ELEMENTARY
ALGEBRA

HAROLD R. JACOBS

$$r^2 = x^2 + y^2 + z^2$$

in the beginning...

REVISED EDITION

ELEMENTARY
ALGEBRA

HAROLD R. JACOBS

Please consider requesting that a copy of this volume be purchased by your local library system.

Printed in the United States

Please visit our website for other great titles:
www.masterbooks.com

For information regarding author interviews,
please contact the publicity department at (870) 438-5288

Introductory Comments

The exercises for each lesson in Elementary Algebra are arranged in four sets. Those in Set I review ideas from preceding lessons, affording continual practice with material considered earlier in the course. Sets II and III enable the student to apply the concepts of the new lesson. Because both sets have the same content, you will probably prefer to work one set or the other. Answers to all of the Set II exercises are given at the back of the textbook. Those answers, together with those to Sets I, III, and IV, are also included in this book.

Solutions to many of the exercises appear in brackets following the answers. These solutions vary in detail according to the complexity of the exercise. For those exercises that can be solved in a variety of ways, just one example solution is given.

Most of the Set IV exercises are intended to challenge the better students, although some are simple enough that everyone can be successful with them. They should be considered optional and counted as extra credit.

Detailed lesson plans for the course are available in the *Teacher's Guide* to *Elementary Algebra*.

I am grateful to Dale Callahan and Cassidy Cash for their help in preparing this solutions manual.

Harold R. Jacobs

ANSWERS TO EXERCISES

Introduction (pp. 3–4)

1. a)

Think of a number:	1	2	3	4	5
Double it:	2	4	6	8	10
Add six:	8	10	12	14	16
Divide by two:	4	5	6	7	8
Subtract the number you first thought of:	3	3	3	3	3

   b) No.

   c)

   Think of a number:
   Double it: □
   Add six: □□
   Divide by two: □□ ○○○
   Subtract the number you first thought of: □ ○○○

   d) Yes.   ○○○

2. a) Think of a number.
   b) Multiply it by four.
   c) Add eight.
   d) Divide by four.
   e) Add three.
   f) Subtract the number that you first thought of.

3. a) 2.
   b) The result at the end is 4 instead of 2.
   c) The result at the end is 1.
   d) The result at the end now depends on the number first thought of.

4. (One of many possible answers.)
   Think of a number.
   Triple it.
   Add twelve.
   Divide by three.
   Subtract the number that you first thought of.
   The result is four.

Chapter 1, Lesson 1

Set I (page 8)

1. 1,776.
2. 1,107.
3. 1.984.
4. 20.202.
5. 1,370.
6. 1,370.
7. 4.664.
8. 10.631.
9. 2.8.
10. 1.605.

Set II (pages 8–9)

11. a) $10 + 7$ or 17.
    b) $x + 7$.
    c) $10 + y$.
    d) $x + y$.
    e) $4 + 8$ or 12.
    f) $4 + z$.
    g) $2 + 5 + 1$ or 8.
    h) $x + 5 + 1$ or $x + 6$.
    i) $2 + y + 1$ or $y + 3$.
    j) $x + y + 1$.

12. a) $9 + 4$.
    b) 13.
    c) $x + 5$.
    d) 7.
    e) 9.

13. a) $11 + 4 + 5$ or 20.
    b) $x + 6$.
    c) $x + y$.
    d) $5 + 3 + x$ or $8 + x$.
    e) $x + 1 + y + 1$ or $x + y + 2$.
    f) $x + y + z$.

14. a) □○○○○○○ and ○○○○○○□
    b) ○○□○○○○○ and □○○○○○○○
    c) □○○○○□ and ○○○○□□

15. a) $8 + y + 2$ or $y + 10$.
    b) $9 + y + 2$ or $y + 11$.
    c) $x + 3 + 2$ or $x + 5$.
    d) $x + 0 + 2$ or $x + 2$.
    e) $6 + 2 + 2$ or 10.

16. a) 44.
    b) $39 + x$.
    c) $39 + x + 6$ or $x + 45$.
    d) $x + 5$.
    e) $x + y$.
    f) $x + y + z$.

Set III

17. a) $3 + 11$ or 14.
    b) $3 + x$.

  c) $y + 11$.
  d) $y + x$.
  e) $7 + 2$ or 9.
  f) $7 + x$.
  g) $9 + 1 + 4$ or 14.
  h) $x + 1 + 4$ or $x + 5$.
  i) $9 + y + 4$ or $y + 13$.
  j) $x + y + 4$.

18. a) $8 + 3$.
    b) 11.
    c) $x + 6$.
    d) 7.
    e) 11.

19. a) 17.
    b) $x + 14$.
    c) $x + y + 16$.
    d) $x + x + x$.
    e) $x + y + x + y$.

20. a) $\circ\circ\square$ and $\square\circ\circ$
    b) $\circ\circ\circ\circ\circ\circ\circ\square\circ$ and $\square\circ\circ\circ\circ\circ\circ\circ\circ$
    c) $\square\square\circ\circ\circ$ and $\square\circ\circ\circ\square$

21. a) $2 + 1 + y$ or $3 + y$.
    b) $0 + 1 + y$ or $1 + y$.
    c) $x + 1 + 6$ or $x + 7$.
    d) $x + 1 + 9$ or $x + 10$.
    e) $3 + 1 + 7$ or 11.

22. a) 19 miles.
    b) $x + 1$ miles.
    c) $y + 3$ miles.
    d) $y + z$ miles.

Set IV (page 10)

It would happen with any four numbers because:

a	b	$a + b$
x	y	$x + y$
$a + x$	$b + y$	$a + b + x + y$ $= a + x + b + y$

Chapter 1, Lesson 2

Set I (page 12)

1. 20,222.
2. 589.
3. 877.

4. 3.321.
5. 4.221.
6. 0.
7. 0.1.
8. 0.01.
9. 1,793.88.
10. 179.388.

Set II (pages 13–14)

11. a) $10 - 7$ or 3.
    b) $6 - x$.
    c) $x - 6$.
    d) $11 - 3$ or 8.
    e) $x - 1$.
    f) $x - y$.
    g) $4 - x$.
    h) $x - 4$.

12. a) $12 - 7$ or 5.
    b) $14 - x$.
    c) $x - 3$.
    d) $y - x$.
    e) $9 - 2 - 3$ or 4.
    f) $x - y - 1$.

13. a) 2.
    b) 3.
    c) 10.
    d) The value of $x - 4$ gets larger.
    e) 12.
    f) 11.
    g) 5.
    h) The value of $15 - x$ gets smaller.

14. a) 8.
    b) 10.
    c) 8.
    d) 10.
    e) Each expression is $x + y - 3$.

15. a) 4.
    b) $7 - x$.
    c) 6.
    d) $14 - y$.

16. a) $7,000 - x$ pounds.
    b) $7,000 + y$ pounds.

17. a) 24 cents.
    b) $y - x$ cents.
    c) $x + 30$ cents.

d) $95 - y$ cents.

Set III

18. a) $9 - 3$ or 6.
    b) $x - 5$.
    c) $5 - x$.
    d) $20 - 8$ or 12.
    e) $x - 2$.
    f) $y - x$.
    g) $7 - x$.
    h) $x - 7$.

19. a) 5.
    b) $y - 7$.
    c) 3.
    d) $10 - y$.
    e) $y - 15$.
    f) $z - y - 21$.

20. a) 7.
    b) 8.
    c) 18.
    d) The value of $x - 2$ gets larger.
    e) 7.
    f) 6.
    g) 0.
    h) The value of $8 - x$ gets smaller.

21. a) 11.
    b) 14.
    c) 11.
    d) 14.
    e) Each expression is $x + 7 - y$.

22. a) $12 - x$ feet.
    b) $x + y$ feet.
    c) $x - y$ feet.

23. a) $2 - x$ dollars.
    b) $2 + y$ dollars.
    c) $z - 2$ dollars.

24. a) $100 - x$.
    b) $100 + y$.
    c) $100 + y - z$.

Set IV (page 14)
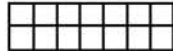
The clerk is giving a customer the change for an \$8.47 purchase: \$20 − \$8.47 = \$11.53. The problem is being solved by addition.

Chapter 1, Lesson 3
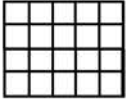
1. 36,000.
2. 714,285.
3. 77,777.
4. 1.
5. 12.345.
6. 12.345.
7. 1.001.
8. 10.01.
9. 100.1.
10. 10,000.

Set II (pages 17–18)

11. a) for $4 \cdot 3$ and for $3 \cdot 4$.
    b)
    c)

12. a) $5 \cdot 6$ or 30.
    b) $5 + 6$ or 11.
    c) $5x$.
    d) $5 + x$.
    e) xy.
    f) $x + y$.
    g) xx.
    h) $8x$.
    i) $x - 8$.
    j) $2 + 7 + x$ or $9 + x$.
    k) $2 \cdot 7 \cdot x$ or $14x$.
    l) $10 + y + 3$ or $y + 13$.
    m) $10 \cdot y \cdot 3$ or $30y$.
    n) $4 + x + y$.
    o) $4xy$.

13. a) $6 \cdot 2$.
    b) $2 \cdot 6$.
    c) $5x$.
    d) $11 \cdot 7$.
    e) $x \cdot 7$ or $7x$.
    f) xy.
    g) $17 + 17 + 17$.
    h) $x + x + x + x$.
    i) $2 + 2 + \cdots + 2$ (y of them).
    j) $z + z + \cdots + z$ (y of them).

14. a) $7 \cdot 8$ or 56.
    b) $10x$.
    c) xy.

d) *xx*.

15. a) 140.
    b) 354*x*.

16. a) 7*x*.
    b) 24*x*.
    c) 1,440.
    d) 1,440*x*.
    e) 10,080*x*.
    f) 100*x*.
    g) 1,200*x*.

17. a) 165.
    b) 11*x*.

Set III

18. a)

    b)

    c)

19. a) 21.
    b) 10.
    c) 7*x*.
    d) 7 + *x*.
    e) *xy*.
    f) *x* + *y*.
    g) *yy*.
    h) 5*x*.
    i) *x* − 5.
    j) 10 + *x*.
    k) 24*x*.
    l) *y* + 17.
    m) 60*y*.
    n) *x* + *y* + 2.
    o) 2*xy*.

20. a) 3 · 10.
    b) 10 · 3.
    c) 3*x*.
    d) 15 · 4.
    e) 4*x*.
    f) *xy*.
    g) 19 + 19.
    h) *x* + *x* + *x* + *x* + *x*.
    i) 3 + 3 + ⋯ + 3 (*y* of them).
    j) *x* + *x* + ⋯ + *x* (*x* of them).

21. a) 72.

b) 16*x*.
c) 7*xy*.
d) *xxx*.

22. a) 48.
    b) 4*x*.
    c) 48*y*.

23. a) 20*x*.
    b) *xx*.

24. a) 1,000.
    b) 1,000*x*.
    c) 3,000.
    d) 1,000*y*.
    e) 100,000.
    f) 100,000*y*.
    g) 1,000,000*y*.

Set IV (page 19)

1.
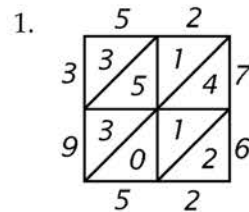
   Yes: 3952 is correct.

2.
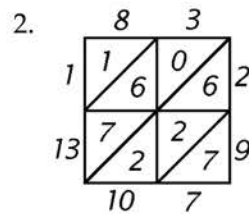
   Yes. If we "carry" from each slanting column to the next going clockwise, we get 2407.

Chapter 1, Lesson 4

Set I (page 21)

1. 50.
2. 0.02.
3. 1234.
4. 3003.
5. 1.6.
6. 16.
7. 0.625.
8. 0.0625.

Set II (pages 21–22)

9. a) $\frac{12}{3}$ or 4.

   b) 12 − 3 or 9.

c) $\dfrac{7}{x}$.

d) $\dfrac{x}{7}$.

e) $\dfrac{x}{2}$.

f) $x \cdot 2$ or $2x$.

g) $\dfrac{10}{x}$.

h) $10 - x$.

i) $\dfrac{x}{y}$.

j) xy.

10. a) $\dfrac{12}{3} = 4$ and $\dfrac{12}{4} = 3$.

    b) $\dfrac{16}{4} = 4$.

    c) $\dfrac{18}{3} = 6$ and $\dfrac{18}{6} = 3$.

11. a) $5 \cdot 3 = 15$.
    b) $23 \cdot 4 = 92$.
    c) $12 \cdot 0 = 0$.
    d) $(7.5)(1) = 7.5$.
    e) $10 \cdot 7 = x$.
    f) $x \cdot 12 = 36$.
    g) $4x = 20$.
    h) $y \cdot 2 = x$.

12. a) 45. $[9 \cdot 5.]$
    b) 63. $[9 \cdot 7.]$
    c) 108. $[9 \cdot 12.]$
    d) The value of $9x$ gets larger.

    e) 1. $[\dfrac{4}{4}.]$

    f) 5. $[\dfrac{20}{4}.]$

    g) 25. $[\dfrac{100}{4}.]$

    h) The value of $\dfrac{x}{4}$ gets larger.

    i) 15. $[\dfrac{30}{2}.]$

j) 6. $[\dfrac{30}{5}.]$

k) 0.5. $[\dfrac{30}{60}.]$

l) The value of $\dfrac{30}{x}$ gets smaller.

13. a) 20. $[\dfrac{300}{15}.]$

    b) $\dfrac{300}{x}$.

14. a) $170x$ dollars.

    b) 600. $[\dfrac{102{,}000}{170}.]$

    c) $\dfrac{x}{170}$.

15. a) $12x$ inches or x feet.

    b) 50. $[\dfrac{600}{12}.]$

    c) $\dfrac{x}{12}$.

16. a) 10.6. $[\dfrac{159}{15}.]$

    b) $\dfrac{159}{x}$.

Set III

17. a) $\dfrac{8}{2}$ or 4.

    b) $8 - 2$ or 6.

    c) $\dfrac{x}{5}$.

    d) $\dfrac{5}{x}$.

    e) $\dfrac{3}{x}$.

    f) $3x$.

    g) $\dfrac{x}{12}$.

    h) $x - 12$.

i) $\dfrac{y}{x}$.

j) yx.

18. a) 21.
b) 42.
c) 77.
d) The value of $7x$ gets larger.
e) 0.
f) 4.
g) 17.
h) The value of $\dfrac{x}{3}$ gets larger.
i) 2.
j) 1.8.
k) 0.4.
l) The value of $\dfrac{18}{x}$ gets smaller.

19. a) 7. $[\dfrac{63}{9}.]$

b) 8. $[\dfrac{8x}{x}.]$

c) $\dfrac{15}{x}$.

d) $\dfrac{x}{4}$.

e) $\dfrac{x}{y}$.

20. a)
```
  966
 −840      40 twenty-ones
 ─────
  126
 −126       6 more twenty-ones
 ─────
    0      46 twenty-ones subtracted
```
b)
```
    125
 7)875     875
 − 700   − 700     100 sevens
 ─────   ─────
   175     175
 − 140   − 140      20 more sevens
 ─────   ─────
    35      35
 −  35   −  35       5 more sevens
 ─────   ─────
     0       0     125 sevens subtracted
```

21. a) 20 minutes. $[\dfrac{2,000}{100}.]$

b) $\dfrac{2000}{x}$ minutes.

22. a) xy.

b) $\dfrac{80}{x}$.

c) $\dfrac{x}{8}$.

23. a) 150 square inches. $[6 \cdot 25.]$
b) $6x$ square inches.
c) 16 square inches. $[\dfrac{96}{6}.]$
d) $\dfrac{y}{6}$ square inches.

24. a) 1.08 dollars. $[\dfrac{10.80}{10}.]$

b) 0.09 dollars. $[\dfrac{10.80}{120}.]$

c) $\dfrac{x}{10}$ dollars.

d) $\dfrac{x}{120}$ dollars.

Set IV (page 23)

There were twelve loaves of bread altogether. In sharing them equally, the three travelers each got four loaves. This means that the third traveler got three loaves from the pilgrim who had seven loaves and one loaf from the pilgrim who had five loaves.

Because $\dfrac{3}{3+1} = \dfrac{3}{4}$ of the traveler's bread came from the first pilgrim, he should receive $\dfrac{3}{4}$ of the twelve coins:

$\dfrac{3}{4}$ of 12 coins = 9 coins.

Chapter 1, Lesson 5

Set I (page 25)

1. 25.
2. 32.
3. 1,000.
4. 10,000,000.
5. 1.
6. 1.
7. 1.69.
8. 9.61.

9. 0.064.
10. 0.004096.

Set II (pages 26–27)

11. a) "x squared" and "x to the second power."
    b) An exponent.

12. a) 3^2.
    b) 5^2.
    c) x^2.
    d) 4^3.
    e) x^3.

13. a) 7^2.
    b) 2^6.
    c) x^3.
    d) x^8.
    e) 3^x.
    f) x^y.

14. a) 7^4.
    b) 4^7.
    c) x^6.
    d) 2^{12}.
    e) 2^x.
    f) x^y.
    g) $8 \cdot 8 \cdot 8 \cdot 8 \cdot 8$.
    h) $x \cdot x \cdot x$.
    i) $3 \cdot 3 \cdot \cdots \cdot 3$ (x of them).
    j) $y \cdot y \cdot \cdots \cdot y$ (x of them).

15. a) 2,401.
    b) 49^2.

16. a) 3^6.
    b) 2^6.
    c) 4^3.
    d) 8^2.
    e) 10^4.
    f) 10^9.
    g) Because all powers of 1 are equal to 1.

17. a) 512. [$2 \cdot 256$.]
    b) 14,641. [$11 \cdot 1,331$.]
    c) 2,187. [$3 \cdot 3 \cdot 243$.]
    d) 390,625. [$5 \cdot 5 \cdot 5 \cdot 3,125$.]
    e) x.
    f) x^2.

Set III

18. a) "x cubed" and "x to the third power."
    b) An exponent.

19. a) 6^2.
    b) 4^2.
    c) y^2.
    d) 5^3.
    e) y^3.

20. a) 2^3.
    b) 5^{10}.
    c) x^2.
    d) x^4.
    e) 9^y.
    f) y^x.

21. a) 6^5.
    b) 11^3.
    c) y^7.
    d) 3^{10}.
    e) 3^x.
    f) x^y.
    g) $1 \cdot 1 \cdot 1 \cdot 1$.
    h) $x \cdot x \cdot x \cdot x \cdot x$.
    i) $5 \cdot 5 \cdot \cdots \cdot 5$ (x of them).
    j) $x \cdot x \cdot \cdots \cdot x$ (y of them).

22. a) 7^3.
    b) 81^2.
    c) 9^4.
    d) 3^8.
    e) 10^3.
    f) 10^7.
    g) Because all powers of 1 are equal to 1.

23. a) The last digit of 6^{100} is 6.
    b) $5^2 = \quad 25$
       $5^3 = \quad 125$
       $5^4 = \quad 625$
       $5^5 = \quad 3,125$
       $5^6 = \quad 15,625$
    c) The last two digits of 5^{100} are 25.
    d) The last digit of 9^{100} is 1.

Set IV (page 27)

This table shows Ollie's study times:

Week	Minutes
1	1
2	2
3	$2 \cdot 2 = 2^2$
4	$2 \cdot 2 \cdot 2 = 2^3$
5	$2 \cdot 2 \cdot 2 \cdot 2 = 2^4$
6	$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5$
⋮	⋮
20	2^{19}

According to Ollie's plan, he would study 2^{19} minutes in the last week, but $2^{19} = 524{,}288$ minutes. Altogether there are $7 \cdot 24 \cdot 60 = 10{,}080$ minutes in a week, so Ollie wouldn't be able to do it!

Chapter 1, Lesson 6

Set I (page 30)

1. a) □
   b) □ 88
   c) □ □ □ 888888
   d) □ □ □ 8₀
   e) □ ₀
   f) ₀

2. a) $3x$.
   b) $3 + x$.
   c) $3 - x$.
   d) $\dfrac{3}{x}$.
   e) x^3.
   f) 3^x.

3. a) $2a$.
   b) $b + b + b + b + b$.
   c) c^3.
   d) $d \cdot d \cdot d \cdot d$.
   e) xe.
   f) f^y.

Set II (pages 30–31)

4. a) The sum of a number and zero is the number.
   b) The difference between a number and zero is the number.
   c) The product of a number and zero is zero.
   d) The product of a number and one is the number.
   e) The quotient of zero and a nonzero number is zero.
   f) The quotient of a number and zero is not defined.
   g) The quotient of a number and one is the number.

5. a) 45.
   b) 0.
   c) 11.
   d) 1.

6. a) $1^2 = 1$ because 1 times 1 is 1.
   b) 1.
   c) 1.

7. a) 0.
   b) 10 has a "higher value" than 1.

8. a) x.
   b) 0.
   c) x.
   d) $x + 1$ cannot be simplified.
   e) x.
   f) 0.
   g) $\dfrac{x}{0}$ is not defined.
   h) x.

9. a) $x + y$.
   b) x.
   c) y.
   d) 0.
   e) $x + y$.
   f) 0.
   g) x.
   h) y.

10. a) 1.
    b) 0.
    c) Even.

Set III

11. a) 1.
    b) 0.
    c) 1.
    d) 1.
    e) $\dfrac{0}{0}$ is meaningless.

f) 0.

g) x.

h) 0.

12. a) The product $2 \cdot 5$ would have two rows.

b) The product $1 \cdot 5$ would have one row.

c) No; $0 \cdot 5$ would have no rows.

13. a) 0 because zero times zero is zero.

b) 0.

c) 0.

14. a) x.

b) $y - x$.

c) y.

d) 0.

e) $y + x$.

f) 0.

g) y.

h) x.

15. a) 45

$\dfrac{-5}{40}$

b) The answer is 4 tens and 0 ones.

16. a) $x + 1$.

b) $x - 1$.

c) 1.

17. a) If $\dfrac{7}{0}$ were equal to 0, then $0 \cdot 0$ would

be equal to 7.

b) No. If $\dfrac{7}{0}$ were equal to 7, then $0 \cdot 7$

would be equal to 7.

c) $\dfrac{0}{7} = 0$ because $7 \cdot 0 = 0$.

d) No; because we could also say $\dfrac{0}{0} = a$ for

any number a.

18. a) No, because no matter how many times zero is subtracted from 12, the result is always 12.

b) The calculator endlessly subtracts zero from zero without getting any answer. [This is the way that electrically driven calculators work. An electronic calculator will give an error message.]

If x is not equal to 0, then $x^0 = 1$. If x is equal to 0, it would seem that $0^0 = 0$.

Chapter 1, Lesson 7

Set I (page 35)

1. a) 5^3.

b) Not possible.

c) 2^6.

2. a) $x + 1$.

b) $17 - x$.

c) $x + 26$.

3. a) $7x$.

b) $\dfrac{1000}{x}$ days.

c) $15{,}000 + 10x$.

Set II (pages 35–36)

4. a) Figure 4.

b) Figure 5.

c) Figure 2.

d) Figure 6.

e) Figure 3.

f) Figure 4.

g) Figure 1.

5. a) 50. $[10 + 40 = 50.]$

b) 32. $[2 + 20 + 10 = 32.]$

c) 48. $[3 \cdot 16 = 48.]$

d) 19. $[3 + 16 = 19.]$

e) 9. $[25 - 16 = 9.]$

f) 400. $[25 \cdot 16 = 400.]$

g) 57. $[42 - 12 + 27 = 57.]$

h) 57. $[42 + 27 - 12 = 57.]$

i) 27. $[42 - 27 + 12 = 27.]$

j) 27. $[42 + 12 - 27 = 27.]$

k) 18. $[\dfrac{16}{8} + \dfrac{64}{4} = 2 + 16 = 18.]$

l) 12. $[\dfrac{16}{4} + \dfrac{64}{8} = 4 + 8 = 12.]$

m) 18. $[\dfrac{64}{4} + \dfrac{16}{8} = 16 + 2 = 18.]$

n) 19. $[11 - 6 + 14 = 19.]$

o) 52. $[11 - 8 + 49 = 52.]$

p) 39. $[11 \cdot 8 - 49 = 88 - 49 = 39.]$

q) 531. $[11 \cdot 49 - 8 = 539 - 8 = 531.]$

6. a) $x^2 + y^2$.
   b) $10 - 5x$.
   c) $\dfrac{x}{5} - 10$.
   d) $8x^3$.
   e) $y^4 - y$.
   f) $\dfrac{12}{x} + 2$.
   g) $x + xy$.

7. a) 2. $[1^2 + 3 \cdot 1 - 2 = 1 + 3 - 2 = 2.]$
   b) 26. $[4^2 + 3 \cdot 4 - 2 = 16 + 12 - 2 = 26.]$
   c) 128. $[10^2 + 3 \cdot 10 - 2 = 100 + 30 - 2 = 128.]$
   d) 458. $[20^2 + 3 \cdot 20 - 2 = 400 + 60 - 2 = 458.]$

8. a) 19. $[2 \cdot 6 + 7 = 12 + 7 = 19.]$
   b) 9. $[15 - 3 \cdot 2 = 15 - 6 = 9.]$
   c) 101.
      $[1 + 4 \cdot 5^2 = 1 + 4 \cdot 25 = 1 + 100 = 101.]$
   d) 900. $[10^3 - 10^2 = 1000 - 100 = 900.]$
   e) 84. $[3^4 + 3 = 81 + 3 = 84.]$
   f) 82. $[5 \cdot 4^2 - 4 + 6 = 5 \cdot 16 - 4 + 6 = 80 - 4 + 6 = 82.]$

9. a) 845 cents or $8.45.
      $[7 \cdot 80 + 3 \cdot 95 = 560 + 285 = 845.]$
   b) $80x + 95y$ cents.

Set III

10. a) Figure 1.
    b) Figure 6.
    c) Figure 4.
    d) Figure 3.
    e) Figure 5.
    f) Figure 6.
    g) Figure 2.

11. a) 17. $[14 + 3 = 17.]$
    b) 2. $[20 - 18 = 2.]$
    c) 117. $[120 - 3 = 117.]$
    d) 17. $[45 - 28 = 17.]$
    e) 34. $[5 + 36 - 7 = 34.]$
    f) 66. $[2 + 64 = 66.]$
    g) 128. $[2 \cdot 64 = 128.]$
    h) 48. $[16 \cdot 3 = 48.]$
    i) 25. $[26 + 24 - 25 = 25.]$
    j) 25. $[26 - 25 + 24 = 25.]$
    k) 27. $[26 - 24 + 25 = 27.]$
    l) 27. $[26 + 25 - 24 = 27.]$

    m) 20. $[\dfrac{64}{2} - \dfrac{36}{3} = 32 - 12 = 20.]$
    n) 0.5. $[\dfrac{8}{4} - \dfrac{9}{6} = 2 - 1.5 = 0.5.]$
    o) 6. $[36 - 30 = 6.]$
    p) 147. $3 \cdot 81 - 3 \cdot 32 = 243 - 96 = 147.]$
    q) 147. $81 \cdot 3 - 32 \cdot 3 = 243 - 96 = 147.]$

12. a) $7x + 1$.
    b) $x^3 - y^3$.
    c) $3x - 3y$.
    d) $12 + \dfrac{x}{6}$.
    e) $5x^2$.
    f) $x + y^5$.
    g) $\dfrac{1}{xy}$.

13. a) 4. $[0^3 - 2 \cdot 0 + 4 = 0 - 0 + 4 = 4.]$
    b) 8. $[2^3 - 2 \cdot 2 + 4 = 8 - 4 + 4 = 8.]$
    c) 119. $[5^3 - 2 \cdot 5 + 4 = 125 - 10 + 4 = 119.]$
    d) 333. $[7^3 - 2 \cdot 7 + 4 = 343 - 14 + 4 = 333.]$

14. a) 44. $[5 \cdot 8 + 4 = 40 + 4 = 44.]$
    b) 11. $[(17 - 2 \cdot 3 = 17 - 6 = 11.]$
    c) 49. $[1 + 3 \cdot 4^2 = 1 + 3 \cdot 16 = 1 + 48 = 49.]$
    d) 1,100. $[10^2 + 10^3 = 100 + 1,000 = 1,100.]$
    e) 620. $[5^4 - 5 = 625 - 5 = 620.]$
    f) 5. $[6 \cdot 1^2 + 1 - 2 = 6 + 1 - 2 = 5.]$

15. a) 51 square centimeters.
       $[10^2 - 7^2 = 100 - 49 = 51.]$
    b) $y^2 - x^2$ square centimeters.

Set IV (page 37)

1. 72. $[60 - 2 + 14 = 72.]$
2. 40. $[12 \cdot 5 = 60; 60 - 8 = 52; 52 \div 4 = 13; 13 + 7 = 20; 20 \cdot 2 = 40.]$
3. The calculator would do the operations in order from left to right.
4. Write down some of the intermediate steps.

Chapter 1, Lesson 8

Set I (page 40)

1. a) 0.
   b) 100.
   c) 0.
   d) Not possible.
   e) 0.01.

f) 100.

2. a) 7. $[\frac{35}{5}.]$

b) 6. $[\frac{6x}{x}.]$

c) x. $[\frac{x^2}{x}.]$

d) $\frac{20}{x}$.

e) x. $[\frac{x}{1}.]$

f) $\frac{y}{x}$

3. a) 5. $[\frac{50}{10}.]$

b) $\frac{x}{10}$.

Set II (pages 41–42)

4. a) Yes. $[16 + 2 = 18;\ 11 + 7 = 18.]$
   b) No. $[6 - 2 = 4;\ 11 - 3 = 8.]$
   c) Yes. $[16 - 2 = 14;\ 11 + 3 = 14.]$
   d) No. $[6 + 2 = 8;\ 11 - 7 = 4.]$
   e) Yes. $[72 \cdot 3 = 216;\ 12 \cdot 18 = 216.]$
   f) Yes. $[12 + 18 = 30;\ 12 + 18 = 30.]$
   g) No. $[12 + 18 = 30;\ 18 \cdot 3 = 54.]$
   h) Yes. $[\frac{18}{3} = 6;\ \frac{18}{3} = 6.]$

5. a) 63. $[7 \cdot 9 = 63.]$
   b) 441. $[21^2 = 441.]$
   c) 15. $[4 + 6 + 5 = 15.]$
   d) 23. $[6 \cdot 3 + 5 = 18 + 5 = 23.]$
   e) 20. $[4 + 2 \cdot 8 = 4 + 16 = 20.]$
   f) 48. $[6 \cdot 8 = 48.]$
   g) 1. $[15 - 12 - 2 = 1.]$
   h) 24. $[12 \cdot 2 = 24.]$
   i) 5. $[15 - (12 - 2) = 15 - 10 = 5.]$
   j) 9. $[15 - 3 \cdot 2 = 15 - 6 = 9.]$
   k) 6. $[3 + 3 = 6.]$
   l) 3. $[\frac{36}{12} = 3.]$
   m) 9. $[3 \cdot 3 = 9.]$
   n) 9. $[\frac{180}{20} = 9.]$
   o) 5. $[25 - 5 \cdot 4 = 25 - 20 = 5.]$
   p) 80. $[(25 - 5) \cdot 4 = 20 \cdot 4 = 80.]$
   q) 225. $[(25 - 10)^2 = 15^2 = 225.]$

6. a) Figure 2.
   b) Figure 3.
   c) Figure 1.
   d) Figure 5.
   e) Figure 4.
   f) Figure 6.
   g) Figure 1.
   h) Figure 6.

7. a) $(x - 5)^3$.
   b) $x \cdot 6 + y$ or $6x + y$.
   c) $(y + 6)x$.
   d) $\frac{10}{x} - y$.
   e) $\frac{10 - y}{x}$.
   f) $(x + 2)(x + 7)$.
   g) $\frac{x - y}{2x}$.
   h) $11 - (3x)^2$.
   i) $(11 - 3x)^2$.
   j) $(x^3 + y^3)8$ or $8(x^3 + y^3)$.

8. a) 0. $(3^2 + 2 \cdot 3 - 15 = 9 + 6 - 15 = 0.)$
   b) 9. $(4^2 + 2 \cdot 4 - 15 = 16 + 8 - 15 = 9.)$
   c) 105.
      $[10^2 + 2 \cdot 10 - 15 = 100 + 20 - 15 = 105.]$
   d) 2,585. $[50^2 + 2 \cdot 50 - 15 =$
      $2,500 + 100 - 15 = 2,585.]$
   e) 0. $[0 \cdot 8 = 0.]$
   f) 9. $[1 \cdot 9 = 9.]$
   g) 105. $[7 \cdot 15 = 105.]$
   h) 2,585. $[47 \cdot 55 = 2,585.]$

Set III

9. a) Yes. $[20 + 1 = 21;\ 14 + 7 = 21.]$
   b) Yes. $[20 - 1 = 19;\ 14 + 5 = 19.]$
   c) No. $[8 - 1 = 7;\ 14 - 5 = 9.]$
   d) No. $[8 + 1 = 9;\ 14 - 7 = 7.]$
   e) Yes. $[20 \cdot 4 = 80;\ 10 \cdot 8 = 80.]$
   f) No. $[10 + 8 = 18;\ 12 \cdot 4 = 48.]$
   g) Yes. $[(10 + 8 = 18;\ 10 + 8 = 18.]$
   h) Yes. $[\frac{8}{4} = 2;\ \frac{8}{4} = 2.]$

10. a) 48. $[3 \cdot 16 = 48.]$
    b) 144. $[12^2 = 144.]$
    c) 29. $[8 + 24 - 3 = 29.]$

d) 85. $[11 \cdot 8 - 3 = 88 - 3 = 85.]$
e) 23. $[8 + 3 \cdot 5 = 8 + 15 = 23.]$
f) 55. $[11 \cdot 5 = 55.]$
g) 1. $[12 - 10 - 1 = 1.]$
h) 40. $[10 \cdot 4 = 40.]$
i) 3. $[12 - (10 - 1) = 12 - 9 = 3.]$
j) 4. $[12 - 2 \cdot 4 = 12 - 8 = 4.]$
k) 6. $[8 - 2 = 6.]$
l) 14. $[\frac{28}{2} = 14.]$

m) 16. $[8 \cdot 2 = 16.]$

n) 16. $[\frac{128}{8} = 16.]$

o) 13. $[49 - 9 \cdot 4 = 49 - 36 = 13.]$
p) 160. $[(49 - 9) \cdot 4 = 40 \cdot 4 = 160.]$
q) 961. $[(49 - 18)^2 = 31^2 = 961.]$

11. a) Figure 1.
    b) Figure 3.
    c) Figure 2.
    d) Figure 5.
    e) Figure 4.
    f) Figure 6.
    g) Figure 2.
    h) Figure 6.

12. a) $(x + 11)y$.
    b) $11y + x$.

    c) $\frac{x}{3} - 1$.

    d) $\frac{x-1}{3}$.

    e) $(x + y)^2$.

    f) $x^2 + y^2$.

    g) $(x - y)x$.

    h) $9 - (2x)^3$.

    i) $(9 - 2x)^3$.

    j) $\frac{x+y}{5y}$.

13. a) 0. $[2^2 + 4 \cdot 2 - 12 = 4 + 8 - 12 = 0.]$
    b) 20. $[4^2 + 4 \cdot 4 - 12 = 16 + 16 - 12 = 20.]$
    c) 128.
       $[10^2 + 4 \cdot 10 - 12 = 100 + 40 - 12 = 128.]$
    d) 273.
       $[15^2 + 4 \cdot 15 - 12 = 225 + 60 - 12 = 273.]$
    e) 0. $[8 \cdot 0 = 0.]$
    f) 20. $[10 \cdot 2 = 20.]$
    g) 128. $[16 \cdot 8 = 128.]$

h) 273. $[21 \cdot 13 = 273.]$

Set IV (page 42)

1. All multiplication signs except for the first one which should be an addition sign:
   $1 + 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 \cdot 10$.

2. 3,628,801.

3. Use the same symbols of operation as before, but put parentheses around the $1 + 2$: $(1 + 2) \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 \cdot 10$.

4. 5,443,200.

Chapter 1, Lesson 9

Set I (page 45)

1. a) $5a$.
   b) b^3.
   c) $a + b$.
   d) 0.

2. a) $20 + x$.
   b) $5x$.
   c) $\frac{y}{20}$ minutes.

3. a) $1000 - x$ pounds.
   b) $\frac{1000}{y}$ pounds.
   c) $1000 - 10z$ pounds.

Set II (pages 45–46)

4. a) $3(6 + 2) = 3(6) + 3(2)$.
   b) $4(7 - 3) = 4(7) - 4(3)$.
   c) $5(1 + 8) = 5(1) + 5(8)$.
   d) $6(5 - 1) = 6(5) - 6(1)$.

5. a) $4x^3$.
   b) $7(2x)$.
   c) $3(x + 1)$.
   d) $9(x + y)$.
   e) $x^4 + x^4$.
   f) $3x + 3x + 3x + 3x + 3x$.
   g) $(x + 7) + (x + 7) + (x + 7) + (x + 7)$.

6. a) $3(x + 5) = (x + 5) + (x + 5) + (x + 5) = x + x + x + 5 + 5 + 5 = 3x + 15$.
   b) $2(x + y) = (x + y) + (x + y) = x + x + y + y = 2x + 2y$.

c) $4(x^2 + 1) =$
$(x^2 + 1) + (x^2 + 1) + (x^2 + 1) + (x^2 + 1) =$
$x^2 + x^2 + x^2 + x^2 + 1 + 1 + 1 + 1 =$
$4x^2 + 4$.

7. a) $8x + 24$.
    b) $5y - 10$.
    c) $x^2 + x$.
    d) $xy - y^2$.
    e) $2x + 18$.
    f) $4y + xy$.
    g) $7y - 7x$.
    h) $x^2 - 6x$.
    i) $10x^2 + 40$.
    j) $x^4 - x$.

8. a) $\begin{array}{r} 72 \\ \times\ 43 \\ \hline 216 \\ 2880 \\ \hline 3096 \end{array}$

    b) $43 \cdot 72 = (40 + 3)72$
       $= 40 \cdot 72 + 3 \cdot 72$
       $= 2880 + 216$
       $= 3096$

    c) $\begin{array}{r} 43 \\ \times\ 72 \\ \hline 86 \\ 3010 \\ \hline 3096 \end{array}$

    d) $72 \cdot 43 = (70 + 2)43$
       $= 70 \cdot 43 + 2 \cdot 43$
       $= 3010 + 86$
       $= 3096$

9. a) $4(x + 5)$ and $4x + 20$.
    b) $x(10 + x)$ and $10x + x^2$.
    c) $3(x + y + 3)$ and $3x + 3y + 9$.
    d) $x(x^2 + x + 1)$ and $x^3 + x^2 + x$.

10. a) $x + y$.
    b) $2(x + y)$.
    c) $2x$.
    d) $2y$.
    e) $2x + 2y$.

Set III

11. a) $2(5 + 4) = 2(5) + 2(4)$.
    b) $7(3 + 1) = 7(3) + 7(1)$.
    c) $4(8 - 5) = 4(8) - 4(5)$.
    d) $5(4 - 1) = 5(4) - 5(1)$.

12. a) $7x^2$.
    b) $3(5x)$ or $15x$.
    c) $2(x + 7)$.
    d) $10(x + y)$.
    e) $x^3 + x^3 + x^3 + x^3$.
    f) $7x + 7x$.
    g) $(x + 8) + (x + 8) + (x + 8)$.

13. a) $2(x + 6) = (x + 6) + (x + 6) =$
       $x + x + 6 + 6 = 2x + 12$.
    b) $4(x + y) =$
       $(x + y) + (x + y) + (x + y) + (x + y) =$
       $x + x + x + x + y + y + y + y = 4x + 4y$.
    c) $3(x^2 + 2) = (x^2 + 2) + (x^2 + 2) + (x^2 + 2) =$
       $x^2 + x^2 + x^2 + 2 + 2 + 2 = 3x^2 + 6$.

14. a) $2x + 10$.
    b) $4y - 28$.
    c) $3x + x^2$.
    d) $y^2 - y$.
    e) $10x + 80$.
    f) $6x + x^2$.
    g) $5y - 20$.
    h) $xy - y^2$.
    i) $3x^2 + 27$.
    j) $x^3 - 2x^2$.

15. a) $\begin{array}{r} 84 \\ \times\ 21 \\ \hline 84 \\ 1680 \\ \hline 1764 \end{array}$

    b) $21 \cdot 84 = (20 + 1)84$
       $= 20 \cdot 84 + 1 \cdot 84$
       $= 1680 + 84$
       $= 1764$

    c) $\begin{array}{r} 21 \\ \times\ 84 \\ \hline 21 \\ 1680 \\ \hline 1764 \end{array}$

    d) $84 \cdot 21 = (80 + 4)21$
       $= 80 \cdot 21 + 4 \cdot 21$
       $= 1680 + 84$
       $= 1764$

16. a) $3(x + 7)$ and $3x + 21$.
    b) $y(2 + y)$ and $2y + y^2$.
    c) $4(x + y + 6)$ and $4x + 4y + 24$.
    d) $x(y^2 + y + 1)$ and $xy^2 + xy + x$.

17. a) $x + y$.
   b) $3.50(x + y)$ dollars.
   c) $3.50x$ dollars.
   d) $3.50y$ dollars.
   e) $3.50x + 3.50y$ dollars.

Set IV (page 47)

1. a) 4. $[(2 + 0)^2 = 2^2 = 4.]$
   b) 36. $[(0 + 6)^2 = 6^2 = 36.]$
   c) 49. $[(3 + 4)^2 = 7^2 = 49.]$
   d) 100. $[(9 + 1)^2 = 10^2 = 100.]$
   e) 4. $[2^2 + 0^2 = 4 + 0 = 4.]$
   f) 36. $[0^2 + 6^2 = 0 + 36 = 36.]$
   g) 25. $[3^2 + 4^2 = 9 + 16 = 25.]$
   h) 82. $[9^2 + 1^2 = 81 + 1 = 82.]$

2. $(x + y)^2$ and $x^2 + y^2$ are sometimes equal and sometimes not equal.

Chapter 1, Review

Set I (pages 50–51)

1. a) $4 \cdot 7$.
   b) 7^4.
   c) $x + x$.
   d) $y \cdot y \cdot y \cdot y \cdot y \cdot y$.

2. a) w^2.
   b) $3x$.
   c) $17 - y$.
   d) z^5.

3. a) Think of a number: □
   Multiply by five: □□□□□
   Add eight: □□□□□⬛⬛⬛⬛
   Subtract three: □□□□□⬛⬛
   Divide by five: □⬛
   Subtract the number you first thought of: ∘
   b) The number 1.
   c) Steps 3 and 4.
   d) Add 5.

4. a) Figure 3.
   b) Figure 1.
   c) Figure 2.

5. a) $4^2 =$ 16
   $4^3 =$ 64
   $4^4 =$ 256
   $4^5 = 1,024$
   $4^6 = 4,096$

   b) The last digit of 4^{100} is 6.

6. a) 64.
   b) 8^2.

7. a) Perimeter, 22; area, 28.
   b) Perimeter, $2x + 6$; area, $3x$.
   c) Perimeter, $4y$; area, y^2.

8. a) 14. $[21 - 7 = 14.]$
   b) 28. $[30 - 2 = 28.]$
   c) 65. $[1 + 64 = 65.]$
   d) 125. $[5^3 = 125.]$

9. a) $x + 151$. $[9 + x + 142.]$
   b) $160 - x - y$ or $160 - (x + y)$.

10. a) If $\dfrac{2}{0} = a$, then $0 \cdot a = 2$. But $0 \cdot a = 0$.
    b) No.

11. a) $600x$.
    b) $\dfrac{10,000}{x}$.

12. a) $5x + 1$.
    b) $(x + 3)^2$.
    c) $x^6 - 7$.

13. a) $7a + 14$.
    b) $b - b^2$.
    c) $5c + 45$.

14. a) $3x$ carbon catoms and $8x$ hydrogen atoms.
    b) $3x + 5x$.
    c) 11.
    d) $11x$.

Set II (pages 51–52)

1. a) $3 \cdot 11$.
   b) 2^7.
   c) $x + x + x + x + x$.
   d) $y \cdot y \cdot y \cdot y$.

2. Step 1.   Think of a number.
   Step 2.   Add 1.
   Step 3.   Multiply by 4.
   Step 4.   Add 8.
   Step 5.   Divide by 4.
   Step 6.   Subtract the number that you first thought of.

3. a) $a - 5$.
    b) b^3.
    c) $2 + c$.
    d) $\dfrac{1}{d}$.

4. a) 2^5.
       $[2^1 = 2,\ 2^2 = 4,\ 2^3 = 8,\ 2^4 = 16,\ 2^5 = 32.]$
    b) It is impossible to express 3 as a power
       of 1. $[1^1 = 1,\ 1^2 = 1,\ 1^3 = 1, \ldots]$
    c) 10^6. $[10^1 = 10,\ 10^2 = 100,\ 10^3 = 1{,}000,$
       $10^4 = 10{,}000,\ 10^5 = 100{,}000,$
       $10^6 = 1{,}000{,}000.]$

5. a) $x - 3$.
    b) $x + 5$.

6. a) 9. $[\dfrac{72}{8}.]$
    b) $\dfrac{20}{x}$.
    c) $\dfrac{x}{y}$.

7. a) $72 + x$.
    b) $72 - y$.
    c) $3 + x$. $[75 + x - 72.]$

8. a) 600. $[6 \cdot 100 = 600.]$
    b) 3,600. $[60^2 = 3{,}600.]$
    c) 45. $[9 \cdot 5 = 45.]$
    d) 55. $[2 + 56 - 3 = 55.]$

9. a) $3^2 - 1^2 = 8 = 2^3$
          $6^2 - 3^2 = 27 = 3^3$
          $10^2 - 6^2 = 64 = 4^3$
          $15^2 - 10^2 = 125 = 5^3$
    b) $21^2 - 15^2 = 216 = 6^3$.

10. a) $(x - 6)2$ or $2(x - 6)$.
    b) $\dfrac{x}{8} + 4$.
    c) $150 - x^3$.

11. a) $8v + 88$.
    b) $3w - 18$.
    c) $xy + xz$.

12. a) $3(x + 4) = 3x + 12$.
    b) $(6 + x)x = 6x + x^2$.
    c) $y(x + 1) = yx + y$.

13. a) $3x$.
    b) $200 - 3x$ kilograms.
    c) $\dfrac{x}{3}$.

14. a) 0. $[2^2 + 5 \cdot 2 - 14 = 4 + 10 - 14 = 0.]$
    b) 10. $[3^2 + 5 \cdot 3 - 14 = 9 + 15 - 14 = 10.]$
    c) 136.
       $[10^2 + 5 \cdot 10 - 14 = 100 + 50 - 14 = 136.]$
    d) 0. $[9 \cdot 0 = 0.]$
    e) 10. $[10 \cdot 1 = 10.]$
    f) 136. $[17 \cdot 8 = 136.]$