

TEACHER GUIDE

High School

Algebra I



Weekly Lesson Schedule



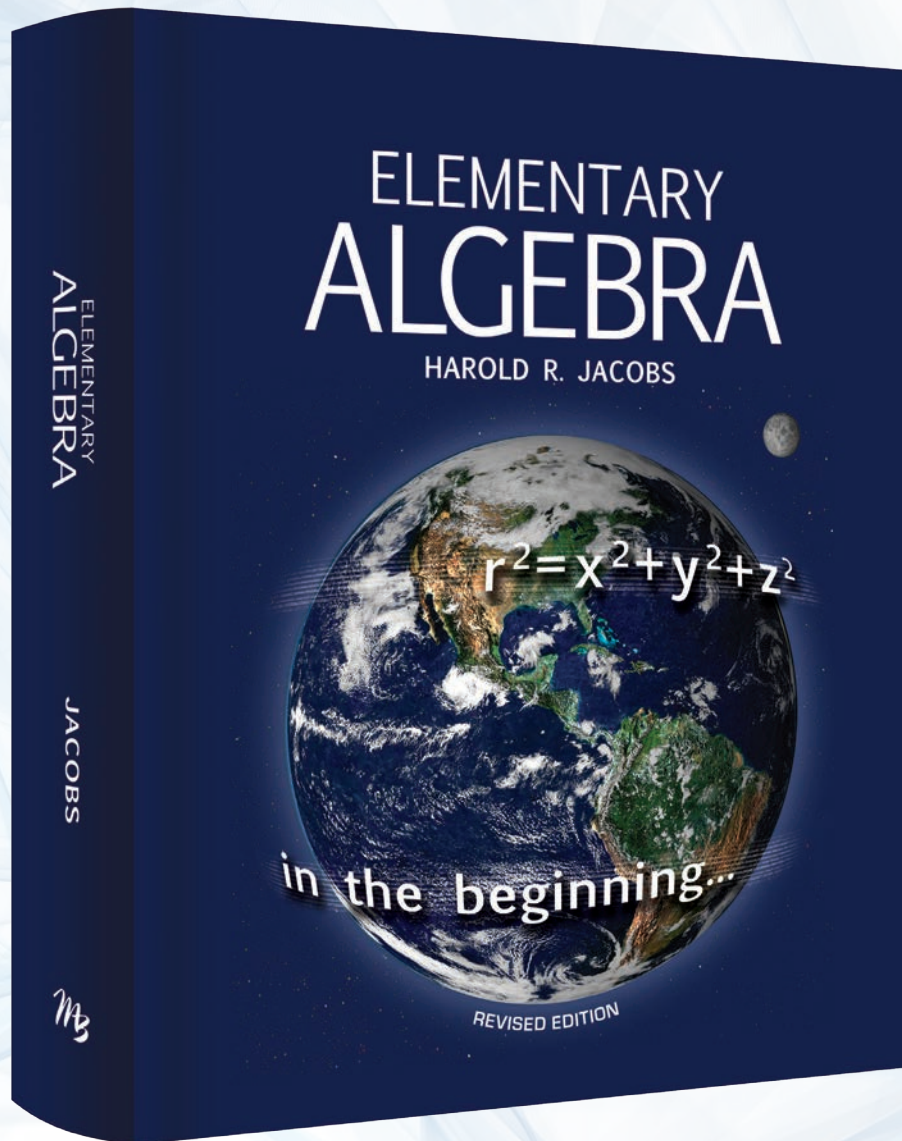
Chapter Tests & Final Exam



Set III Worksheets



Answer Keys



TEACHER GUIDE

9th–12th Grade

Includes Student
Worksheets

Algebra I



Answer Keys



Weekly Lesson Schedule



Set III Worksheets



Tests

Elementary Algebra



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Unless otherwise noted, Scripture quotations are from the New King James Version of the Bible.

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Author Bio:

Harold R. Jacobs is teacher of mathematics and science, writer, and well-respected speaker. He received his B.A. from U.C.L.A. and his M.A.L.S from Wesleyan University. His other publications include *Mathematics: A Human Endeavor*, *Geometry: Seeing, Doing, Understanding* and articles for *The Mathematics Teacher* and the *Encyclopedia Britannica*. Mr. Jacobs has received the Most Outstanding High School Mathematics Teacher in Los Angeles award, and the 1988 Presidential Award for Excellence in Science and Mathematics Teaching.

Using This Teacher Guide

Features: The suggested weekly schedule enclosed has easy-to-manage lessons that guide the reading, course exercises, and all assessments. The pages of this guide are perforated and three-hole punched so materials are easy to tear out, hand out, grade, and store. Teachers are encouraged to adjust the schedule and materials needed in order to best work within their unique educational program.

Lesson Scheduling: Students are instructed to read the pages for each lesson in the student textbook and then complete the corresponding sets of exercises. Assessments include lesson exercises, chapter tests, a mid-term review, and final exam given at regular intervals with space to record each grade. Space is provided on the weekly schedule for assignment dates, and flexibility in scheduling is encouraged. Teachers may adapt the scheduled days per each unique student situation. As the student completes each assignment, this can be marked with an “X” in the box.



Approximately 45 to 60 minutes per lesson, five days a week



Includes answer keys for chapter tests, mid-term and final exams



Set III exercise worksheets for each lesson – an optional part of the course used for additional review or extra practice



Multiple versions of the chapter tests are included, mid-term and final exams are also included to help reinforce learning and offer flexibility for teaching the course and provide assessment opportunities.



Designed for grades 9 to 12 in a one-year course to earn 1 math or Algebra I credit

Course Objectives:

- ✓ *Elementary Algebra* is designed to prepare the student with a foundational understanding of basic principles in Algebra. By understanding the nature of equations and solutions, the student will be better able to apply them and build upon this core knowledge in future advanced coursework including Algebra II.
- ✓ *Elementary Algebra* has come to be highly regarded in the education market. This curriculum provides a full year of mathematics in a clearly written format with guidance for teachers as well as for students who are self-directed.

Course Description

The course is divided into 17 sections, covering functions and graphs, integers, rational numbers, exponents, polynomials, factoring, fractions, and more. As the student works through the textbook, the answers to the Set II exercise are provided in the back of the student textbook, with full solutions to all of the exercises available in the *Solutions Manual* for the *Elementary Algebra* course.

Additional Materials Needed:

- ✓ *Elementary Algebra* (student textbook).
- ✓ A notebook and graph paper are essential. It is highly recommended that students use a 3-ring binder, loose-leaf paper, and graph paper to complete the coursework. Tab dividers to separate your work by lesson are also recommended.

How To Use This Course:

There is flexibility in how the course is structured for the student, where the teacher can assign various sets of exercises based on the focus and intensity of the course. Both the student textbook and the *Solutions Manual* are crucial in completing the course.

- ✓ Each chapter is segmented into lessons, with each lesson having informative material to explain and analyze critical concepts with examples for analysis; four sets of exercises for each lesson are available for applying this knowledge, as well as a corresponding chapter exam available for assessment purposes.
- ✓ Set I exercises review ideas from the previous lessons which allows for an ongoing application of material from earlier in the course.
- ✓ Set II and Set III are similar in that they allow the student to apply concepts from the new lesson.
- ✓ Set IV exercises are meant to challenge students who are excelling in the course, although many can be done by any student taking the course. These should be considered optional and can also be used for extra credit.
- ✓ Extra testing materials can be used as pre-tests, reviews, or bonus material.

Optional Exercises and Assessment Materials

This *Teacher Guide* for *Elementary Algebra* has optional Set III exercise worksheets which can be used by the student for additional practice or for bonus purposes. The Set III worksheets are an optional feature. The course is robust even without their inclusion.

Testing

There are four versions available of each chapter test. The teacher can utilize these in various ways based on assessment needs. Some can be used as review sheets or as practice work in prep for the chapter exams. They can also be used as bonus materials. Mid-term and final exams are also included.

For ease in applying the testing materials, all of the Test A versions are recommended, though versions B, C, and D are grouped in a supplemental section of this *Teacher Guide*.

Grading

It is always the prerogative of an educator/parent to assess student grades however he or she might deem best. The following is only a suggested guideline based on the material presented through this course:

- ✓ Each lesson's coursework is worth 100 points. Because of variations in how this course may be taught – based on the numbered Set Exercises you choose to include, it is recommended you use the formula for grading explained below.
- ✓ All tests within the course are worth 100 points each.

To calculate the percentage of the worksheets, chapter tests, mid-term or final exam, the parent/educator may use the following guide.

Divide total number of questions correct (example: 43) by the total number of questions possible (example: 46) to calculate the percentage out of 100 possible. $43/46 = 93$ percent correct. The suggested grade values are noted as follows:

90 to 100 percent = A

80 to 89 percent = B

70 to 79 percent = C

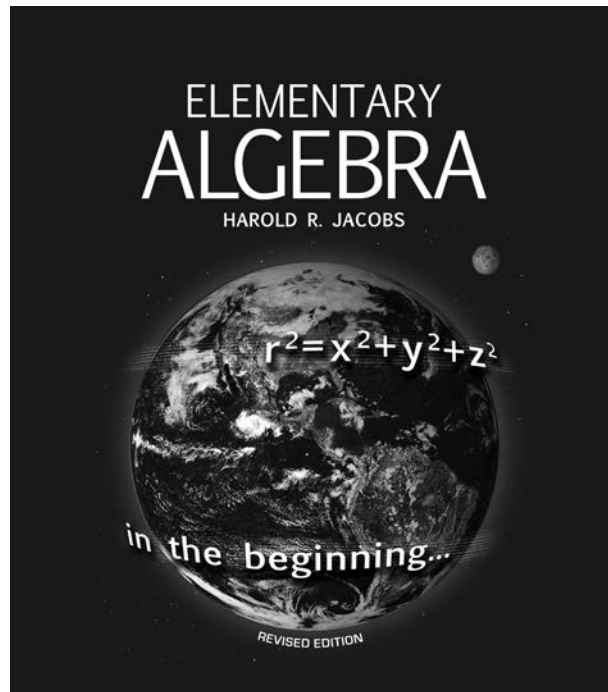
60 to 69 percent = D

0 to 59 percent = F

First Semester Suggested Daily Schedule

Date	Day	Assignment	Due Date	✓	Grade
First Semester-First Quarter					
Week 1	Day 1	Read pages xi-3 of <i>Elementary Algebra</i> textbook (EA) • Complete the exercise on pages 3-4 • See this <i>Teacher Guide</i> (TG) for exams.			
	Day 2	Read Chapter 1, Lesson 1: Addition, pages 6-7 (EA) Complete Exercises I, II, and IV on pages 7-10.			
	Day 3	Read Chapter 1, Lesson 2: Subtraction, pages 11-12 (EA) Complete Exercises I, II, and IV on pages 12-14.			
	Day 4	Read Chapter 1, Lesson 3: Multiplication, pages 15-16 (EA) Complete Exercises I, II, and IV on pages 17-19.			
	Day 5	Read Chapter 1, Lesson 4: Division, pages 20-21 (EA) Complete Exercises I, II, and IV on pages 21-23.			
Week 2	Day 6	Read Chapter 1, Lesson 5: Raising to a Power, pages 24-25 (EA) Complete Exercises I, II, and IV on pages 25-27.			
	Day 7	Read Chapter 1, Lesson 6: Zero and One, pages 28-29 (EA) Complete Exercises I, II, and IV on pages 30-31.			
	Day 8	Read Chapter 1, Lesson 7: Several Operations, pages 32-34 (EA) Complete Exercises I, II, and IV on pages 35-37.			
	Day 9	Read Chapter 1, Lesson 8: Parentheses, pages 38-40 (EA) Complete Exercises I, II, and IV on pages 40-42.			
	Day 10	Read Chapter 1, Lesson 9: The Distributive Rule, pages 43-44 (EA) Complete Exercises I, II, and IV on pages 45-47.			
Week 3	Day 11	Read Chapter 1: Summary and Review, pages 48-49 (EA) Complete Exercises I and II on pages 50-52.			
	Day 12	Chapter 1 Test Study Day			
	Day 13	Chapter 1 Test A, page 161 (TG)			
	Day 14	Read Chapter 2, Lesson 1: An Introduction to Functions, pages 54-55 (EA) • Complete Exercises I, II, and IV on pages 55-58.			
	Day 15	Read Chapter 2, Lesson 2: The Coordinate Graph, pages 59-60 (EA) • Complete Exercises I, II, and IV on pages 60-62.			
Week 4	Day 16	Read Chapter 2, Lesson 3: More on Functions, pages 63-64 (EA) Complete Exercises I, II, and IV on pages 65-67.			
	Day 17	Read Chapter 2, Lesson 4: Direct Variation, pages 68-69 (EA) Complete Exercises I, II, and IV on pages 70-72.			
	Day 18	Read Chapter 2, Lesson 5: Linear Functions, pages 73-74 (EA) Complete Exercises I, II, and IV on pages 75-77.			
	Day 19	Read Chapter 2, Lesson 6: Inverse Variation, pages 78-79 (EA) Complete Exercises I, II, and IV on pages 80-82.			
	Day 20	Read Chapter 2, Summary and Review, page 83 (EA) Complete Exercises I and II on pages 84-88.			
Week 5	Day 21	Chapter 2 Test Study Day			
	Day 22	Chapter 2 Test A, page 163 (TG)			
	Day 23	Review text and work for Chapters 1-2. Use this time to build your skills or work on concepts you may be struggling to understand or master.			
	Day 24	Read Chapter 3, Lesson 1: The Integers, pages 90-91 (EA) Complete Exercises I, II, and IV on pages 91-93.			
	Day 25	Read Chapter 3, Lesson 2: More on the Coordinate Graph, pages 94-95 (EA) • Complete Exercises I, II, and IV on pages 96-98.			

Date	Day	Assignment	Due Date	✓	Grade
Week 6	Day 26	Read Chapter 3, Lesson 3: Addition, pages 99-100 (EA) Complete Exercises I, II, and IV on pages 101-102.			
	Day 27	Read Chapter 3, Lesson 4: Subtraction, pages 103-104 (EA) Complete Exercises I, II, and IV on pages 105-106.			
	Day 28	Read Chapter 3, Lesson 5: Multiplication, pages 107-109 (EA) Complete Exercises I, II, and IV on pages 109-110.			
	Day 29	Read Chapter 3, Lesson 6: Division, pages 111-112 (EA) Complete Exercises I, II, and IV on pages 112-113.			
	Day 30	Read Chapter 3, Lesson 7: Several Operations, pages 114-115 (EA) Complete Exercises I, II, and IV on pages 115-117.			
Week 7	Day 31	Read Chapter 3, Summary and Review, pages 118-119 (EA) • Complete Exercises I and II on pages 119-122.			
	Day 32	Chapter 3 Test Study Day			
	Day 33	Chapter 3 Test A, page 165 (TG)			
	Day 34	Read Chapter 4, Lesson 1: The Rational Numbers, pages 124-126 (EA) • Complete Exercises I, II, and IV on pages 126-127.			
	Day 35	Read Chapter 4, Lesson 2: Absolute Value and Addition, pages 128-130 (EA) • Complete Exercises I, II, and IV on pages 131-132.			
Week 8	Day 36	Read Chapter 4, Lesson 3: More on Operations with Rational Numbers, pages 133-134 (EA) • Complete Exercises I, II, and IV on pages 135-136.			
	Day 37	Read Chapter 4, Lesson 4: Approximations, pages 137-139 (EA) Complete Exercises I, II, and IV on pages 139-140.			
	Day 38	Read Chapter 4, Lesson 5: More on Graphing Functions, pages 141-143 (EA) • Complete Exercises I, II, and IV on pages 143-145.			
	Day 39	Read Chapter 4, Summary and Review, pages 146-147 (EA) Complete Exercises I and II on pages 147-149.			
	Day 40	Chapter 4 Test Study Day			
Week 9	Day 41	Chapter 4 Test A, page 167 (TG)			
	Day 42	Review text and work for Chapters 3-4. Use this time to build your skills or work on concepts you may be struggling to understand or master.			
	Day 43	Read Chapter 5, Lesson 1: Equations, pages 152-153 (EA) • Complete Exercises I, II, and IV on pages 153-155.			
	Day 44	Read Chapter 5, Lesson 2: Inverse Operations, pages 156-158 (EA) • Complete Exercises I, II, and IV on pages 158-161.			
	Day 45	Read Chapter 5, Lesson 3: Equivalent Equations, pages 162-165 (EA) • Complete Exercises I, II, and IV on pages 165-167.			
First Semester-Second Quarter					
Week 1	Day 46	Read Chapter 5, Lesson 4: Equivalent Expressions, pages 168-171 (EA) Complete Exercises I, II, and IV on pages 171-172.			
	Day 47	Read Chapter 5, Lesson 5: More on Solving Equations, pages 173-176 (EA) • Complete Exercises I, II, and IV on pages 176-178.			
	Day 48	Read Chapter 5, Lesson 6: Length and Area, pages 179-181 (EA) Complete Exercises I, II, and IV on pages 181-184.			
	Day 49	Read Chapter 5, Lesson 7: Distance, Rate and Time, pages 185-186 (EA) • *Complete Exercises I, II, and IV on pages 186-188.			
	Day 50	Read Chapter 5, Lesson 8: Rate Problems, pages 189-191 (EA) Complete Exercises I, II, and IV on pages 192-193.			



Set III Exercise Worksheets

for use with

Elementary Algebra

NOTE: The Set III Exercise problems retain their sequential numbering between the Set II and Set IV exercises in the Elementary Algebra student textbook.



17. Write a number or expression for each of the following.

- The sum of 3 and 11.
- The sum of 3 and x .
- The sum of y and 11.
- The sum of y and x .
- Seven increased by 2.
- Seven increased by x .
- The sum of 9, 1, and 4.
- The sum of x , 1, and 4.
- The sum of 9, y , and 4.
- The sum of x , y , and 4.

18. In the figures below, the box represents any number and the sets of circles represent specific numbers.



Figure 1

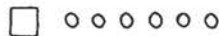
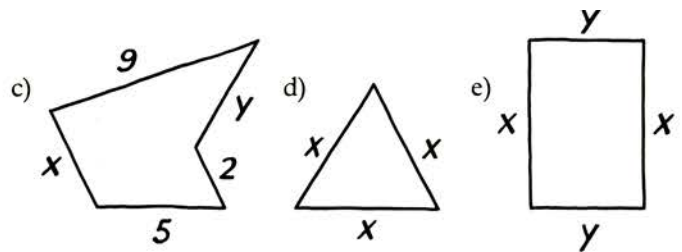
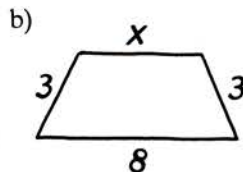
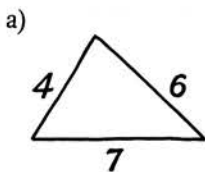


Figure 2

- What addition problem is illustrated by Figure 1?
 - What is the answer to the problem?
 - Write an algebraic expression to represent the addition problem illustrated by Figure 2.
 - What is the answer to the problem if the box represents 1?
 - What is the answer to the problem if the box represents 5?
19. The *perimeter* of a figure is the sum of the lengths of its sides. What is the perimeter of each of these figures?



20. The figure below can be used to show that $4 + 5$ and $5 + 4$ are the same number, depending on whether the figure is read from left to right or from right to left.



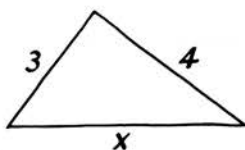
Draw boxes and circles to show that

- $2 + x$ and $x + 2$ mean the same thing.
 - $8 + x + 1$ and $x + 9$ mean the same thing.
 - $x + x + 3$ and $x + 3 + x$ mean the same thing.
21. The expression $x + 1 + y$ represents the sum of x , 1, and y . If x is 4, it can be written as $4 + 1 + y$ or $5 + y$. How can $x + 1 + y$ be written if
- x is 2?
 - x is 0?
 - y is 6?
 - y is 9?
 - x is 3 and y is 7?
22. Each week, Dashing Dan jogs one mile farther than he did the week before.
- If he jogs 18 miles this week, how far will he jog next week?
 - If he jogs x miles this week, how far will he jog next week?
 - If he jogged y miles three weeks ago, how far will he jog this week?
 - If he jogged y miles z weeks ago, how far will he jog this week?



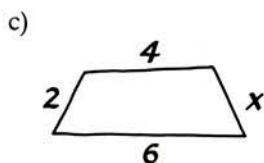
18. Write a number or expression for each of the following.
- The difference between 9 and 3.
 - Five taken away from x .
 - Five decreased by 2.
 - Eight less than 20.
 - Two less than x .
 - The difference between y and x .
 - The result of subtracting x from 7.
 - Seven subtracted from x .

19. The perimeter of a figure is the sum of the lengths of its sides.

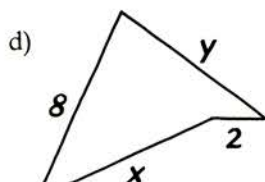


- How long is the side marked x in this triangle if the triangle's perimeter is 12?
- How long is it if the triangle's perimeter is y ?

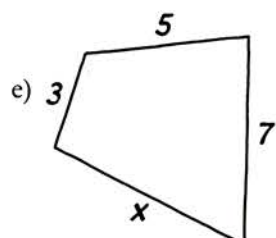
Use the perimeters given below each of the following figures to tell the length of the side marked x .



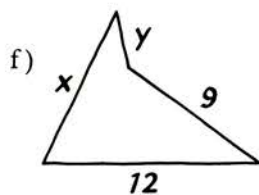
Perimeter is 15



Perimeter is 20



Perimeter is y



Perimeter is z

20. Find the value of each of the following expressions for the numbers given.
- $x - 2$ if x is 9.
 - $x - 2$ if x is 10.
 - $x - 2$ if x is 20.
 - What happens to the value of $x - 2$ as x gets larger?
 - $8 - x$ if x is 1.
 - $8 - x$ if x is 2.

- $8 - x$ if x is 8.
- What happens to the value of $8 - x$ as x gets larger?

21. Find the value of each of the following for the numbers given.

The sum of x and $7 - y$

- if x is 5 and y is 1.
- if x is 13 and y is 6.

The difference between $x + 7$ and y

- if x is 5 and y is 1.
- if x is 13 and y is 6.

- Can you explain why the answers to parts c and d are the same as those to parts a and b?

22. A log is cut into two pieces.

- If the log was 12 feet long and one piece is x feet long, how long is the other piece?
- If the two pieces are x feet and y feet long, how long was the log?
- If the log was x feet long and one piece is y feet long, how long is the other piece?

23. Laverne DeFazio has 2 dollars in her checking account.

- If she writes a check for x dollars, how much money will remain in her account?
- If she makes a deposit of y dollars, how much money will she have in her account?
- If her account increases to z dollars, how much money has she added to it?

24. The Swinging Singles Tennis Club has 100 members.

- If x of them are men, how many are women?
- If y people join the club, how many members will it have?
- If y people join the club and z people resign, how many members will it have?



18. Draw figures as indicated.
- A figure with circles to show that $2 \cdot 6$ and $6 \cdot 2$ are the same number.
 - A figure with boxes to illustrate $3x$ if each box represents x .
 - A rectangle divided into squares to illustrate $4 \cdot 5$.

19. Write a number or expression for each of the following.
- The product of 7 and 3.
 - The sum of 7 and 3.
 - The product of 7 and x .
 - The sum of 7 and x .
 - The product of y and x .
 - The sum of y and x .
 - The product of y and y .
 - Five multiplied by x .
 - Five subtracted from x .
 - The sum of 4, 6, and x .
 - The product of 4, 6, and x .
 - The sum of 5, y , and 12.
 - The product of 5, y , and 12.
 - The sum of x , y , and 2.
 - The product of x , y , and 2.

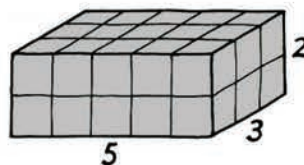
20. The multiplication problem $5 \cdot 8$ and the addition problem $8 + 8 + 8 + 8 + 8$ are equivalent. Write a multiplication problem equivalent to each of the following addition problems.

- $10 + 10 + 10$
- $3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3$
- $x + x + x$
- $\underbrace{4 + 4 + \cdots + 4}_{15 \text{ of them}}$
- $\underbrace{4 + 4 + \cdots + 4}_{x \text{ of them}}$
- $\underbrace{x + x + \cdots + x}_{y \text{ of them}}$

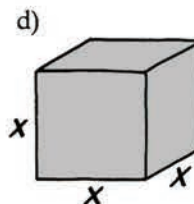
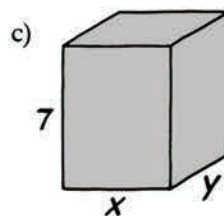
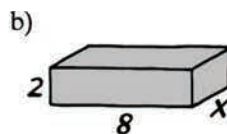
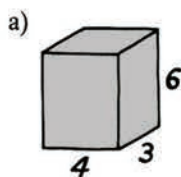
Write an addition problem equivalent to each of the following multiplication problems.

- $2 \cdot 19$
- $5x$
- $y \cdot 3$
- $x \cdot x$

21. The volume of a rectangular box is the product of its length, width, and height. The volume of the box shown here, for example, is $5 \cdot 3 \cdot 2 = 30$ because it contains 30 cubes.



What is the volume of each of these boxes?



22. Jelly Belly jelly beans are each 4 calories.

- How many calories are a dozen of these jelly beans?
- How many calories are x jelly beans?
- How many calories are y dozen jelly beans?

23. The trees in an orchard are arranged in rows with an equal number of trees in each row.

- If there are x rows and each row contains 20 trees, how many trees are there in all?
- If there are x rows and each row contains x trees, how many trees are there in all?

24. Because there are 100 centimeters in one meter, there are $100x$ centimeters in x meters.
- a) How many millimeters are there in one meter?
 - b) How many millimeters are there in x meters?
 - c) How many meters are there in three kilometers?
 - d) How many meters are there in y kilometers?
 - e) How many centimeters are there in one kilometer?
 - f) How many centimeters are there in y kilometers?
 - g) How many millimeters are there in y kilometers?



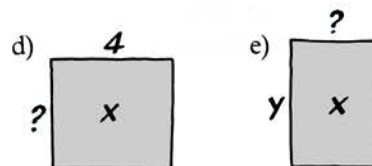
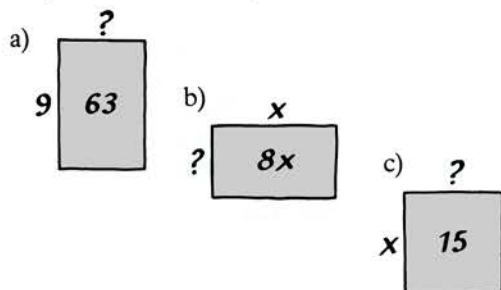
17. Write a number or expression for each of the following.

- The quotient of 8 and 2.
- The difference between 8 and 2.
- Five divided into x .
- Five divided by x .
- The quotient of 3 and x .
- The product of 3 and x .
- The result of dividing x by 12.
- The result of subtracting 12 from x .
- The quotient of y and x .
- The product of y and x .

18. Find the value of each of the following expressions for the numbers given.

- $7x$ if x is 3.
- $7x$ if x is 6.
- $7x$ if x is 11.
- What happens to the value of $7x$ as x gets larger?
- $\frac{x}{3}$ if x is 0.
- $\frac{x}{3}$ if x is 12.
- $\frac{x}{3}$ if x is 51.
- What happens to the value of $\frac{x}{3}$ as x gets larger?
- $\frac{18}{x}$ if x is 9.
- $\frac{18}{x}$ if x is 10.
- $\frac{18}{x}$ if x is 45.
- What happens to the value of $\frac{18}{x}$ as x gets larger?

19. Find the missing dimension for each of these rectangles. (The numbers inside represent their areas.)



20. Most people learn how to do long division without knowing why it works. The method by which it is done is based on repeated subtraction. For example, compare the two methods below:

<i>Long division</i>	<i>Repeated subtraction</i>	
12		
15 $\overline{)180}$	180	
- 150	- 150	10 fifteens
30	30	
- 30	- 30	2 more fifteens
0	0	12 fifteens subtracted

a) Write this long division problem as a repeated subtraction problem.

$$\begin{array}{r} 46 \\ 21 \overline{)966} \\ - 840 \\ \hline 126 \\ - 126 \\ \hline 0 \end{array}$$

b) Divide 875 by 7 using both long division and repeated subtraction.

21. Suppose that a dogcart were to travel at a steady rate for 2,000 meters.

- How long would it take if the dogcart traveled 100 meters each minute?
- How long would it take if the dogcart traveled x meters each minute?

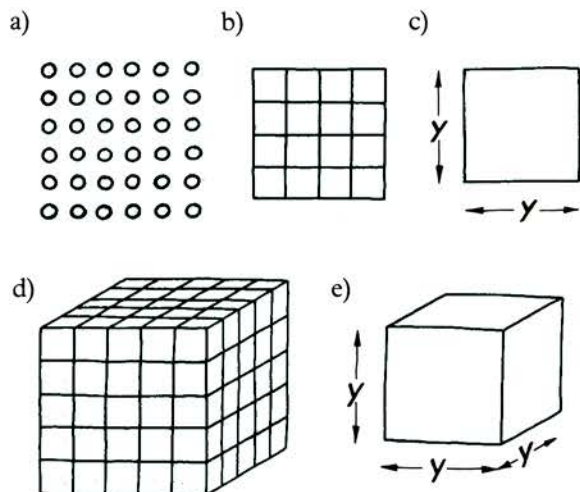
22. The members of the River City School band are marching in a rectangular array of rows and columns.

- If there are x rows and y columns, how many people are in the band?
- If there are 80 people in the band and x rows, how many columns are there?
- If there are x people in the band and 8 columns, how many rows are there?

23. A cube has six faces.
- a) If each face of a cube has a surface area of 25 square inches, what is its total surface area?
 - b) If each face of a cube has a surface area of x square inches, what is its total surface area?
 - c) If the total surface area of a cube is 96 square inches, what is the surface area of one face?
 - d) If the total surface area of a cube is y square inches, what is the surface area of one face?
24. On Monday, Mr. Kitzel made 10 dozen donuts.
- a) If it cost him \$10.80, what was his cost per dozen donuts?
 - b) If it cost him \$10.80, what was his cost per donut?
 - c) If it cost him x dollars, what was his cost per dozen donuts?
 - d) If it cost him x dollars, what was his cost per donut?



18. The expression x^3 can be named in more than one way.
a) Write two different names for it.
b) What is the 3 called?
19. What numbers or expressions do these figures represent?
Express each as a power.



20. Write each of the following in symbols.
a) Two cubed.
b) Five raised to the tenth power.
c) The number x squared.
d) The fourth power of x .
e) Nine raised to the y th power.
f) The x th power of y .
21. The raising-to-a-power problem x^4 and the multiplication problem $x \cdot x \cdot x \cdot x$ are equivalent. Write a power problem equivalent to each of the following.
a) $6 \cdot 6 \cdot 6 \cdot 6 \cdot 6$
b) $11 \cdot 11 \cdot 11$
c) $y \cdot y \cdot y \cdot y \cdot y \cdot y \cdot y$
d) $\underbrace{3 \cdot 3 \cdot \dots \cdot 3}_{10 \text{ of them}}$
e) $\underbrace{3 \cdot 3 \cdot \dots \cdot 3}_x$
f) $\underbrace{x \cdot x \cdot \dots \cdot x}_y$

Write a multiplication problem equivalent to each of the following.

- g) 1^4 h) x^5 i) 5^z j) x^y

22. The number 625 can be written as a power of 5. To find out what power it is, we can make a list of powers of 5 until we come to 625:

$$5^2 = 25, 5^3 = 125, 5^4 = 625$$

Express each of the following numbers as a power of the number given.

- a) 343 as a power of 7.
b) 6,561 as a power of 81.
c) 6,561 as a power of 9.
d) 6,561 as a power of 3.
e) 1,000 as a power of 10.
f) 10,000,000 as a power of 10.
g) It is impossible to express 10 as a power of 1. Explain why.
23. This table shows the values of the second through sixth powers of 6.

$$\begin{aligned} 6^2 &= 36 \\ 6^3 &= 216 \\ 6^4 &= 1,296 \\ 6^5 &= 7,776 \\ 6^6 &= 46,656 \end{aligned}$$

- a) Can you guess what one of the digits of 6^{100} might be?
b) Make a table showing the values of the second through sixth powers of 5.
c) Can you guess what any of the digits of 5^{100} might be?

This table shows the values of the second through sixth powers of 9.

$$\begin{aligned} 9^2 &= 81 \\ 9^3 &= 729 \\ 9^4 &= 6,561 \\ 9^5 &= 59,049 \\ 9^6 &= 531,441 \end{aligned}$$

- d) Can you guess what any of the digits of 9^{100} might be?



11. If possible, tell what number should replace \blacksquare in each of the following equations to make it true.

- a) $x \cdot \blacksquare = x$
 b) $x + \blacksquare = x$
 c) $\frac{x}{\blacksquare} = x$
 d) $\frac{x}{x} = \blacksquare$ (Assume that x is not zero.)
 e) $\frac{0}{\blacksquare} = \blacksquare$
 f) $x - \blacksquare = x$
 g) $x - \blacksquare = 0$
 h) $x \cdot \blacksquare = 0$

12. One way to picture the product $3 \cdot 5$ is shown below.



- a) How could this figure be changed to picture the product $2 \cdot 5$?
 b) How could it be changed to picture the product $1 \cdot 5$?
 c) Could it be changed to picture the product $0 \cdot 5$?
13. The following questions are about powers of zero.
- a) What is the value of 0^2 ? Why?
 b) What is the value of 0^5 ?
 c) What is the value of 0^x , in which x is a counting number larger than one?
14. Each of the following expressions contains two unknown numbers, x and y . Simplify each expression as much as you can. You may assume that neither x nor y is zero.
- a) $1x + 0y$ c) $1y - 0x$
 b) $1y - 1x$ d) $0x - 0y$
- e) $\frac{y}{1} + \frac{x}{1}$
 f) $\frac{0}{x} - \frac{0}{y}$
 g) $\frac{y}{1} - \frac{0}{x}$
 h) $\frac{0}{y} + \frac{x}{1}$

15. In the ninth century, an Arab mathematician wrote:

“When nothing remains in subtraction, put down a small circle so that the place be not empty, but the circle must occupy it, so that the number of places will not be diminished when the place is empty and the second be mistaken for the first.”

- a) Use the problem below to show what he meant.

$$\begin{array}{r} 45 \\ - 5 \\ \hline \end{array}$$

- b) What does the zero in the answer to this problem mean?

16. The following questions are about the counting numbers.
- a) If x represents a counting number, what is the next larger counting number?
 b) If x represents a counting number larger than one, what is the next smaller counting number?
 c) What is the smallest counting number?

17. Obtuse Ollie says that, if you divide a number by zero, the answer is zero.

- a) Explain why $\frac{7}{0}$ is not equal to 0.
 b) Does it make sense to say that $\frac{7}{0}$ is equal to 7?

Acute Alice says that, if you divide zero by a number, the answer is zero.

- c) Explain why $\frac{0}{7}$ is equal to 0.
 d) Does it make sense to say that $\frac{0}{0}$ is equal to 0?

18. Some automatic calculators do division by repeated subtraction, subtracting the dividing number over and over until the result is zero.

- a) If you tried to divide 12 by 0 on such a calculator, would it eventually arrive at zero? Explain.
 b) What do you suppose would happen if you tried dividing 0 by 0 on such a calculator?



10. The figure at the right illustrates the expression $5^2 - 2 \cdot 3$. Which figure below illustrates each of the expressions in parts a through g?

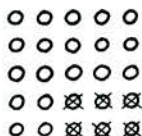


Figure 1

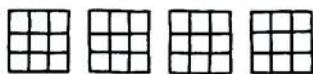


Figure 2

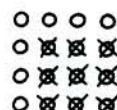


Figure 3



Figure 4



Figure 5

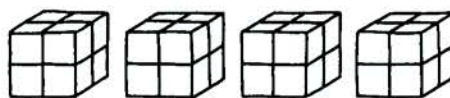


Figure 6

- a) $3 \cdot 2 + 4 \cdot 2$ e) $4 \cdot 2 - 3 \cdot 2$
 b) $2^3 + 2^3 + 2^3 + 2^3$ f) $2^3 \cdot 4$
 c) $3^2 + 4^2$ g) $3^2 \cdot 4$
 d) $4^2 - 3^2$

11. Find the value of each of the following expressions.

- a) $20 - 6 + 3$
 b) $20 - 6 \cdot 3$
 c) $20 \cdot 6 - 3$
 d) $5 \cdot 9 - 4 \cdot 7$
 e) $5 + 9 \cdot 4 - 7$
 f) $2 + 4^3$
 g) $2 \cdot 4^3$
 h) $2^4 \cdot 3$
 i) $26 + 3 \cdot 8 - 5^2$
 j) $26 - 5^2 + 3 \cdot 8$
 k) $26 - 3 \cdot 8 + 5^2$
 l) $26 + 5^2 - 3 \cdot 8$

m) $\frac{4^3}{2} - \frac{6^2}{3}$ n) $\frac{2^3}{4} - \frac{3^2}{6}$

- o) $3 \cdot 3 \cdot 4 - 3 \cdot 2 \cdot 5$
 p) $3 \cdot 3^4 - 3 \cdot 2^5$
 q) $3^4 \cdot 3 - 2^5 \cdot 3$

12. Write an expression for each of the following.

- a) One more than the product of x and 7.
 b) The difference between the cubes of x and y .
 c) Three times x , decreased by three times y .
 d) Twelve increased by the quotient of x and 6.

- e) The product of 5 and the square of x .
 f) The sum of x and the fifth power of y .
 g) The quotient of 1 and the product of x and y .

13. The value of the expression $x^3 - 2x + 4$ depends on the number with which we replace x . For example, if x is 3,

$$\begin{aligned} x^3 - 2x + 4 &= \\ 3^3 - 2 \cdot 3 + 4 &= \\ 27 - 2 \cdot 3 + 4 &= \\ 27 - 6 + 4 &= \\ 21 + 4 &= \\ &25 \end{aligned}$$

Find the value of $x^3 - 2x + 4$ if

- a) x is 0.
 b) x is 2.
 c) x is 5.
 d) x is 7.

14. Find the value of each of the following expressions for the numbers given.

- a) $5x + 4$ if x is 8.
 b) $17 - 2x$ if x is 3.
 c) $1 + 3x^2$ if x is 4.
 d) $x^2 + x^3$ if x is 10.
 e) $x^4 - x$ if x is 5.
 f) $6x^2 + x - 2$ if x is 1.

15. Acute Alice put a square snapshot of her aunt Edna in a square frame.



Find the area of the frame (the white region in the figure above) if

- a) each side of the snapshot is 7 centimeters long and the outer sides of the frame are each 10 centimeters long.
- b) each side of the snapshot is x centimeters long and the outer sides of the frame are each y centimeters long.



9. Tell whether or not the expressions in each of the following pairs are equal.

a) $(14 + 6) + 1$ and $14 + (6 + 1)$

b) $(14 + 6) - 1$ and $14 + (6 - 1)$

c) $(14 - 6) - 1$ and $14 - (6 - 1)$

d) $(14 - 6) + 1$ and $14 - (6 + 1)$

e) $(10 \cdot 2) \cdot 4$ and $10 \cdot (2 \cdot 4)$

f) $10 + 2 \cdot 4$ and $(10 + 2) \cdot 4$

g) $10 + 2 \cdot 4$ and $10 + (2 \cdot 4)$

h) $\frac{10 - 2}{4}$ and $\frac{(10 - 2)}{4}$

10. Find the value of each of these expressions.

a) $3 \cdot 4^2$

b) $(3 \cdot 4)^2$

c) $8 + 3 \cdot 8 - 3$

d) $(8 + 3) \cdot 8 - 3$

e) $8 + 3 \cdot (8 - 3)$

f) $(8 + 3) \cdot (8 - 3)$

g) $12 - 2 \cdot 5 - 1$

h) $(12 - 2) \cdot (5 - 1)$

i) $12 - (2 \cdot 5 - 1)$

j) $12 - 2 \cdot (5 - 1)$

k) $\frac{32}{4} - \frac{4}{2}$

l) $\frac{32 - 4}{4 - 2}$

m) $\frac{32 \cdot 4}{4 \cdot 2}$

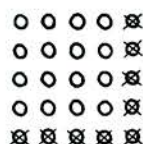
n) $\frac{32 \cdot 4}{4 \cdot 2}$

o) $7^2 - 9 \cdot 2^2$

p) $(7^2 - 9) \cdot 2^2$

q) $(7^2 - 9 \cdot 2)^2$

11. The figure shown here illustrates the expression $(5 - 1)^2$.



Which figure at the top of the next page illustrates each of the expressions in parts a through h?

a) $3^2 - 2^2$

e) $3^2 + 2^2$

b) $(3 - 2)^2$

f) $3(3 + 2)$

c) $3(3 - 2)$

g) $3^2 - 3 \cdot 2$

d) $(3 + 2)^2$

h) $3^2 + 3 \cdot 2$



Figure 1



Figure 2



Figure 3



Figure 4



Figure 5



Figure 6

12. To show that someone is to subtract 2 from x and then multiply the result by 3, we write $3(x - 2)$ or $(x - 2)3$. Write an expression for each of the following sets of operations.

a) Add 11 to x and multiply by y .

b) Multiply 11 by y and then add x .

c) Divide x by 3 and then subtract 1.

d) Subtract 1 from x and then divide by 3.

e) Add x and y and square the result.

f) Add the squares of x and y .

g) Multiply the difference between x and y by x .

h) Cube the product of 2 and x and subtract the result from 9.

i) Subtract the product of 2 and x from 9 and cube the result.

j) Divide the sum of x and y by 5 times y .

13. Find the values of the following expressions for the numbers given.

$x^2 + 4x - 12$

a) if x is 2

b) if x is 4

c) if x is 10

d) if x is 15

$(x + 6)(x - 2)$

e) if x is 2

f) if x is 4

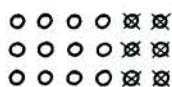
g) if x is 10

h) if x is 15



11. The figure below illustrates the pattern

$$3(6 - 2) = 3(6) - 3(2).$$

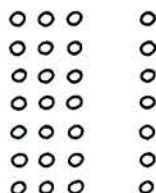


Write a pattern illustrated by each of the following figures.

a)



b)



c)



d)



12. The multiplication problem
- $3x^4$
- and the addition problem
- $x^4 + x^4 + x^4$
- are equivalent. Write a multiplication problem equivalent to each of the following addition problems.

a) $x^2 + x^2 + x^2 + x^2 + x^2 + x^2 + x^2$

b) $5x + 5x + 5x$

c) $(x + 7) + (x + 7)$

d) $(x + y) + (x + y) + \dots + (x + y)$

10 of them

Write an addition problem equivalent to each of the following multiplication problems.

e) $4x^3$

f) $2(7x)$

g) $3(x + 8)$

13. According to the distributive rule,

$5(x + 1) = 5x + 5$. One way to prove this is by writing $5(x + 1)$ as a repeated addition problem and rearranging the numbers being added:

$$\begin{aligned} 5(x + 1) &= (x + 1) + (x + 1) + (x + 1) + (x + 1) + (x + 1) \\ &= x + x + x + x + x + 1 + 1 + 1 + 1 + 1 \\ &= 5x + 5 \end{aligned}$$

Use the same method to prove that

a) $2(x + 6) = 2x + 12$

b) $4(x + y) = 4x + 4y$

c) $3(x^2 + 2) = 3x^2 + 6$

14. Use the distributive rule to write each of the following as a sum or difference.

a) $2(x + 5)$

b) $4(y - 7)$

c) $x(3 + x)$

d) $y(y - 1)$

e) $(x + 8)10$

f) $(6 + x)x$

g) $(y - 4)5$

h) $(x - y)y$

i) $3(x^2 + 9)$

j) $x^2(x - 2)$

15. The way in which you learned to multiply numbers in arithmetic has as its basis the distributive rule. For example, to multiply 62 by 14 we write

$$\begin{array}{r} 62 \\ \times 14 \\ \hline 248 \\ + 620 \\ \hline 868 \end{array}$$

To see how the distributive rule applies, consider the fact that $14 = 4 + 10$ so that

$$14 \cdot 62 = (4 + 10)62 = 4 \cdot 62 + 10 \cdot 62 \\ = 248 + 620 = 868$$

- a) Do the following multiplication problem.

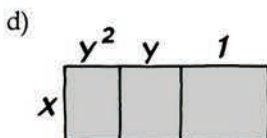
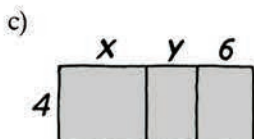
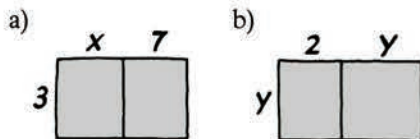
$$\begin{array}{r} 84 \\ \times 21 \\ \hline \end{array}$$

- b) Show, by using the distributive rule, why what you have done is correct.
c) Now do this multiplication problem.

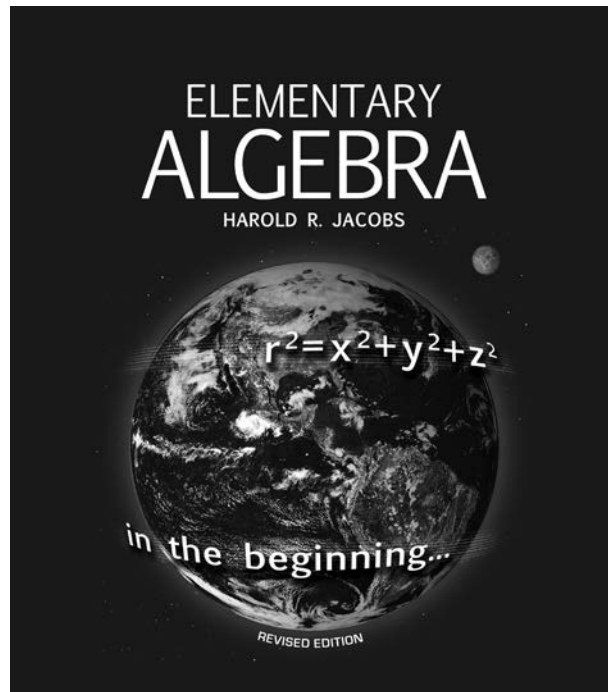
$$\begin{array}{r} 21 \\ \times 84 \\ \hline \end{array}$$

- d) Explain your method by using the distributive rule.

16. Write the total area of each of these rectangles in two different ways.



17. The FOUR N 20 Restaurant sells espresso coffee for \$3.50 a cup. Suppose that one week it sells x cups of the coffee and the next week it sells y cups.
- How many cups does the restaurant sell in all?
 - Write the total amount charged for the coffee during the two weeks as a product.
 - How much did the restaurant charge for the coffee during the first week?
 - How much did the restaurant charge during the second week?
 - Write the total amount charged for the coffee during the two weeks as a sum.



Algebra Tests
for Use with
Elementary Algebra

NOTE: This section also contains the mid-term and final tests. There are extra versions of the chapter tests in the next section of this teacher guide. Special thanks to Molly Crocker for her work on the tests.

Introductory Comments

This book contains chapter tests, a midyear examination, and a final examination that can be used with *Elementary Algebra*. There are four versions, identified by the letters A, B, C and D in each chapter test. The midyear and final examinations consist of eighty problems each; both are four pages long, with twenty problems on each page.

The chapter tests are designed for an examination period of approximately 45 minutes; the midyear and final are designed for an examination period of approximately 110 minutes. Complete answers for all of the tests are in a separate section at the end of this book.



Write another expression equivalent to each of the following.

- $x + x + x$
- $7 \cdot 7 \cdot 7 \cdot 7$
- y^2

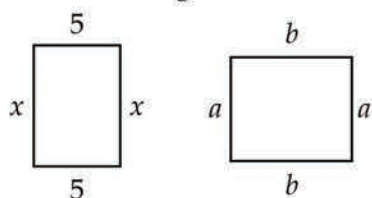
If possible, express each of the following as a power of the number given.

- 10,000,000 as a power of 10.
- 4 as a power of 1.
- 32,768 as a power of 8.

The following problems are about division and zero.

- Does it make sense to divide 0 by 15?
- Explain why or why not.

The perimeter of a rectangle is the sum of the lengths of its sides. The area of a rectangle is the product of its length and width. What is the perimeter and area of each of these rectangles?



- Perimeter of the left rectangle.
- Area of the left rectangle.
- Perimeter of the right rectangle.
- Area of the right rectangle.

Write an expression for each of the following.

- The number x multiplied by 6.
- The difference between 2 and y .
- The fourth power of z .

Which figure below illustrates each of these expressions?

16. $2 \cdot 3^2$

17. $2 + 3^2$

18. $(2 + 3)^2$

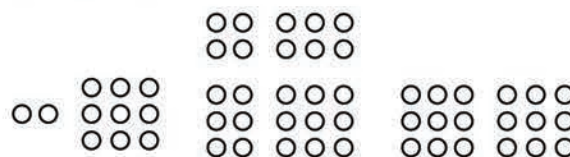


Figure 1

Figure 2

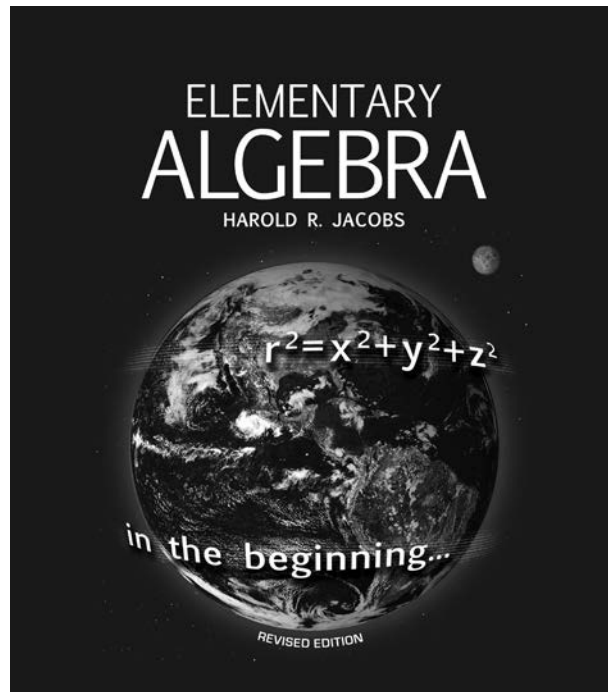
Figure 3

Max and Minnie are taking a test.

- If Max's score is 2 more than the average score, x , how many questions did he answer correctly?
- If there are y questions and Minnie answers 20 of them correctly, how many did she get wrong?

Write an expression for each of the following sets of operations.

- Multiply 5 by 7 and then cube the result.
- Subtract 4 from 12 and then multiply by 6.



Answers for Tests
for Use with
Elementary Algebra

Chapter 1 Test Answers

Problem	Test 1A	Test 1B	Test 1C	Test 1D
1	$3x$	$5 - x$	Figure 3.	$2(1 + 8)$
2	7^4	y^3	Figure 2.	$40 - 6^2$
3	$y \cdot y$	$9z$	Figure 1.	$x - 2$
4	10^7	80	$x + y$	$20 - y$
5	Not possible.	31	x	8^5
6	8^5	24	1000x dollars.	$3y$
7	Yes.	27	$\frac{y}{1000}$	$x \cdot x \cdot x \cdot x \cdot x \cdot x$
8	$\frac{0}{15} = 0$, because $15 \cdot 0 = 0$.	□	$5x$	$3a + 21$
9	$2x + 10$	□□□	y^2	$x^2 - xy$
10	$5x$	□□□ ○○○○○○	$1 - z$	0
11	$2a + 2b$	□○○○	41	y
12	ab	□○○○○	24	$\frac{x}{1000}$
13	$6x$	○○○○	100	1000y dollars.
14	$2 - y$	Four.	33	□
15	z^4	10^5	$45 + 5a$	□□
16	Figure 3.	11^4	$xy - y^2$	□□ ○○○○○○
17	Figure 1.	Not possible.	$20 - x$	□○○○○○
18	Figure 2.	$4(3 + 15)$	$y - 2$	□○○○○○○
19	$x + 2$	$(4 \cdot 6)^2$	$2a + 4$	○○○○○○

Chapter 1 Test Answers

Problem	Test 1A	Test 1B	Test 1C	Test 1D
20	$y - 20$	No.	$2a$	Six.
21	$(5 \cdot 7)^3$	No number times 0 equals 10.	$4x$	$\frac{2}{x}$
22	$6(12 - 4)$	$42 + 6a$	x^2	$9 - y$
23	\square	$cd - d^2$	10^4	z^3
24	$\square \square \square \square$	x	Not possible.	256
25	$\square \square \square \square \begin{matrix} \circ \circ \circ \circ \circ \circ \\ \circ \circ \circ \circ \circ \circ \end{matrix}$	0	9^4	1,024
26	$\square \square \square \square \circ \circ \circ \circ$	Figure 2.	No.	4,096
27	$\square \circ$	Figure 3.	No number times 0 equals 20.	Even.
28	\circ	Figure 1.	$3 \cdot 5$	$2a + 12$
29	One.	$x - 20$	3^4	$6a$
30	$4a + 32$	$y + 2$	$x + x$	$2x + 2y$
31	$c^2 - cd$	243	\square	xy
32	$\frac{y}{1000}$	729	$\square \circ \circ \circ \circ$	9
33	$1000x$ dollars.	2,187	$\square \square \square \begin{matrix} \circ \circ \circ \circ \circ \circ \\ \circ \circ \circ \circ \circ \circ \end{matrix}$	147
34	34	Odd.	$\square \square \square \circ \circ \circ$	53
35	63	$2x + 8$	$\square \circ$	30
36	60	$4x$	\circ	Figure 2.
37	25	$4y$	One.	Figure 1.
38	y	y^2	$24 - 2^3$	Figure 3.
39	$x - y$	$x + x + x$	$3(7 + 5)$	10^8

Chapter 1 Test Answers

Problem	Test 1A	Test 1B	Test 1C	Test 1D
40	x nitrogen atoms and $3x$ hydrogen atoms.	$2 \cdot 8$	216	12^4
41	$x + 3x$	2^5	1,296	Not possible.
42	4	1000 y dollars.	7,776	Yes.
43	$4x$	$\frac{x}{1000}$	Even.	$\frac{0}{25} = 0$, because $25 \cdot 0 = 0$.
44	125	9	16	20
45	625	4	9	12
46	3,125	1	4	6
47	Odd.	x carbon atoms and $2x$ oxygen atoms.	x carbon atoms and $4x$ hydrogen atoms.	$2x$ nitrogen atoms and x oxygen atoms.
48	12	$x + 2x$	$x + 4x$	$2x + x$
49	6	3	5	3
50	2	$3x$	$5x$	$3x$
Extra Credit	$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 \cdot 9$	$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 \cdot 9$	$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 \cdot 9$	$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 \cdot 9$