

PRE-CALCULUS

TABLE OF CONTENTS

CHAPTER ONE — ANALYTIC GEOMETRY

TOPIC A: Distance From a Point to a Line	1
TOPIC B: Conic Sections	
PART 1: Parabolas	
◆ Properties of the Parabola	2
◆ Finding the Equation of the Parabola	5
◆ Sketching the Graph of the Parabola	12
PART 2: Ellipses	
◆ Properties of the Ellipse	17
◆ Finding the Equation of the Ellipse	19
◆ Sketching the Graph of the Ellipse	22
◆ Semi-Ellipses	26
PART 3: Hyperbolas	
◆ Properties of the Hyperbola	28
◆ Finding the Equation of the Hyperbola	33
◆ Sketching the Graph of the Hyperbola	37
◆ Equilateral Hyperbolas	40
◆ Semi-Hyperbolas	43
PART 4: Circles	
◆ Finding the Center and Radius of the Circle	44
◆ Finding the Equation of the Circle	47
◆ Graphing the Equation of the Circle	59
◆ Writing the Equation of the Circle Given Three Points	64
◆ Semi-Circles	65
◆ Point and Imaginary Circles	67
PART 5: Identification and Classification of Conic Sections	68
PART 6: Systems of Non-linear Equations and Inequalities	
◆ Solving Systems Algebraically	71
◆ Solving Systems Graphically	75

CHAPTER TWO — POLAR COORDINATES

TOPIC A: Plotting Points in the Polar Coordinate System	87
TOPIC B: Polar and Rectangular Conversions	
PART 1: Converting Points	
◆ Rectangular to Polar Point Conversions	91
◆ Polar to Rectangular Point Conversions	92
PART 2: Converting Equations	
◆ Rectangular to Polar Equation Conversions	93
◆ Polar to Rectangular Equation Conversions	93
TOPIC C: Distance Between Points in the Polar Coordinate System	94
TOPIC D: Sketching Polar Graphs	
PART 1: Using a Graphing Calculator or Graphing Utility	94
PART 2: Graphs of Polar Equations	97
TOPIC E: Polar Form of a Complex Number	
PART 1: Converting $a + bi$ to $r \operatorname{cis} \theta$	107
PART 2: Converting $r \operatorname{cis} \theta$ to $a + bi$	108
PART 3: Product and Quotient of Complex Numbers in Polar Form	109
PART 4: De Moivre's Theorem	
◆ Powers of Complex Numbers	111
◆ Roots of Complex Numbers	112

CHAPTER THREE — VECTORS & PARAMETRIC EQUATIONS

TOPIC A: Basic Properties of Vectors	115
TOPIC B: Vector Addition and Subtraction	116
TOPIC C: Scalar Multiplication.....	118
TOPIC D: Dot Product.....	119
TOPIC E: Real World Applications.....	121
TOPIC F: Parametric Equations	
PART 1: Properties of Parametric Equations	122
PART 2: Graphing Parametric Equations.....	123

CHAPTER FOUR — MATRICES

TOPIC A: Basic Matrix Operations	
PART 1: Dimension	127
PART 2: Equality	129
PART 3: Sum and Difference.....	130
PART 4: Scalar Product	132
PART 5: Matrix Multiplication	133
TOPIC B: Identity and Inverse Matrices.....	136
TOPIC C: Solving Systems Using Inverse Matrices.....	138
TOPIC D: Determinants and Cramer's Rule	
PART 1: Finding the Determinant.....	140
PART 2: Solving by Cramer's Rule	143
TOPIC E: Augmented Matrices and Gaussian Elimination	144

CHAPTER FIVE — SEQUENCES & SERIES

TOPIC A: Introduction to Sequences	
PART 1: Finding Terms in a Given Sequence	147
PART 2: Arithmetic vs. Geometric Sequences	150
TOPIC B: Arithmetic Sequences and Series	
PART 1: Arithmetic Sequences.....	151
PART 2: Arithmetic Series.....	154
TOPIC C: Geometric Sequences and Series	
PART 1: Geometric Sequences	157
PART 2: Finite Geometric Series.....	161
PART 3: Infinite Geometric Sequences and Series	
◆ Limits of Infinite Sequences	163
◆ Sum of Infinite Geometric Series.....	164
TOPIC D: Using Sigma with Series.....	166
TOPIC E: Mathematical Induction.....	170

CHAPTER SIX — FUNCTION THEORY

TOPIC A: Basics of Functions	
PART 1: Properties of Functions.....	173
PART 2: Algebra of Functions	177
PART 3: Composition of Functions	180
PART 4: Inverse of a Function.....	185
PART 5: Common Functions	
◆ Square Root Functions	194
◆ Greatest Integer Functions	196
◆ Absolute Value Functions	197
◆ Piecewise Functions.....	206
TOPIC B: Polynomial Functions of Higher Degree	
PART 1: Properties of Polynomial Functions	215
PART 2: Graphing Polynomial Functions.....	220
TOPIC C: Rational Functions	
PART 1: Properties of Rational Functions	
◆ Domain.....	235
◆ x- and y-Intercepts	236
◆ Vertical and Horizontal Asymptotes	236

PART 2: Graphing Rational Functions	237
TOPIC D: Power Functions with Rational Exponents	
PART 1: Properties of Power Functions	246
PART 2: Graphing Power Functions	247

CHAPTER SEVEN — THEORY OF EQUATIONS

TOPIC A: Division of Polynomials	
PART 1: Long Division	251
PART 2: Synthetic Division	252
TOPIC B: Roots of a Polynomial Equation	
PART 1: Factor Theorem.....	253
PART 2: Remainder Theorem	254
PART 3: Rational and Irrational Roots	256
PART 4: Complex Roots	259
PART 5: Descartes' Rule of Signs.....	260
TOPIC C: Partial Fractions.....	262

CHAPTER EIGHT — EXPONENTS & LOGARITHMS

TOPIC A: Exponential Functions and Their Graphs	263
TOPIC B: Logarithmic Functions and Their Graphs.....	281
TOPIC C: Properties of Logarithms	
PART 1: Logarithmic Form vs. Exponential Form.....	292
PART 2: Laws of Logarithms	297
TOPIC D: Exponential and Logarithmic Equations	305
TOPIC E: Exponential and Logarithmic Models	
PART 1: Growth and Decay	307
PART 2: Compound Interest.....	311

CHAPTER NINE — CIRCULAR TRIGONOMETRY

TOPIC A: Angles in Standard Position	
PART 1: Graphing Angles and Coterminal Angles	315
PART 2: Converting Between Degrees and Radians	
◆ Converting From Degrees to Radians.....	321
◆ Converting From Radians to Degrees.....	324
PART 3: Length of an Arc and Area of a Sector	327
TOPIC B: The Unit Circle	
PART 1: Definition of the Six Trigonometric Functions.....	333
PART 2: Locating the Angle in the Correct Quadrant	340
PART 3: Evaluating All Six Trigonometric Functions	342
PART 4: Using Technology to Evaluate Trigonometric Functions of Any Angle	347
TOPIC C: Special Angles and Quadrantal Angles	349
TOPIC D: Reference Angles	354
TOPIC E: Cofunctions and Reciprocal Relationships	360

CHAPTER TEN — GRAPHS OF TRIGONOMETRIC FUNCTIONS

TOPIC A: Properties of Trigonometric Functions	
PART 1: Domain, Range, Increase, and Decrease.....	365
PART 2: Amplitude, Frequency, Period, and Phase Shift	
◆ Amplitude	367
◆ Frequency	370
◆ Period	372
◆ Phase Shift and Displacement	374
◆ Equations of Sine and Cosine Curves.....	377
TOPIC B: Graphs and Equations of Trigonometric Functions	
PART 1: Transformations and Sketching Sine and Cosine Functions	381
PART 2: Writing Equations of Trigonometric Functions Given a Periodic Graph.....	395
PART 3: Graphs of Functions for $\tan x$, $\cot x$, $\sec x$, and $\csc x$	403
TOPIC C: Inverse Trigonometric Functions	
PART 1: Graphs of Inverse Trigonometric Functions	410
PART 2: Evaluating Inverse Trigonometric Expressions	417

CHAPTER ELEVEN — SOLVING TRIANGLES

TOPIC A: Right Triangle Trigonometry	
PART 1: Definitions of the Six Trigonometric Functions.....	425
PART 2: Finding the Measure of Missing Sides and Angles of Right Triangles.....	430
TOPIC B: Law of Cosines	436
TOPIC C: Law of Sines	443
TOPIC D: Forces.....	452
TOPIC E: Area of a Triangle or Parallelogram.....	454

CHAPTER TWELVE — TRIGONOMETRIC EQUATIONS

TOPIC A: Solving First-Degree Trigonometric Equations.....	461
TOPIC B: Solving Quadratic Trigonometric Equations.....	464
TOPIC C: Verifying and Applying Trigonometric Identities	
PART 1: Quotient, Reciprocal, and Pythagorean Identities	467
PART 2: Verifying Trigonometric Identities	474
PART 3: Sum and Difference of Two Angles.....	476
PART 4: Double and Half Angles.....	481
PART 5: Product to Sum and Sum to Product	485
PART 6: Solving Trigonometric Equations Involving Identities	486

APPENDIX—HOW TO ADD MATH GRIDS & TABLES TO AN EXAM 493