[JR HIGH – YEAR 2 – STUDENT]

BOOK 2

PRINCIPLES OF MATHEMATICS

BIBLICAL WORLDVIEW CURRICULUM

[Katherine A. Loop]

Cover by Diana Bogardus

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Acknowledgments

I don't believe you would be holding this curriculum in your hands today if it were not for some very special people. It's with much appreciation that I'd like to acknowledge and thank

Have You Completed Book 1?

The book you're holding in your hands builds on the foundational concepts covered in Book 1 of *Principles of Mathematics: Biblical Worldview Curriculum.*

Book 1 covers the core principles of arithmetic and geometry (along with some statistics), while Book 2 builds on those principles as it introduces the core principles of algebra, probability, and trigonometry (along with more statistics).

In Book 1, students learned to find the height of a tree without leaving the ground, use negative numbers to describe the force on objects, explore historical multiplication methods, apply math to music, and much more! In Book 2, students will build on that knowledge so they'll be prepared to understand and see the purpose of algebra and other upper math courses. From exploring genetics to force to sequences in music, they'll continue seeing how math truly is a real-life tool we can use to explore God's creation and serve Him.
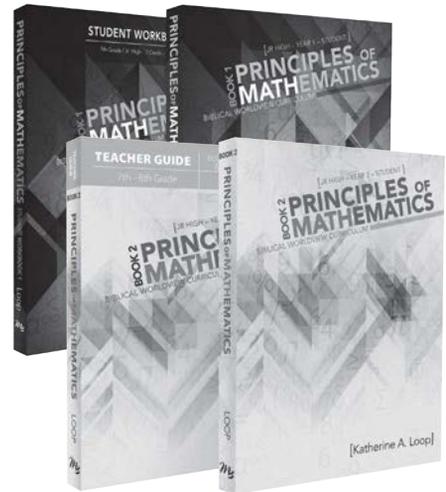
About the Author

Katherine Loop is a homeschool graduate from northern Virginia. Understanding the biblical worldview in math made a tremendous difference in her life and started her on a journey of researching and sharing on the topic. For over a decade now, she's been researching, writing, and speaking on math, along with other topics. Her books on math and a biblical worldview have been used by various Christian colleges, homeschool groups, and individuals.

Website and Blog: www.ChristianPerspective.net
(Check out the free e-newsletter.)

Facebook: https://www.facebook.com/katherinealoop
(Join the conversation!)

YouTube: https://www.youtube.com/user/mathisnotneutral

About This Curriculum

This is Book 2 of a two-book math course designed to give students a firm mathematical foundation, both academically and spiritually. Not only does the curriculum build mathematical thinking and problem-solving skills, it also shows students how a biblical worldview affects our approach to math's various concepts. Students learn to see math, not as an academic exercise, but as a way of exploring and describing consistencies God created and sustains. The worldview is not just an addition to the curriculum, but is the starting point. Science, history, and real life are integrated throughout.

How Does a Biblical Worldview Apply to Math . . . and Why Does It Matter?

Please see Lesson 1.1 for a brief introduction to how a biblical worldview applies to math and why it matters.

Who Is This Curriculum For?

This curriculum is aimed at grades 6–8, fitting into most math approaches the year or two years prior to starting high school algebra. If following traditional grade levels, Book 1 should be completed in grade 6 or 7, and Book 2 (the book in your hands) in grade 7 or 8.

The curriculum also works well for high school students looking to firm up math's foundational concepts and grasp how a biblical worldview applies to math. High school students may want to follow the alternate accelerated schedule provided in the *Teacher Guide* and complete each year of the program in a semester, or use the material alongside a high school course.

Where Do I Go Upon Completion?

Upon completion of Book 1, students will be ready to move on to Book 2. Upon completion of both years, students should be prepared to begin or return to any high school algebra course.

Are There Any Prerequisites?

Book 1: Students should have a **basic knowledge of arithmetic** (basic arithmetic will be reviewed, but at a fast pace and while teaching problem-solving skills and a biblical worldview of math) and **sufficient mental development** to think through the concepts and examples given. Typically, anyone in 6th grade or higher should be prepared to begin.
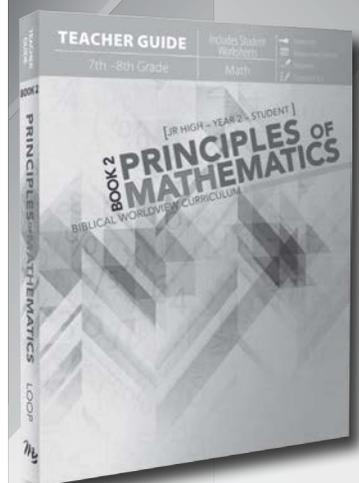
Teacher Guide

Book 2: It is strongly recommended that students complete Book 1 before beginning Book 2, as math builds on itself, and the principles of arithmetic and geometry are essential for understanding the principles covered in Book 2. Before beginning Book 2, students need to be comfortable with basic math skills (including rounding and working with decimals), fractions, unit conversion, negative numbers, geometry formulas (finding perimeter, area, and volume), and exponents; they also need to have problem-solving skills.

What Are the Curriculum's Components?

The curriculum consists of the ***Student Textbook*** and the ***Teacher Guide***. The *Student Textbook* contains the lessons, and the *Teacher Guide* contains all the worksheets, quizzes, and tests, along with an Answer Key and suggested schedule.

How Do I Use This Curriculum?

General Structure — This curriculum is designed to be self-taught, so students should be able to read the material and complete assignments on their own, with a parent or teacher available for questions. If teaching in a classroom, the text can serve as the basis for the teacher's presentations. This *Student Textbook* is divided into chapters and then into lessons. The number system used to label the lessons expresses this order. The first lesson is labeled 1.1 because it is Chapter 1, Lesson 1.

Worksheets, Quizzes, and Tests — The accompanying *Teacher Guide* includes worksheets, quizzes, and tests to go along with the material in this book, along with a suggested schedule and answer key.

Answer Key — A complete Answer Key is located in the *Teacher Guide*.

Schedule — A suggested schedule for completing the material in one year, along with an accelerated one-semester schedule, is located in the *Teacher Guide*.

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Preface

In Book 1 of *Principles of Mathematics*, we explored together the core concepts of arithmetic and geometry, seeing how math's very existence declares God's praises, and how it serves as a real-life tool. While in that book we looked mainly *backwards* at aspects of math with which you were probably already familiar, in this course we'll be looking *ahead* at concepts into which you'll dig into more depth in future math courses. As we do, we'll see that all of these new concepts, like the ones we looked at in the last book, work because of God's faithfulness and are tools we can use to serve Him.

Researching for this second book has been a fun yet stretching process for me, as it has forced me to revisit many concepts that I saw as meaningless when I first learned them. The more I continued to learn and research, though, the more amazed I grew at how useful different concepts are in describing the intricacies of God's creation. I've joked that you know you're writing a math curriculum when you begin to "see" math everywhere...which for me was quite a switch.

You see, I didn't grow up loving math or seeing it as a way of describing God's creation. As I mentioned in the preface to Book 1, while I was good at math, I viewed it as a subject of rules to be memorized, applied, and forgotten. It wasn't until I read *Mathematics: Is God Silent?* by James D. Nickel as a senior in high school that I realized that math wasn't neutral or confined to a textbook—it was a way of describing God's creation that loudly proclaimed the Creator's praises.

Seeing the biblical worldview in math was transformational for me—so much so that I wanted to share it with everyone I could. It led to more than a decade of research on the topic, several books on teaching math from a biblical worldview, and lastly this curriculum series.

It's been such a joy to hear from users of Book 1 and my previous books about how they have blessed them. Hearing that one mom found her child reading extra lessons on her own, watching students' eyes light up at conventions when they realize math really is more than apparently meaningless bookwork, finding out that a biblical worldview of math encouraged a mom to overcome her fears of the

subject, and listening as a longtime teacher shared how excited she was to finally know how to teach math biblically has warmed my heart.

My earnest hope and prayer is that God will use this curriculum as well to bless readers. While the curriculum is labeled for junior high, it really applies to anyone of any age who wants to really "get" what math is all about from a biblical worldview. Whatever you're age and mathematical leaning (or lack thereof), I hope you'll find the material friendly and helpful. I hope you will find it a fascinating journey of discovery, helping you discover God's handiwork in math. Above all, I hope you'll leave encouraged that God can be trusted completely.

Writing this curriculum has been quite a journey for me, both in exploring math to a deeper level and of watching God faithfully provide. There were times I never thought the curriculum would actually reach completion, but it has. The acknowledgements don't begin to describe how blessed I was to be supported by a lot of talented people who were willing to let me hound them with questions, read countless drafts, work with me on refining and formatting concepts, check changes, and pray for and encourage me.

May the One who holds all creation together by the Word of His power (Hebrews 1:3) be honored and glorified in every page.

By God's Grace,

Katherine Loop

The Big Picture

1.1 Overview of Mathematics

You probably learned to write numbers and count at a young age. At some point, you mastered the basic operations: addition, subtraction, multiplication, and division. Then came fractions, ratios, and percentages. Along the way, you learned to recognize different shapes, measure them, and find their perimeter, area, and volume.

$$86\%$$

$$1 + 1 = 2$$

$$8 - 2 = 6$$

$$928.32$$

$$88 \div 2 = 44$$

$$5\frac{1}{2} + 10\frac{1}{2} = 16$$

We've become so familiar with math that it's easy to lose sight of math's purpose. What does $1 + 1 = 2$ really mean? Why does $1 + 1$ equal 2? Why do we have fractions, ratios, and percentages? What's the purpose of exploring shapes?

Before we jump into new concepts, we are going to pause and look at math's overall purpose — at the "big picture," so to speak. First, though, we need to talk about worldviews, as our worldview determines how we see both the "big picture" and the details of mathematics.

Worldview Matters

Whether we realize it or not, our perspective on any area of life is determined by what is called a worldview. In *Understanding the Times*, David Noebel (founder of Summit Ministries) defines a worldview this way: "A worldview is like a pair of glasses — it is something through which you view everything. And the fact is,

everyone has a worldview, a way he or she looks at the world."[1] In other words, a worldview is a set of truths (or falsehoods we believe to be true) through which we interpret life.

As we look at the "big picture" of math and study math together, we'll be seeking to start with the Bible — God's revealed Word to man — as our source of truth, using it as the "glasses" through which we look at math. We'll discover that the Bible gives us solid answers to where math originated, why math is possible, what we should expect as we use math, and how we should use math. With that in mind, let's begin looking at the "big picture" of math.

Where Did Math Originate?

Have you ever stopped to think about where math originated? How did we get math? Did men invent it? Has it just always been there?

To understand *where* math came from, we have to first consider *what* math really is. Is math just a collection of facts in a textbook? While it may feel that way at times, math is hardly confined to a textbook. The main reason math is such an essential subject is because it applies outside of a textbook.

Take a moment to think about how many times you see numbers each day. Price tags, exit signs, clocks, calendars, addresses, expiration dates, nutritional data, Bible verses, speed-limit signs, rulers, radio dials, TV remotes, thermometers, store hours — we see numbers all the time.

Every occupation utilizes math to some degree or another. Scheduling airplane flights, generating computer graphics, managing businesses, designing bridges and buildings, laying out roads, producing digital sounds — all of these tasks require math. Even non-mathematical fields employ math. Artists deal with proportions, musicians work with a musical notation based on fractions, airplane pilots compute a weight and balance to make sure their plane will handle the cargo, veterinarians have to know how much medicine to give — math proves useful on the job, whatever that job might be.

Math is also used behind the scenes to develop or produce the technology and resources we use on a daily basis. Computers, cars, ovens, medical treatments, airplanes, cell phones, toys, GPS units — name something you use, and chances are math helped in developing or producing it. Pick a topic from science class — be it the distance to the stars or how an atom works — and most likely math has been used in its exploration.

Departures			17:59
DESTINATION	BOARDING TIME	GATE	STATUS
DUBAI	17:35	15	FINAL CALL
SYDNEY	17:50	04	BOARDING
SYDNEY	18:05	16	BOARDING IN 5 MINS
DUBAI	18:10	02	BOARDING IN 10 MINS
BUENOS AIRES	18:25	07	BOARDING IN 25 MINS
RAROTONGA	18:35	01	BOARDING IN 35 MINS
LOS ANGELES		06	BOARDING IN 35 MINS
SAN FRANCISCO	:50		BOARDING IN 50 MINS

All that to say, math is clearly more than a textbook exercise. **Math** is a way of describing the quantities and consistencies all around us — quantities and consistencies the Bible tells us the triune God created and sustains.

> *In the beginning God created the heaven and the earth. And the earth was without form, and void; and darkness was upon the face of the deep. And the Spirit of God moved upon the face of the waters (Genesis 1:1–2).*

> *In the beginning was the Word, and the Word was with God, and the Word was God. The same was in the beginning with God. All things were made by him; and without him was not any thing made that was made. . . . And the Word was made flesh, and dwelt among us, (and we beheld his glory, the glory as of the only begotten of the Father,) full of grace and truth (John 1:1–3, 14).*

> *Jesus answered them. . . . I and my Father are one (John 10:25, 30).*

> *For by him [Jesus] were all things created, that are in heaven, and that are in earth, visible and invisible, whether they be thrones, or dominions, or principalities, or powers: all things were created by him, and for him: And he is before all things, and by him all things consist (Colossians 1:16–17).*

> *[referring to Jesus] . . . upholding all things by the word of his power . . . (Hebrews 1:3).*

In other words, while men (using their God-given abilities) have developed the different symbols and techniques used in math over the years, the underlying principles those symbols and techniques describe are principles God created and sustains. In fact, as we'll see next, it's God's faithfulness that makes math possible.

Ignoring the Question

The fundamental question of why math works is one that has baffled secular philosophers for years, as it doesn't make sense apart from a biblical worldview. Consider this quote: "Some of the most abstruse concepts of mathematics have an uncanny way of becoming essential tools in physics. Many physicists have been so impressed by the usefulness of mathematics that they have attributed to it almost mystical power. . . . In this article I shall not attempt any deep philosophical discussion of the reasons why mathematics supplies so much power to physics. . . . The vast majority of working scientists, myself included, find comfort in the words of the French mathematician Henri Lebesgue: 'In my opinion a mathematician, in so far as he is a mathematician, need not preoccupy himself with philosophy.'"[2] In other words, this scientist is saying that he's going to avoid the fundamental question of why math works and just use it. As Christians, we don't have to ignore the question — the Bible gives us an answer.
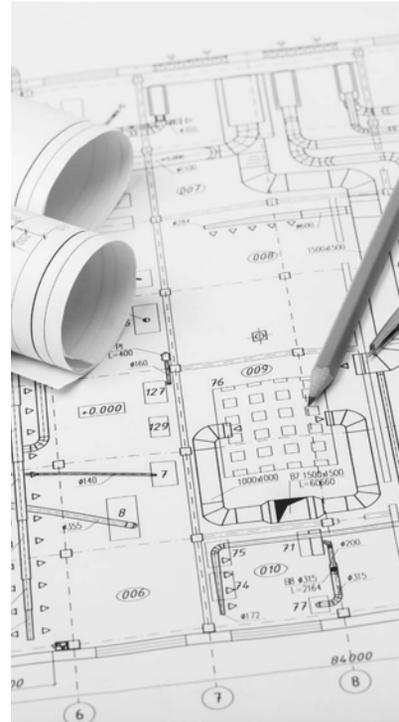
Why Is Math Possible?

People who love math often say they love it because it's always predictable. You can count on 1 + 1 to equal 2. There's no subjectivity to math — it works the way it does.

Why is this? Again, the Bible gives us the answer. In Jeremiah 33:25, God tells us that He is keeping His covenant with the "ordinances" of heaven and earth.

> *Thus saith the LORD; If my covenant be not with day and night, and if I have not appointed the ordinances of heaven and earth;*

Day in and day out, our faithful, consistent Creator is holding all things together in such a predictable and reliable way that we can use math to describe it. Math is reliable *because* God is reliable. In fact, God uses the "ordinances" of heaven and earth to remind the Israelites that He'll be that reliable to His covenant with them.

> *Then will I cast away the seed of Jacob and David my servant, so that I will not take any of his seed to be rulers over the seed of Abraham, Isaac, and Jacob: for I will cause their captivity to return, and have mercy on them (Jeremiah 33:26).*

Water Droplets and Addition

It's important to note that when we say the universe is consistent, we don't mean that it all operates exactly the same way. For example, one water droplet plus another water droplet does *not* equal two water droplets — instead, they merge to form one larger droplet. The apparent contradiction in how liquids combine reminds us that, no matter how well we think we have things figured out, God's laws and universe are more complex than we can imagine. God has different, though equally consistent, principles for governing liquids at the visible level than He does solids.

Note, though, that while we visibly see one larger droplet instead of two, that is not what occurs on the atomic level. At the atomic level, the starting atoms plus the final atoms add together following the rules of regular addition — if we started with 1 atom and added 1 more atom, the ending larger droplet would have 2 atoms.

Men have often sought to make either experience or man's intellect the foundation for the underlying truth in math. Yet neither of these foundations hold up to scrutiny. Raindrops baffle those who would say truth in math is determined solely intellectually,[3] as it's experience that tells us one raindrop plus another equals a larger one. Yet experience can't be the source of truth either, as we use concepts in math we've never experienced (such as infinite numbers). In a biblical worldview, though, we have a solid answer for why we're able to learn both experientially and intellectually — God both created the world and gave us the ability to explore it.

The Bible also gives us a reason for understanding why we're able to discern and record the orderly world God created. It tells us God created man in His image and gave man dominion over the earth.

> *And God said, Let us make man in our image, after our likeness: and let them have dominion over the fish of the sea, and over the fowl of the air, and over the cattle, and over all the earth, and over every creeping thing that creepeth upon the earth. So God created man in his own image, in the image of God created he him; male and female created he them (Genesis 1:26–27).*

Thus we're capable of exploring the universe and developing different symbols and techniques to describe it.

What Should We Expect as We Use Math?

Not only does the Bible give us principles that explain where math came from and why it works, but it also helps us know what to expect as we use math to describe the quantities and consistencies around us. The Bible tells us that God created a perfect world, but sin marred that world.

> *And God saw every thing that he had made, and, behold, it was very good. . . . (Genesis 1:31)*

> *Wherefore, as by one man sin entered into the world, and death by sin; and so death passed upon all men, for that all have sinned: (Romans 5:12).*

Thus we should expect to find evidence of design (pointing us to the wisdom and care of the Creator), yet at the same time a world marred from that perfect order (reminding us of the Fall). And guess what? That is exactly what we find.

For a simple example, we can observe that cats have four legs. Math helps us describe this aspect of the design God placed in cats. Yet if you were to look at enough cats, you'd eventually find a cat that was missing a leg, as we live in a fallen world.

How Should We Use Math?

The Bible tells us to do all that we do in the name of Jesus and with thanksgiving to God.

> *And whatsoever ye do in word or deed, do all in the name of the Lord Jesus, giving thanks to God and the Father by him (Colossians 3:17).*

Notice that this verse doesn't say, "and whatsoever ye do *but math*." It says "and whatsoever ye do." Math is not exempted. Within math, as in every other area of life, we have an opportunity to depend on and praise God. We can also use math, along with other skills we learn, to help us in the tasks God has given us to do.

Term Time
Here are a few terms we'll use extensively as we explore the principles of mathematics together. Be sure to familiarize yourself with them before we get started.
Expression — An expression is "a collection of symbols that jointly express a quantity."[4] For example, 4 + 5 is an expression — 4, 5, and + are a collection of symbols that together express the quantity 9.
Equation — An equation is "a statement that the values of two mathematical expressions are equal (indicated by the sign =)."[5]
Simplify — When we refer to simplifying an expression or equation, we mean to express it as simply as possible. For example, 5 + 6 simplifies to 11.

Keeping Perspective

Wow — that's quite an amazing "big picture," isn't it? Think about it for a minute. Every time you use math and see that it works it's reminding you that you can trust God. Math's an exploration of God's handiwork — a tool we can use to explore His creation while worshiping the Creator.

I hope you're excited about digging deeper into math this year. As we do, we'll see over and over again how math helps us describe the quantities and consistencies God created and sustains. We'll learn how to use concepts outside a textbook, applying what we learn to help us with the tasks God's given us to do. Above all, we'll reflect on what a faithful, wise, infinite Creator we serve — a Creator worthy of all our trust.

1.2 The Language of Mathematics — Symbols and Conventions

Before we jump into the details of math, we're going to take the rest of this chapter to continue building the "big picture." In this lesson, we're going to look at the language side of mathematics.

Many aspects of math can be thought of as a language system. Mathematical symbols, notations, and conventions are like a language system to communicate about the quantities and consistencies God created and sustains.

Symbols

For years, you've been using symbols to **name** quantities. For example, you might use "5" to represent five cups of flour, five inches, or five additional CDs. You've also used symbols to **identify** (the numbers in a telephone number identify a specific telephone line) and to **order** (such as the 5th most popular song of the year).

Sample from Chapter 3

$$ending\ \$ = beginning\ \$ + \$\ gained - \$\ spent$$

$$e = b + g - s$$

Understanding equalities is going to help us better understand how to work with many real-life relationships. In fact, in the next lesson, we'll begin using the principle of equality to help us solve problems containing unknowns.

Keeping Perspective

God's faithfulness ensures that objects will consistently add, subtract, multiply, and divide. Thus, **when we add, subtract, multiply, or divide *both sides* of an equation by the *same quantity*, the sides of the equation remain equal.** And since equal means equal, **we can swap the *entire* expression on the left of an equal sign with the *entire* expression on the right, or vice versa.**

As you solve the problems on Worksheet 3.2, ponder God's faithfulness. Just as He is faithful to keep His covenant with the "ordinances" of heaven and earth (making it possible for us to rely on addition, subtraction, multiplication, and division to work consistently), He will be faithful to everything else He says in His Word. He will be faithful to save . . . and faithful to punish.

> *But the Lord is faithful, who shall stablish you, and keep you from evil (2 Thessalonians 3:3).*

3.3 Equalities and Unknowns

It's time now to look at equalities that include unknowns. We're going to use the *same principle* of adding, subtracting, multiplying, or dividing *both sides* of an equation by the *same quantity* to find the value of an unknown.

Let's say we are trying to save $19 for a purchase, and we've saved $13 so far. How much do we have left to save?

While this is a problem you could solve mentally, let's look at it on a scale for a minute. We know that $13 plus some quantity (which we'll represent with an *x*) equals $19.

$$\$13 + x \qquad = \qquad \$19$$
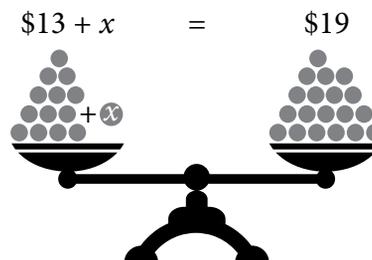
So the question is, how do we find the value of *x*? Let's take a look.

Finding an Unknown

Remember, if we add, subtract, multiply, or divide *both sides* of an equation by the *same quantity*, both sides will stay equal. So if we subtract $13 from both sides, we'll be left with *x* on the left side. Since both sides of our equation will still be equal, the right side will then show us what *x* equals!

Subtracted $13 from Both Sides

−13   −13
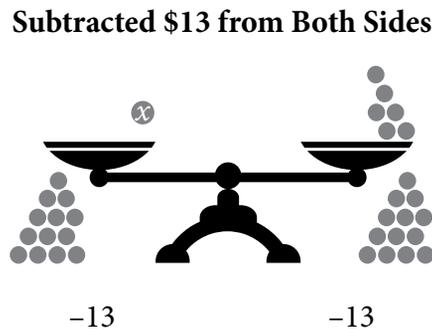
We now know that the value of *x* is $6. Note that if we substitute this value back into our original equation, it holds true. $13 + $6 does indeed equal $19.

Value We Found for *x* Inserted into the Original Equation

$13 + $6   = $19
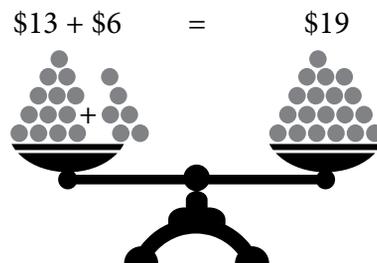
Now, I hope right now you're pausing for a minute and going *wow*. Because of the consistent way God holds this universe together, we can rely on operations to operate (pun intended) so consistently that we can subtract the same quantity from both sides of an equation and be so confident that the answer will still be in balance that we can use that answer to tell us the value of an unknown!

This technique of separating an unknown on one side of an equation in order to find its value is one we're going to be using extensively. While we illustrated it using subtraction, since that was the easiest to show visually on a scale, the same principle applies with other operations.

> To find an unknown in an equation, **get the unknown on a side by itself.** Do this by performing the *same operation* using the *same quantity* to *both sides* of the equation.

While you can find many unknowns mentally, knowing how to find an unknown by isolating it on one side of an equation will help you solve problems that would

otherwise be difficult or impossible. It's a technique you will use frequently — and one that ultimately works because of the consistent way God causes objects to add, subtract, multiply, and divide.

Looking at the Details

I'm sure you'll agree that it's not practical to draw a scale every time we want to find the value of an unknown — nor is it necessary. The scale was simply a visual to show the process. Let's now walk through finding a couple of unknowns on paper.

Example: $x + \$13 = \$19$

> To find the value for x, we want to isolate x on a side by itself. We know *x plus $13* equals $19. So if we *subtract $13* from both sides, we'll end up finding the value for x.
>
> $$x + \$13 - \mathbf{\$13} = \$19 - \mathbf{\$13}$$
>
> Now let's simplify. $13 – $13 equals 0, leaving us $x + 0$, or more simply, x on the left. And $19 – $13 equals $6, leaving us $6 on the right.
>
> $$x = \mathbf{\$6}$$
>
> *Note: We could have used cross outs to show how subtracting $13 canceled out the + $13.*
>
> $$x + \cancel{\$13 - \mathbf{\$13}} = \$19 - \mathbf{\$13}$$
>
> $$x = \mathbf{\$6}$$
>
> Now we can check our work by substituting $6 for x in the original equation. If we found the correct value, the equation will hold true.
>
> Original equation:
>
> $$x + \$13 = \$19$$
>
> Substituting $6 for x:
>
> $$\mathbf{\$6} + \$13 = \$19$$
>
> This is true . . . $6 + $13 does equal $19. We did the math correctly.
>
> We just did on paper the same thing we'd shown before on a scale.

Starting Equation			Subtracted $13 from Both Sides		
$\$13 + x$	=	$19	x	=	$6

| = | | | – $13 | – $13 |
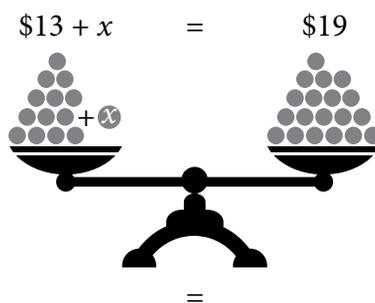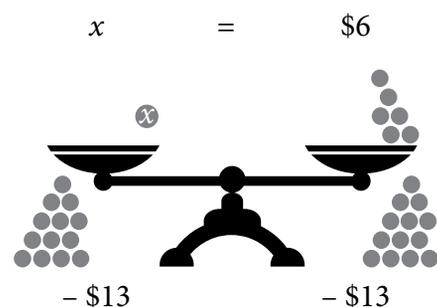
Understanding the Why

While it was easy on the scale to see how subtracting $13 from both sides found the value for *x*, some of you might be wisely wondering how that works mathematically. After all, the order of operations says to add and subtract from left to right. So in *x* + $13 – $13 we should add *x* and $13 first and *then* subtract $13, right?

True . . . except remember that we can think of subtraction as an addition of a negative number . . . and that addition is commutative and associative. Even though we didn't write out the plus sign (and many people don't), **we were really viewing the subtraction of $13 as an addition of –$13.** Thus we were able to add $13 and –$13, which equals 0.

$$x + \$13 + -13$$

$$x + 0$$

You might also wonder how we knew mathematically what number to subtract. Well, as we saw back in Lesson 2.4, the additive inverse is the name we give to the number that, added to another, equals 0. We know that +$13 and –$13 are **additive inverses**. So by subtracting $13 (which we can think of as adding –$13) we got 0. And since adding 0 doesn't change the value of a number (there's the **identity property of addition**), we knew adding +$13 and –$13 would leave us with *x*.

$$x + 0 \text{ simplifies to } x$$

The properties and principles we looked at in the first two chapters are ones we'll rely on again and again. And why can we rely on these properties and principles to work? Because God is governing all things in a consistent, predictable way. Once again, our ability to use math ultimately goes back to God's faithfulness.

Example: $x - 2 = 4$

To find the value for *x*, we want to isolate *x* on a side by itself. We know *x minus 2* equals 4. So if we *add 2* to both sides, we'll end up finding the value for *x*.

$$x - 2 + \mathbf{2} = 4 + \mathbf{2}$$

Now, let's simplify. We know that –2 and + 2 are additive inverses, so they will equal 0. And since adding 0 doesn't change the value of a number, our left-hand side will simplify to *x*. On the right, we'll have 4 + 2, or 6.

$$x = 6$$

Note: We could have used cross outs to show how adding 2 canceled out the subtraction of 2.

$$x \cancel{- 2 + 2} = 4 + 2$$

$$x = 6$$

Now we can check our work by substituting 6 for *x* in the original equation. If we found the correct value, the equation will hold true.

Original equation:

$$x - 2 = 4$$

Substituting 6 for x:

$$\mathbf{6} - 2 = 4$$

This is true . . . 6 – 2 does equal 4. We did the math correctly.

Keeping Perspective

If we perform the same operation (using the same quantity) to both sides of an equation, the sides remain equal. Why? Because God holds creation together in a consistent, predictable enough fashion that we can rely on the fact that if we perform the *same operation* using the *same quantity* to both sides of an equation, they'll stay equal.

3.4 More with Equalities and Unknowns

In the last lesson, we saw that we can find an unknown in an equation by getting the unknown on a side by itself. We do this by performing the *same operation* using the *same quantity* to *both sides* of the equation.

While all of our examples in the last lesson were of problems involving addition and subtraction, in real life, we'll encounter problems that require using other operations too. In this lesson, let's take a look at examples involving multiplication and division.

Example: $2 \cdot x = 8$

> We'll start by dividing *both sides* by 2 (the *same quantity*). We'll show this using a fraction line.
>
> $$\frac{2 \cdot x}{2} = \frac{8}{2}$$
>
> Now, since we only have multiplication in the numerator, we can simplify $\frac{2 \cdot x}{2}$ the same way we have been simplifying fractions. In this case, we'll divide both the numerator and the denominator by 2, leaving us with x equaling 4.
>
> $$\frac{\cancel{2} \cdot x}{\cancel{2}} = \frac{\cancel{8}^{4}}{\cancel{2}}$$
>
> $$x = 4$$
>
> Now we can check our work by substituting 4 for x in the original equation. If we found the correct value, the equation will hold true.

$\frac{2 \cdot x}{2} = \frac{8}{2}$ is another way of writing $(2 \cdot x) \div 2 = 8 \div 2$. Both the fraction line and the $\div$ symbol show division.

Sample from Chapter 5

$$d = \frac{40 \text{ mi}}{60 \text{ min}} \cdot 10 \text{ min}$$

Now the units will cross out and we can simplify to find the value of *d*.

$$d = \frac{{}^{2}40 \text{ mi}}{{}_{3}60 \text{ min}} \cdot 10 \text{ min}$$

$$d = \frac{20 \text{ mi}}{3}$$

$$d = 6.67 \text{ mi}$$

Note: We could have converted the minutes to hours instead — it just would have been a longer process.

Convert the minutes:

$$10 \text{ min} \cdot \frac{1 \text{ hr}}{60 \text{ min}} = \frac{1}{6} \text{ hr}$$

Solve, using $\frac{1}{6}$ hr instead of 10 min for *t*:

$$d = s \cdot t$$

$$d = \frac{40 \text{ mi}}{1 \text{ hr}} \cdot \frac{1}{6} \text{ hr}$$

$$d = \frac{40 \text{ mi}}{1 \text{ hr}} \cdot \frac{1}{6} \text{ hr}$$

$$d = 6.67 \text{ mi}$$

Important Note

If we hadn't written the units out when solving the example problem, we might not have realized the need to convert, leaving us with a terribly wrong answer! Including units is very important — especially in more complicated problems where mistakes are harder to spot.

Keeping Perspective

We all deal with time all the "time" . . . and math can help us find the answers we need in a "timely" way! As you explore time today, remember that math's very existence reminds you that you serve a God you can trust all the time. (And with that, it's "time" to stop making time puns.)

5.4 Proportions and the Pressure and Volume of a Gas

Let's take a quick look at another application of proportions found in God's creation. When a gas is forced into a smaller volume, it exerts more pressure. The smaller the volume, the greater the pressure the gas exerts. The greater the volume,

the less pressure the gas exerts (after all, the gas molecules have a larger volume in which to float around).

Since this relationship is so constant, we can set up a proportion comparing the pressure and volume of the *same gas* at two different times. Assuming the gas's temperature doesn't change, the pressure and volume at the first measurement will relate to the final pressure and volume of the next measurement like this:

$$\frac{P_1}{P_2} = \frac{V_2}{V_1}$$

> **Subscript Numbers**
>
> In the formula $\frac{P_1}{P_2} = \frac{V_2}{V_1}$, P_1 and V_1 stand for the pressure and volume of the gas at the first measurement, while P_2 and V_2 stand for the pressure and volume of the same gas at the second measurement. **The subscripts $_1$ and $_2$ differentiate which pressure and volume we mean.** Using subscripts to differentiate between different pressures, volumes, distances, velocities, etc., is a common convention that you'll encounter.

We can use this proportion — which is a mathematical expression of a consistency God holds in place — to help us solve problems. As we do, we'll also employ cross multiplication.

Example: If a toy balloon contains 2.5 quarts of gas (V_1) and exerts a pressure of 1 atmosphere (P_1), to what size will it shrink if its pressure changes to 5 atmospheres (P_2), assuming the temperature stays the same?

This example is based on one given in *Secondary Arithmetic*, an early 1900s math book.[2]

What do we know? We've been told our initial volume (V_1) is 2.5 quarts, and our initial pressure (P_1) is 1 atmosphere. We want to know the final volume (V_2) if the pressure changes to 5 atmospheres.

Now we can substitute these values into the proportion. We'll use the abbreviation atm for atmosphere and qt for quarts.

$$\frac{\textbf{1 atm}}{\textbf{5 atm}} = \frac{V_2}{\textbf{2.5 qt}}$$

Cross multiply:

$$\frac{\text{1 atm}}{\text{5 atm}} \diagup\!\!\!\!\diagdown \frac{V_2}{\text{2.5 qt}}$$

$$1 \text{ atm} \bullet 2.5 \text{ qt} = 5 \text{ atm} \bullet V_2$$

Simplify:

$$\textbf{2.5 atm} \bullet \textbf{qt} = 5 \text{ atm} \bullet V_2$$

Solve for the unknown by dividing both sides by 5 atm:

$$\frac{\text{2.5 atm} \bullet \text{qt}}{\text{5 atm}} = \frac{\text{5 atm} \bullet V_2}{\text{5 atm}}$$

$$\frac{\textbf{2.5 qt}}{\textbf{5}} = V_2$$

$$\textbf{0.5 qt} = V_2$$

Tada! We have our answer.

Since our purpose is to show math's usefulness rather than to explore pressure and volume, we only touched on the relationship between pressure and volume of gases. It's a relationship you'll probably explore more in science . . . only now you have a taste for how math aids us in exploring it! While science books don't always explain the math behind the concepts they teach, math applies throughout science.

Keeping Perspective

The beginning of a physics book from 1910 points out the reliance of science upon the unfailing regularity of creation.

> *The sun set last night and rose again this morning, and we are sure that it will set again to-night and rise again to-morrow morning. In fact, from all our experiences with Nature we have learned that she always acts in such a perfectly regular way that we can predict what will happen under a given set of circumstances.*[3]

This physics book is making the case that science depends on the regularity all around us — a regularity that is as constant as the sun's rising and setting. It's this regularity that lets us record the relationship between the pressure and volume of a gas and expect it to hold true, no matter the actual values.

Now, we could argue with the book in that some aspects of creation are so incredibly complex that we can't predict them (for example, we can't predict an electron's exact location in an atom). We could also point out that there are often more details involved than a formula takes into account (for instance, more than pressure and volume may affect a gas). Nevertheless, the principle that we live in an amazingly consistent, predictable universe holds true. All of modern science is based upon the fact that creation is consistent.

The Bible tells us it's God who holds this amazing regularity together. From the rising of the sun to the way the volume of a gas affects its pressure to the way speed and time relate, He governs things reliably. The fact that we can reduce things such as pressure and volume to a math equation emphasizes once again just how reliable creation is . . . and in turn how reliable the One holding creation together is. You can rely on God absolutely.

> *The heavens declare the glory of God; and the firmament sheweth his handywork. Day unto day uttereth speech, and night unto night sheweth knowledge. There is no speech nor language, where their voice is not heard (Psalm 19:1–3).*

Sample from Chapter 6

Example: Solve $F = m \bullet a$ for m.

We can solve for m (i.e., find the mass) by dividing both sides by the acceleration.

$$\frac{F}{a} = \frac{m \bullet a}{a}$$

$$\frac{F}{a} = \frac{m \bullet \cancel{a}}{\cancel{a}}$$

$$\frac{F}{a} = m$$

We now have rearranged the formula so it's easy to see how the mass relates to the force and the acceleration. What if we wanted to know how the acceleration relates to the force and mass? We could rearrange the formula so that a was on a side of the equation by itself. In fact, you'll get a chance to do just that when you complete Worksheet 6.4.

Keeping Perspective

Stop and ponder the miracle of $F = m \bullet a$ for a moment. God governs this universe so consistently that we can mathematically record and predict how much force will be needed to move an object based on its mass. We can reduce movement to a formula and apply it to thousands of situations, knowing it will work because *God is a faithful God.*

Just as God is faithful to hold the universe together, He is faithful to His promises to you too. Reflect today on how God has kept His Word to you and to those around you.

6.5 Chapter Synopsis and Sir Isaac Newton

We sure covered a lot of ground in this chapter, didn't we? We started with revisiting **percents** (a shorthand way to express ratios as fractions of 100). We then reviewed how to find a **percentage**, and then eventually learned how to find other missing numbers in a percent problem (such as the **base** or the **rate**). We practiced the same skill while exploring force and motion.

One key point to take away from this chapter is that, because of God's faithfulness in holding all things together, **we can rearrange formulas as needed to find missing information.** It doesn't matter if we know the amount we're adding, subtracting, multiplying, or dividing both sides of the equation by, or whether we're dealing with all unknowns — as long as we change both sides of the equation the same way, we know our equation will be in balance. After all, whatever number it is, God is keeping this universe together in a consistent way, and it will add, subtract, multiply, or divide the same way on both sides of the equation.

Original Equation			Added y to Both Sides		
x	$=$	x	$x + y$	$=$	$x + y$
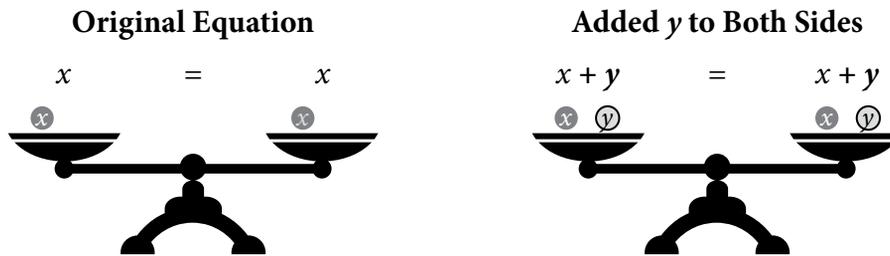
*Our equation is still in balance, as y — whatever its value — is being added to **both sides**.*

Sir Isaac Newton

Since we looked at force and motion a little in the last lesson, let's take a look at the man who discovered the laws of motion. Gifted with an incredible ability to apply math practically, Sir Isaac Newton, commonly referred to as Newton, clearly saw this universe as a complex machine created and held together by God.

Newton concludes his work the *Principia* (in which he explained the law of gravity) with several pages of reflections on God. He stated,

> *This most beautiful system of the sun, planets, and comets, could only proceed from the counsel and dominion of an intelligent and powerful Being.[2]*
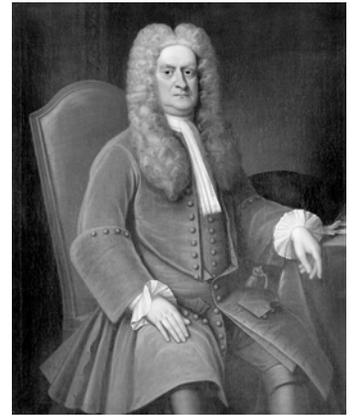
Sir Isaac Newton (1642–1727)[3]

Newton's life serves as a good rebuttal to the belief so prevalent in our culture that a biblical worldview is incompatible with science and progress. Although Newton's theology was not perfect, he clung steadfastly to the Lord and what He believed the Scriptures to say. He recognized that the incomprehensible God of the Bible, not "fate" or "nature" or even a God we could fully understand, had created all things.

Here is another quote from the third book of the *Principia*,

> *And from his true dominion it follows that the true God is a living, intelligent, and powerful Being; and, from his other perfections, that he is supreme, or most perfect. He is eternal and infinite, omnipotent and omniscient; that is, his duration reaches from eternity to eternity; his presence from infinity to infinity; he governs all things and knows all things that are or can be done.[4]*

Newton's childhood was not easy. He was born premature, and people were surprised he survived. His father died before his birth, and his mother remarried when Newton was three, leaving Newton to live with his grandmother for several years. Later, Newton entered the university as "a poor scholar," probably performing many menial tasks to work his way through school.[5] After graduating with his BA, a plague interrupted his further career at the college, forcing the university to close and sending Newton home.

But Newton used this forced time back home to explore God's universe. Those years he spent at home ended up becoming Newton's most productive years — years that laid the groundwork for much of his life's accomplishments. He later

said of those years, "All this was in the two plague years of 1665–1666. For in those days I was in the prime of my age for invention and minded Mathematics and Philosophy more than at any time since."[6] After the plague, Newton returned to the university and became a Fellow and later a Lucasian Professor of Mathematics (at the young age of 27, no less!) — but his diligence *through* the forced time back home during the plague is a wonderful reminder to us to rejoice in whatever God allows into our lives and to seek to honor Him in it.

Fascinating Fact

Newton taught himself many math concepts by studying books on his own.

Sample from Chapter 20

Glimpses of Consumer Math

20.1 Compound Interest

In this chapter, we're going to take a look at some practical applications of math that affect everyone. Rather than learning new math skills, you'll have a chance to apply the skills you already know to new situations.

The applications we're going to look at are from an area of math known as consumer math. The word *consumer* means "a person who purchases goods and services for personal use."[1] So **consumer math** technically means math that you need to purchase goods and services, although it's usually broadened to include other skills needed in daily life too. While we've already covered a lot of consumer math throughout the course, it's time to touch on a couple of concepts we haven't been able to explore yet.

Interest

To start with, let's take a look at the concept of interest. "**Interest** is a fee (or payment) made for the borrowing (or lending) of money."[2] When you put money in a bank, it typically earns interest — that is, the bank pays you for the temporary use of the money while it's in their bank. They can combine your money with thousands of other peoples' money and invest it. As they make money off the invested money, they share a portion of their earnings with you in the form of interest. Because they have so many customers, unless something goes massively wrong, they still have enough cash on hand to give you whatever you withdraw.

Calculating Interest

Most interest is what we call **compound interest** — money is invested, and the investor earns interest on the initial investment *plus all the previous interest earned.* Since the word **compound** means "to put together into a whole; combine;"[3] it

makes sense that *compound* interest means interest that's added to, or combined with, the investment each time.

Let's say you invest $10,000 in a bank that earns 4% interest compounded yearly. This means that every year, an interest of 4% is *compounded*, or *combined with*, the money you've invested in the bank. After the first year, you'll earn 4% of $10,000, or $400, in interest. After the second year, you'll earn 0.04 of *$10,400*, or $416, in interest. Every year that passes you'll earn more interest, as the interest is calculated on the initial investment *plus* all the previous interest earned.

Because money earning compound interest grows exponentially (there's growth on the growth), we can use the *same exponential growth formula* we looked at back in Lessons 15.5 to calculate how money will grow when earning compound interest.

> This formula assumes we don't put any money into or take any money out of the investment (i.e., deposit or withdrawal money), but rather let the initial investment grow untouched.

> **Exponential Growth Formula**
>
> $$P = P_0(1 + r)^t, \text{ where}$$
>
> P = *final amount after growth* (in the case of interest, the ending amount of money, including all the interest)
>
> P_0 = *initial amount before growth* (in the case of interest, the initial investment)
>
> r = *rate of growth over a specified period of time* (in the case of interest, the rate of growth per compound period)
>
> t = *the number of specified periods of time that have passed* (in the case of interest, the number of compound periods)

In order to accurately apply the exponential growth formula to interest rates, it's important to understand what we mean by a **compound period**. A compound period is "the period of time between the compounding of interest."[4] In other words, a compound period of a year means that every year, the interest is added to the account, increasing the amount off which the interest is calculated the next year.

Example: Suppose you invest $10,000 at a 4% interest rate. If the interest is compounded yearly, what will your ending balance be in 10 years, assuming you don't withdraw or deposit any money?

Formula:

$$P = P_0(1 + r)^t$$

Substitute known numbers:

$$P = \mathbf{\$10,000}(1 + \mathbf{0.04})^{\mathbf{10}}$$

Solving inside the parentheses:

$$P = \$10,000(\mathbf{1.04})^{10}$$

Simplify 1.04^{10} using the calculator's exponent button:

$$P = \$10,000(\mathbf{1.480244285})$$

Complete the multiplication:

$14,802.44

As we did with other exponential growth and functions, a coordinate graph can make it easy to see how money grows over time. Notice how this graph gives us a picture for how a $10,000 investment will grow at a 4% yearly interest rate that's compounded yearly.
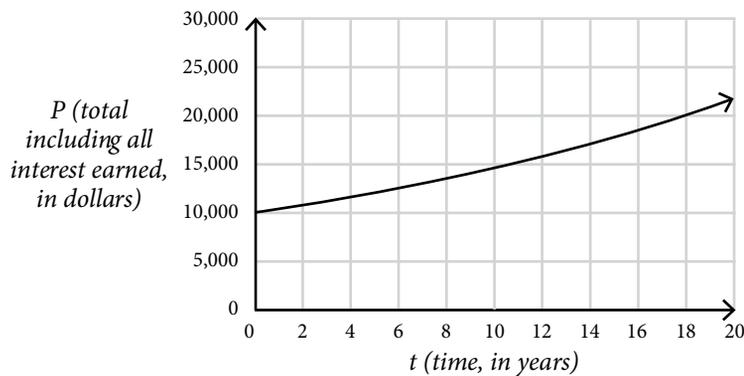
$$P = \$10{,}000(1 + 0.04)^t$$

When referring specifically to interest rates, you'll often see the exponential growth formula rewritten using different letters. For example, you might be given this formula instead:

$$A = P (1 + r)^t$$

Notice that this is the same relationship, only different letters have been used. Don't let different letters confuse you. Both $P = P_0(1 + r)^t$ and $A = P (1 + r)^t$ represent the same relationship.

Example: Find the balance of an initial investment of $300 after 24 months if it is invested at a 0.08% monthly interest rate and the interest is compounded monthly.

Formula:

$$P = P_0(1 + r)^t$$

Substitute known numbers:

$$P = \$300(1 + 0.0008)^{24}$$

Simplify inside the parentheses:

$$P = \$300(1.0008)^{24}$$

Simplify the 1.0008^{24} using the calculator's exponent button:

$$P = \$300(1.019377681)$$

Complete the multiplication:

$$P = \$305.81$$

Notice that 0.08% converts to 0.0008 — be careful when solving interest problems to make sure you convert the percent to a decimal appropriately.

Growing Debt

While the examples showed interest earned on investments, the same principles apply to interest *owed* for money borrowed. Just as the bank pays interest when they receive money, they *charge* interest when they loan money. And that interest can add up quickly!

Ask the person with mounds of credit card debt or some other loan they can't afford, and they'll understand the proverb that "the borrower is servant to the lender."

The rich ruleth over the poor, and the borrower is servant to the lender (Proverbs 22:7).

Keeping Perspective

In this lesson, we looked at **compound interest**, using the mathematic notations and tools you've learned to help calculate the growth of money earning interest. Compound interest is one example of how some of the skills we've been looking at can help with real-life tasks.

As we'll be exploring applications dealing with money in this chapter, it's worth pointing out that while math can help us assess financial decisions (and compare bank accounts), we always need to watch that money doesn't become our master. Treasure in heaven is the treasure that reaps eternal rewards — everything else is temporal. May we live each day with eternity in view.

> *"Do not lay up for yourselves treasures on earth, where moth and rust destroy and where thieves break in and steal; but lay up for yourselves treasures in heaven, where neither moth nor rust destroys and where thieves do not break in and steal. For where your treasure is, there your heart will be also. . . . No one can serve two masters; for either he will hate the one and love the other, or else he will be loyal to the one and despise the other. You cannot serve God and [money]" (Matthew 6:19–24; NKJV).*

20.2 Using the Correct Time and Rate

Throughout this course, we've looked at a lot of different formulas, from geometry to physics to finance. In this lesson, it's time to examine once again the importance of substituting the *correct* values into the formula. After all, a formula only gives the correct result *if* we input the correct information.

As you seek to use math to help make consumer decisions, you'll need to be careful to substitute correct values into the formulas. While we'll focus on the