Mandelbrot set
3
5
8
4
7
1.0 i
0.8 i
0.6 i
0.4 i
0.2 i
−0.6
−0.4
−0.2
0.0
0.2
0.4

The Branches are Smart!

Let us explore the dendrites near the top of the Mandelbrot map. We can zoom in on this area by having the computer check these values at finer resolution than we did previously. The resulting map (figure 1.3) shows that the branches rooted in each circle exhibit some fascinating properties. The branch growing above the largest circle splits into two more, for a total of three that meet at an intersection. The next largest circle to the left has a branch that splits into a total of five. The next largest circle to the left branches into 7, the next 9, then 11, 13, and so on. These branches cover all the odd numbers in perfect sequence to infinity.

Just to the right of the largest circle, on the next largest circle, we count four intersecting branches. The next largest circle to the right has branches that split five ways, the next 6, 7, 8, etc. So the circles on the right side have branches that count all the numbers, both even and odd, from three to infinity! It seems that the branches of the Mandelbrot set know how to count. But wait — there's more!

Consider the largest circle in figure 1.3 (that has three branches) and the next largest to its left (which has five branches). Now examine the largest circle that is between them. It branches into 8 parts. Why is that significant? Eight is three plus five — the sum of the branches on the surrounding circles. In fact, this is the case for all the circles! Each circle that is the largest in between two larger ones has the sum of their branches. It seems that these branches not only know how to count to infinity, but they can also add!

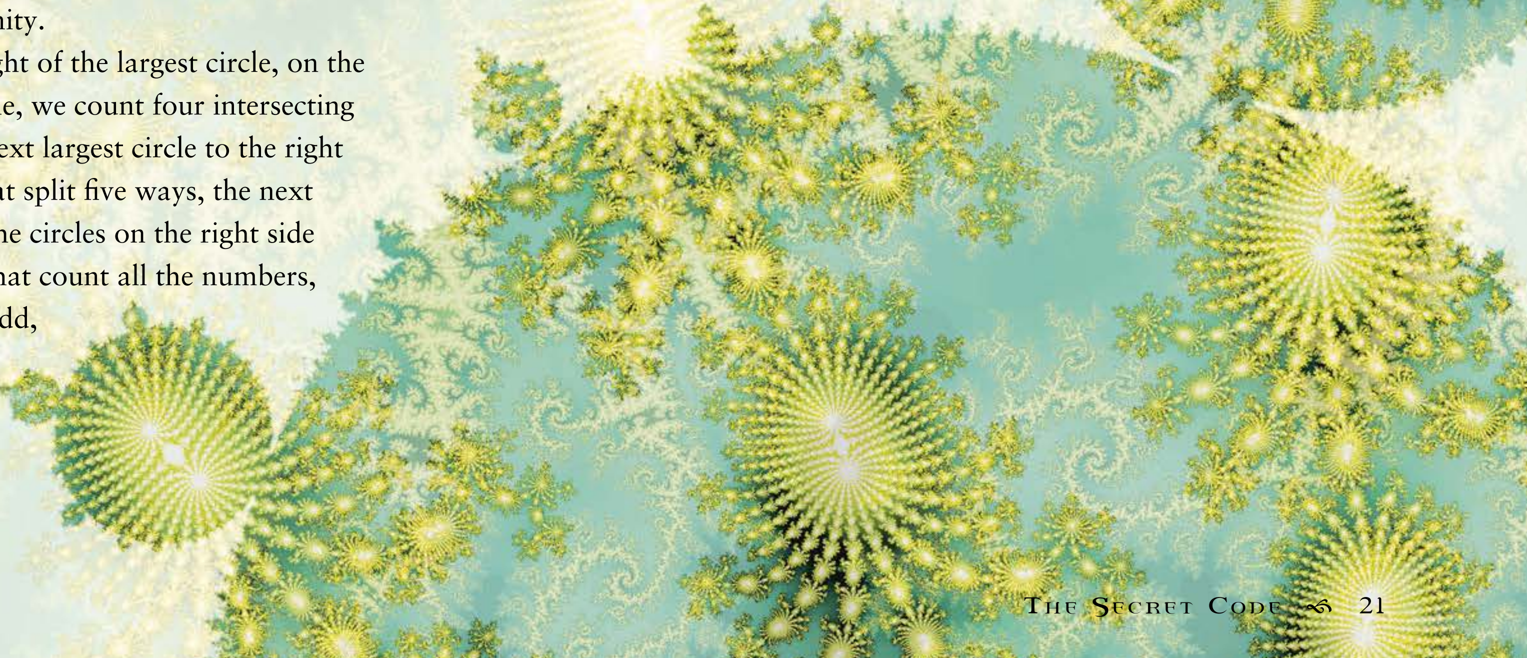
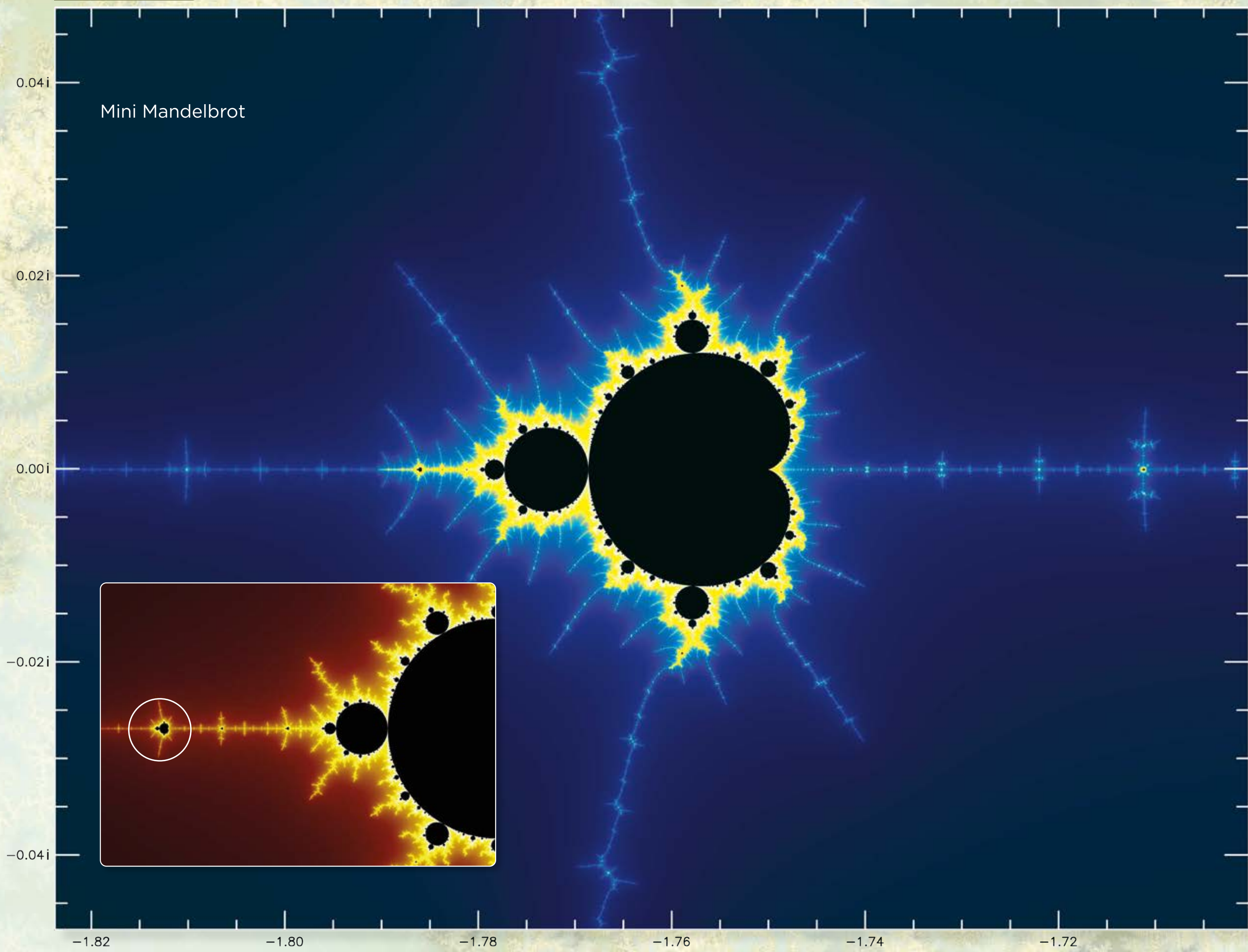


Figure 1.4


Mini Mandelbrot
0.04i
0.02i
0.00i
−0.02i
−0.04i
−1.82
−1.80
−1.78
−1.76
−1.74
−1.72

Mini and Mini-Minis

Returning to figure 1.2, let's now examine one of the most fascinating properties of the Mandelbrot set. Consider the straight, long spike on the left of the image — the only non-wiggly dendrite. About two-thirds of the way to the left is a "bump," with tiny branches extending above and below. When we zoom in on this shape, we find in figure 1.4 that it is a tiny version of the entire Mandelbrot set! This mini-Mandelbrot is nearly identical to the original. It has the large cardioid with circles budding off of it, the largest circle is on the left, and a spike is extending to the left.

But there are slight differences. When we compare figures 1.2 and 1.4, we see that this baby Mandelbrot has extra spikes extending away from it. We zoomed in on the spike of the (large) Mandelbrot and found that the mini version has extra spikes. Can this be a coincidence? Second, we note that this mini Mandelbrot is growing off of another and therefore has a spike entering the cusp of the cardioid. The large Mandelbrot set lacks this trait because it does not stem from a larger structure.

Of course, the mini Mandelbrot also has a spike on its left, just like the original large version. And this spike also has a small bump on it. Zooming in on this tiny bump, we find that it too is a tiny version of the entire Mandelbrot set (figure 1.5). It is a "mini-mini-Mandelbrot"! At this scale, we find it useful to employ more complex color schemes to reveal intricate details. In this case, as the iterations increase, the palette goes from red, to yellow, to light blue, to white.

Again, the main features are identical to the entire Mandelbrot set — the cardioid, the circles, and the branches. But since we zoomed in on the spike of the mini, which was on the spike of the original, this mini-mini-Mandelbrot has extra spikes extending away from it. Apparently, miniature versions of the Mandelbrot set inherit the geometric characteristics of the part of the parent from which they extend. This mini-mini-Mandelbrot also has a spike on its left, which has a bump. Of course, this turns out to be an even smaller version of the Mandelbrot map (figure 1.6). Figures 1.5 and 1.6 were plotted with the same color palette for easy comparison.

The two maps are nearly identical apart from size, and the mini-mini-mini has gained extra spikes that stem off the other spikes. This mini-mini-mini also has an even smaller version budding off of its tail (figure 1.7), which also has an even smaller version budding off of its tail (figure 1.8), and so on. This pattern apparently continues forever, with each smaller version gaining additional spikes and complexity. The Mandelbrot set has an infinite number of smaller versions of itself, built into itself! This type of structure is called a *fractal.*

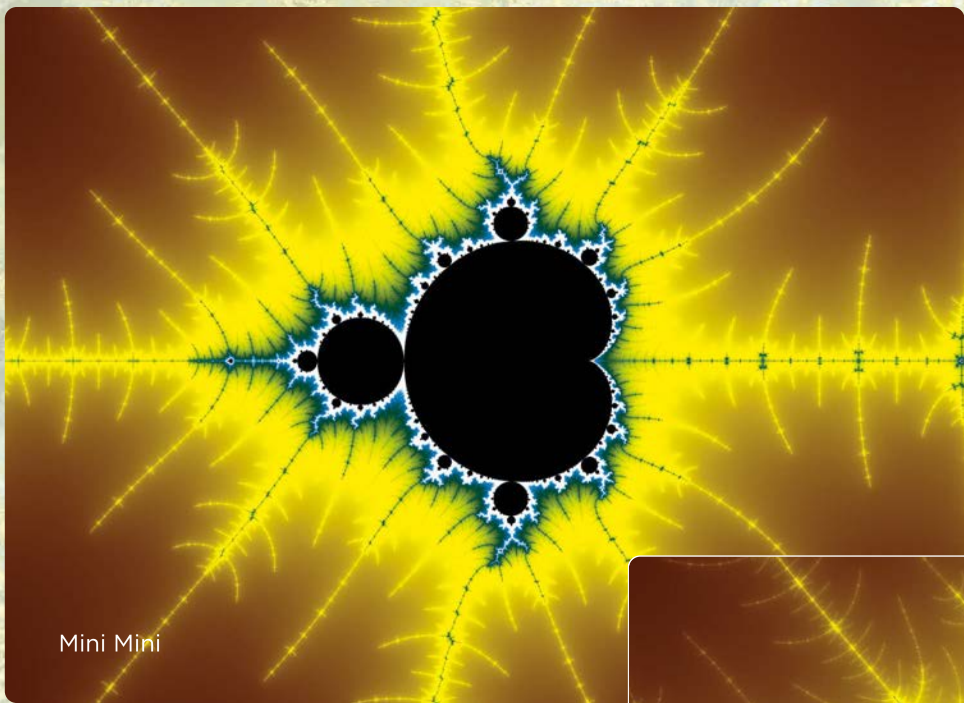
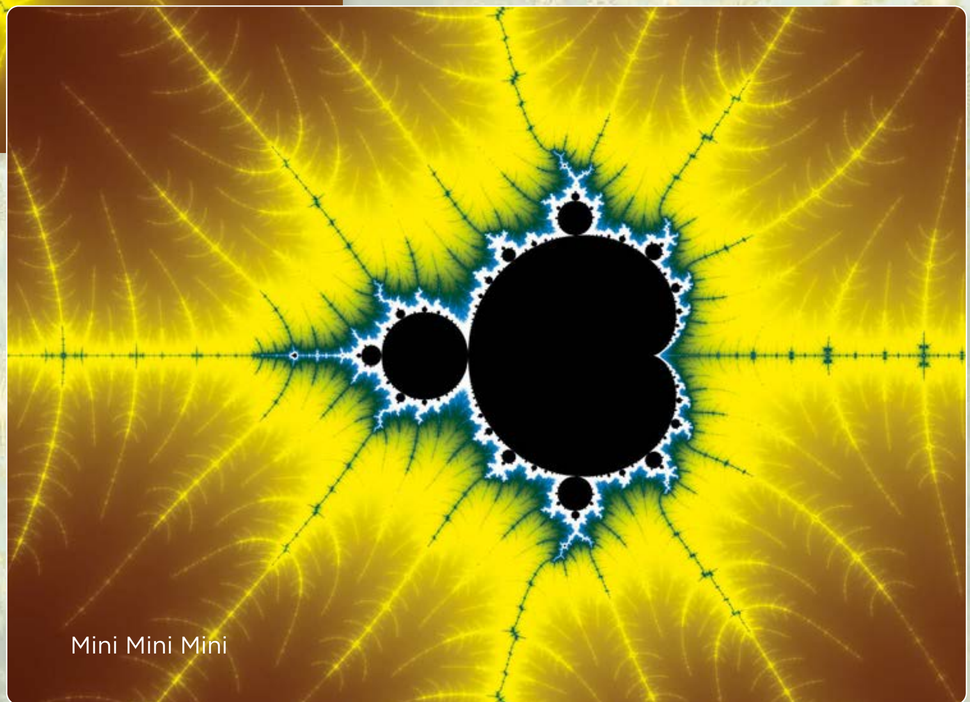


Figure 1.5

A fractal is any geometric shape that has parts that resemble the whole. If we adjusted the contrast on the "mini-Mandelbrots" in figures 1.2 through 1.8 so that the exterior spikes were not visible, they would be virtually indistinguishable from the entire Mandelbrot set. We would not know if we are viewing the

Figure 1.6



entire map or if we have zoomed in by a factor of a billion or a hundred quadrillion. We see the same basic type of shape no matter how much we zoom in. This property of fractals is called *scale-invariance*. In the next chapter we will see that the Mandelbrot set has many sections that exhibit scale-invariance.

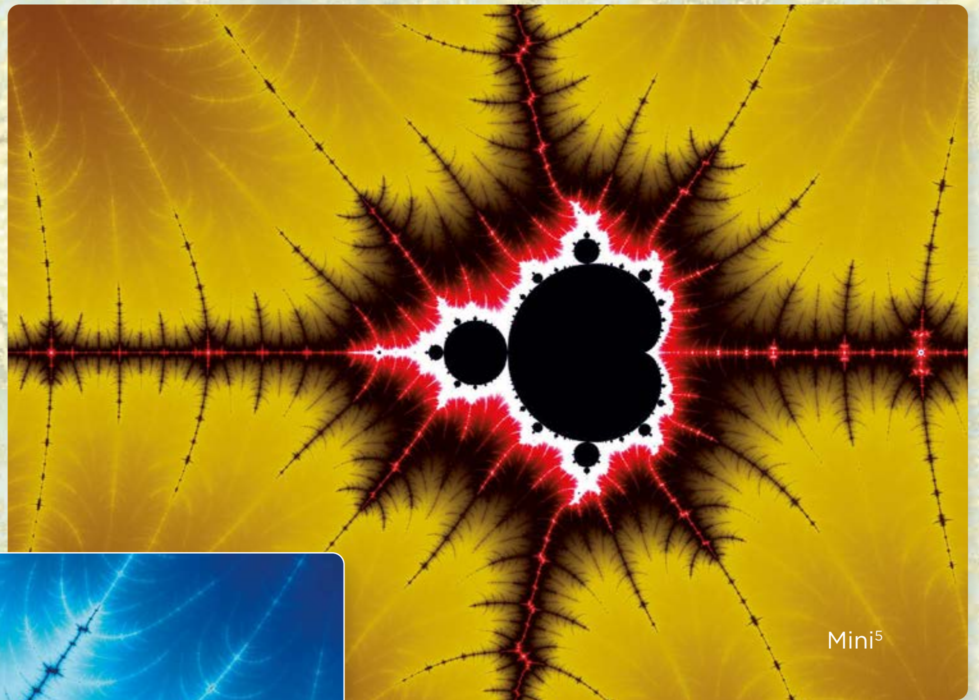


Figure 1.8

Figure 1.7