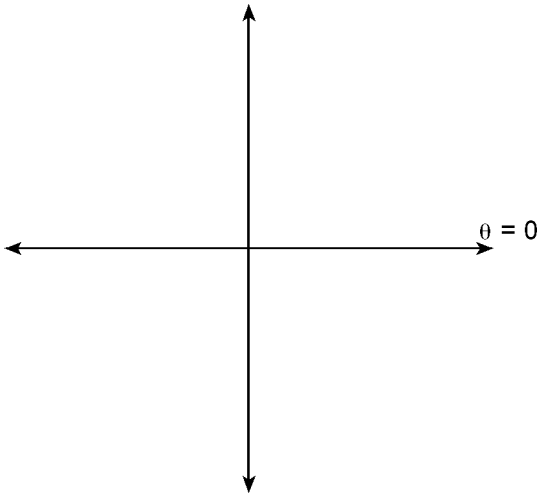
Pre-Calculus Sample Exam

1) On the blank axes below, sketch the graph of the polar equation $r = 2 + 3 \sin \theta$.



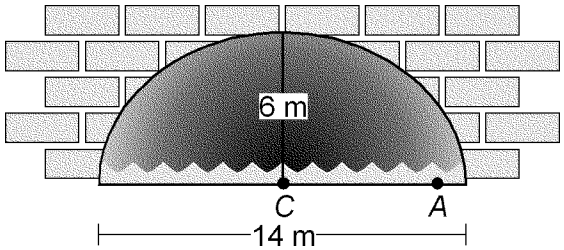
2) Let vector $v = 4i - 3j$, given that i and j are unit vectors. Find all values of scalar k such that $|kv| = 3$.

3) What are the first four terms in the sequence whose general term is $t_n = 3(-1)^{n-1}$?

A) 3, 6, 12, 24
B) 3, -6, 12, -24
C) 3, -3, 3, -3
D) 0, -3, 3, -3

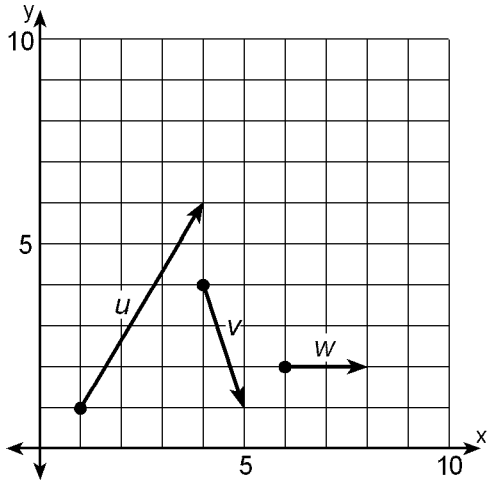
4) Find the indicated sum of the series $\sum_{k=1}^{n} 3(0.25)^k$.

5) A canal tunnel is semi-elliptical in shape, with a maximum height of 6 m and a maximum width of 14 m. Determine the height of the tunnel at point *A*, which is 5 m from the center *C*. [*Accurate to one decimal place.*]

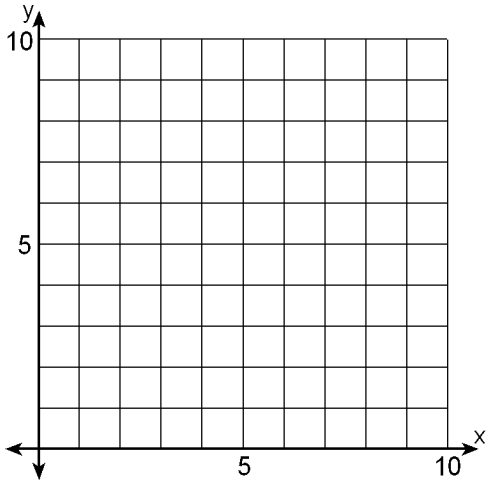


A) 2.0 m
B) 4.2 m
C) 3.6 m
D) 5.4 m
E) 3.1 m

6)



On the blank grid below, plot the vector resulting from $v + u$ shown in the graphed image.



7) Find the limit for the infinite sequence or state that one does not exist.

-3, 9, -27, 81, ...

8) (**a**) On a coordinate grid, sketch a graph of the equation $2x^2 - y^2 - 4x + 4y - 4 = 0$.

(**b**) State the coordinates of the center, vertices, and foci of the graph you plotted in *part (a)*.

(**c**) On the same graph, plot and state the equations of the asymptotes.

(**d**) Determine the value of the graph's eccentricity.

9) Prove by mathematical induction that the statement $\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{(n+1)}$ is true for all natural numbers n.

10) An airplane is traveling at a fixed altitude with no discernible wind velocity. The plane is headed N 60° E at a speed of 420 mph. The plane then encounters a wind with a velocity of 65 mph in the direction N 40° W. Find the resultant direction of the plane. [*Round the answer to the nearest whole number.*]

11) Use mathematical induction to prove the formula for every positive integer n.

$2 + 7 + 12 + 17 + ... + (5n - 3) = (\frac{n}{2})(5n - 1)$

12) For the given parametric equation, eliminate the parameter t and graph the equation. Indicate the direction of increasing t.

$$f(x) = \begin{cases} x = 2 + t^2 \\ y = 1 - t^2 \end{cases} \text{ for all real values of } t$$

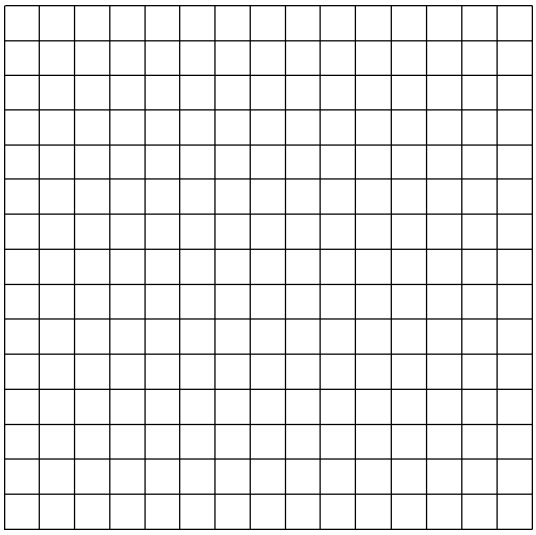
13) Write the complex number in polar form.

$-5i$

14) Find the vertex, focus, and equation for the directrix of the equation. Graph the given equation on the axes provided.

$x = 2y^2 + 4y + 5$

Vertex:
Focus:
Directrix:



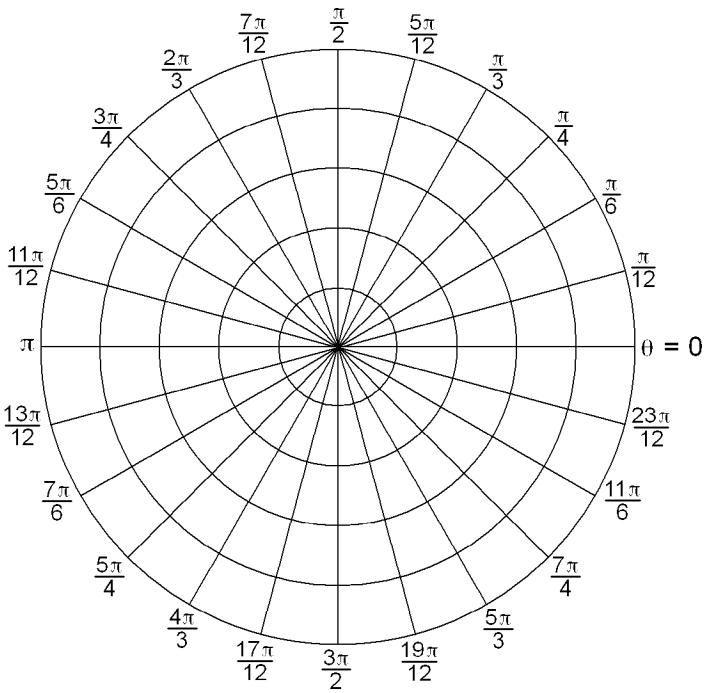
15) Find the product of the given complex numbers and leave the answer in polar form. Convert to polar form first if necessary.

$z = 2(\cos\frac{2\pi}{9} + i\sin\frac{2\pi}{9})$

$w = 4(\cos\frac{\pi}{9} + i\sin\frac{\pi}{9})$

16) Given the polar equation $r = 2\sin 3\theta$, complete the table below by determining the exact value of r and sketch a graph of the equation on the grid provided.

r	θ
	0
	$\frac{\pi}{12}$
	$\frac{\pi}{6}$
	$\frac{\pi}{4}$
	$\frac{\pi}{3}$
	$\frac{5\pi}{2}$
	$\frac{\pi}{2}$



r	θ
	$\frac{7\pi}{12}$
	$\frac{2\pi}{3}$
	$\frac{3\pi}{4}$
	$\frac{5\pi}{6}$
	$\frac{11\pi}{12}$
	π