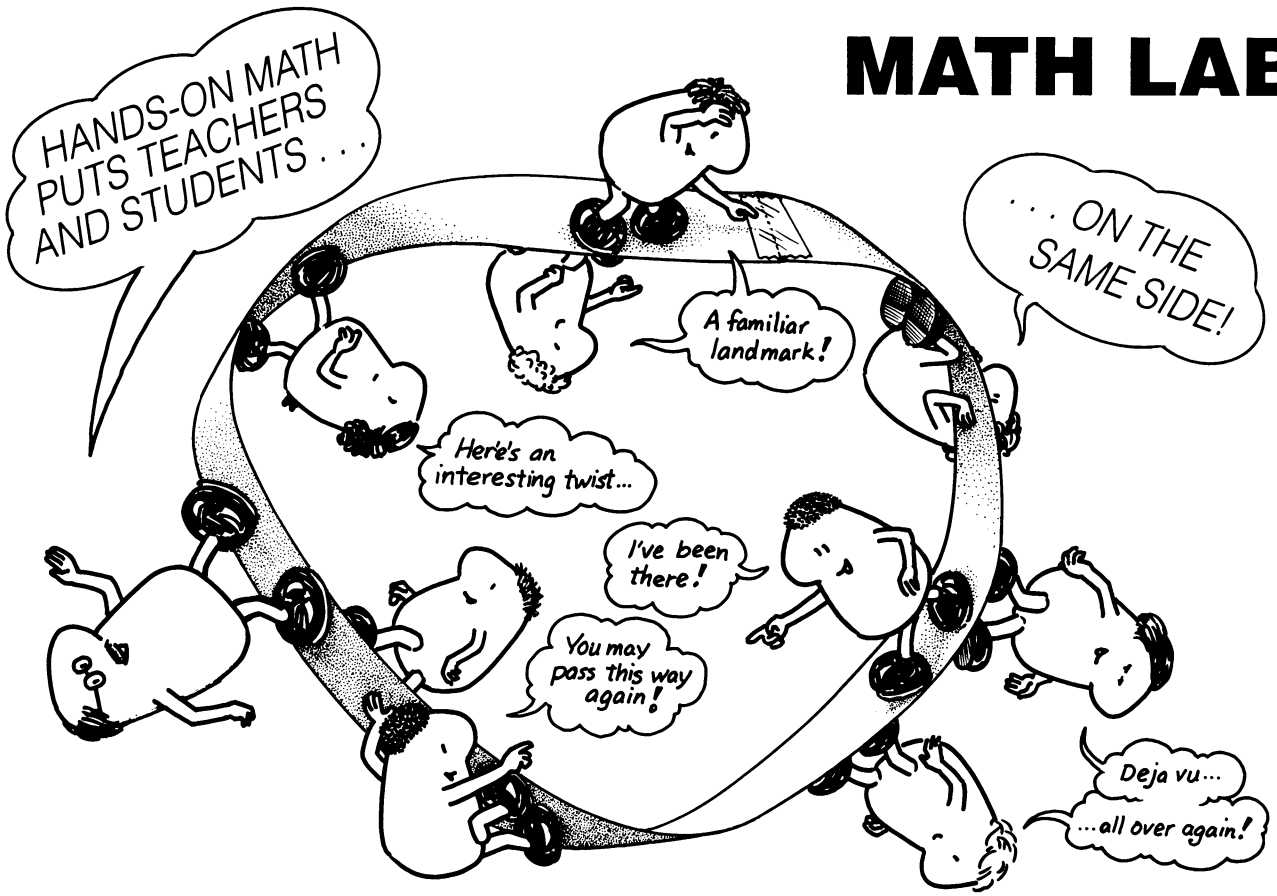
MATH LAB



TASK CARD SERIES

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TOPS LEARNING SYSTEMS

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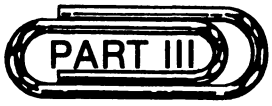


REPRODUCIBLE STUDENT TASK CARDS

Gathering Materials

Listed below is everything you'll need to teach this module. You already have many of these items. The rest are available from your supermarket, drugstore and hardware store. Laboratory supplies may be ordered through a science supply catalog.

Keep this classification key in mind as you review what's needed:

special in-a-box materials: Italic type suggests that these materials are unusual. Keep these specialty items in a separate box. After you finish teaching this module, label the box for storage and put it away, ready to use again the next time you teach this module.	general on-the-shelf materials: Normal type suggests that these materials are common. Keep these basics on shelves or in drawers that are readily accessible to your students. The next TOPS module you teach will likely utilize many of these same materials.
(substituted materials): Parentheses enclosing any item suggests a ready substitute. These alternatives may work just as well as the original, perhaps better. Don't be afraid to improvise, to make do with what you have.	*optional materials: An asterisk sets these items apart. They are nice to have, but you can easily live without them. They are probably not worth an extra trip to the store, unless you are gathering other materials as well.

Everything is listed in order of first use. Start gathering at the top of this list and work down. Ask students to bring recycled items from home. The teaching notes may occasionally suggest additional student activity under the heading "Extensions." Materials for these optional experiments are listed neither here nor in the teaching notes. Read the extension itself to find out what new materials, if any, are required.

Needed quantities depend on how many students you have, how you organize them into activity groups, and how you teach. Decide which of these 3 estimates best applies to you, then adjust quantities up or down as necessary:

$Q_1 / Q_2 / Q_3$

- **Single Student:** Enough for 1 student to do all the experiments.
- **Individualized Approach:** Enough for 30 students working alone or in small groups, all self-paced.
- **Traditional Approach:** Enough for 30 students, working alone, all doing the same lesson.

KEY: *special in-a-box materials* (substituted materials) general on-the-shelf materials *optional materials

$Q_1 / Q_2 / Q_3$:	
1/10/10	rolls of pennies (paper punch — see activity 1)
6/180/180	sheets lined notebook paper
1/30/30	sheets copier paper
1/15/30	index cards
1/15/30	metric rulers
1/15/30	drawing compasses
1/20/30	scissors
1/10/10	rolls clear tape
1/7/15	hand calculators
1/30/30	*sheets dark construction paper*
1/5/15	bottles white glue
1/5/15	*hand lenses
1/1/1	pencil sharpener
1/20/30	*pieces 8 1/2 x 11 inch corrugated cardboard*
1/2/2	spools thread
2/40/60	straight pins
1/20/30	*dark crayons, wrappers removed*
2/20/60	*small styrofoam cups*
1/5/15	*short pieces pencil lead – see activity 12*
4/120/120	sheets scratch paper
1/1/1	roll masking tape
1/2/2	boxes paper clips
1/1/1	roll adding machine tape
3/15/15	*used envelopes with bar codes – see activity 19*

Sequencing Task Cards

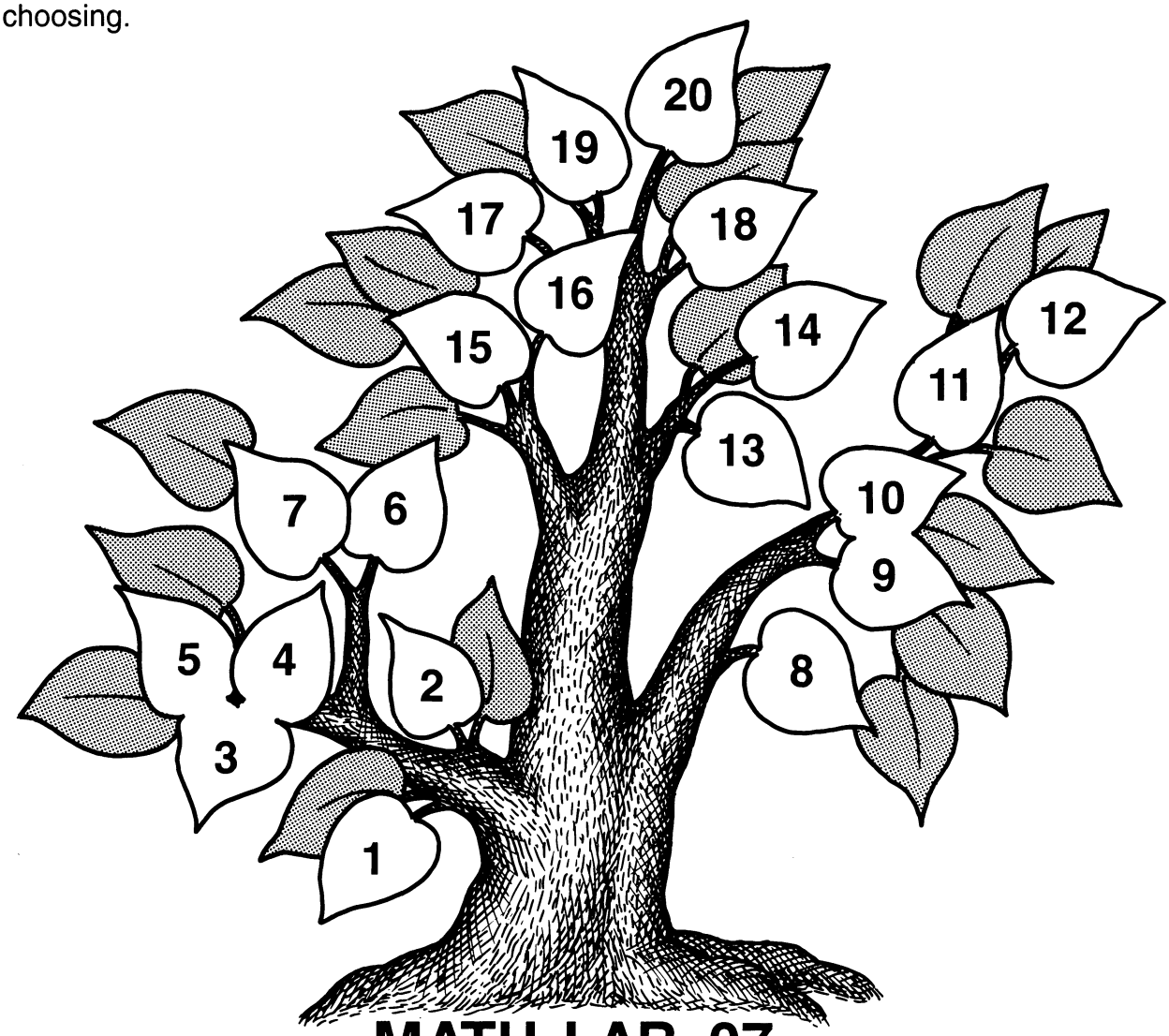
This logic tree shows how all the task cards in this module tie together. In general, students begin at the bottom of the tree and work up through the related branches. As the diagram suggests, upper level activities build on lower level activities.

At the teacher's discretion, certain activities can be omitted, or sequences changed, to meet specific class needs. The only activities that must be completed in sequence are indicated by leaves that open *vertically* into the ones above them. In these cases the lower activity is a prerequisite to the upper.

When possible, students should complete the task cards in the same sequence as numbered. If time is short, however, or certain students need to catch up, you can use the logic tree to identify concept-related *horizontal* activities. Some of these might be omitted, since they serve only to reinforce learned concepts, rather than introduce new ones.

On the other hand, if students complete all the activities at a certain horizontal concept level, then experience difficulty at the next higher level, you might move back down the logic tree to have students repeat specific key activities for greater reinforcement.

For whatever reason, when you wish to make sequence changes, you'll find this logic tree a valuable reference. Parentheses in the upper right corner of each task card allow you total flexibility; they are left blank so you can pencil in sequence numbers of your own choosing.

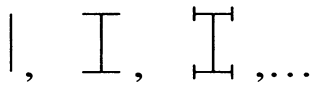


Review / Test Questions

Photocopy this test question page. Cut out those questions you wish to use and tape them onto clean paper. Include questions of your own design, as well. Crowd them all onto a single page for students to answer on another paper, or leave space for student responses after each question, as you wish. Duplicate a class set and your custom-made test is ready to use. Use leftover questions as a class review in preparation for the final exam.

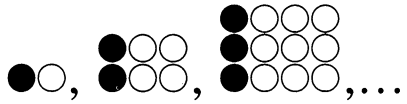
tasks 1-2, 17 A

How many line segments are in the 6th term?



task 1-2, 17 B

a. Draw the 4th term in this sequence with solid circles and hollow circles.



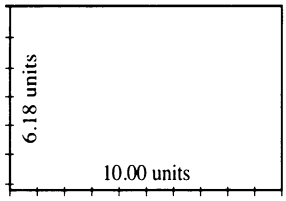
b. Write a general rule for n terms.
c. Use your nth term to find the number of circles in the 5th term.
d. Write out the first 5 terms as a sequence of numbers. Show that the difference between these terms also forms a sequence.

task 3

a. Write out the first 16 terms in the Fibonacci sequence.
b. Add up these 16 terms the easy way. Explain your method.

tasks 4-5 A

Use a calculator to show that this rectangle has "golden mean" proportions:



tasks 4-5 B

Show how Fibonacci numbers approach the "golden mean" (0.618).

tasks 6-7 A

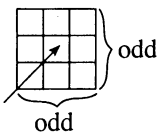
You look under 47 rocks in a meadow and find ants under each one. You generalize that there will be ants under the next rock you find in this same meadow.
a. Is this a reasonable generalization?
b. Is it possible that your prediction is incorrect?

tasks 6-7 B

a. Find the difference between adjacent terms in this sequence of prime numbers: 5, 7, 11, 13, 19, 23...
b. If this pattern holds, what should be the next prime number in this sequence?
c. What can you conclude?

tasks 6-7 C

If both the length and width of a "pool table" have an odd number of squares, a cue ball hit at a 45° angle from 1 corner always exits the opposite corner.



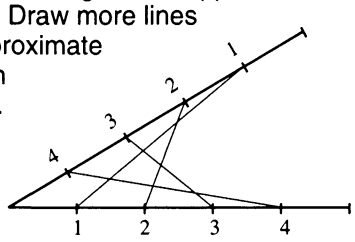
a. Show that this generalization is true for at least 5 different pool tables.
b. Can you be sure that this generalization applies to all pool tables with odd dimensions? Explain.

task 8

A 3-sided regular polygon forms a triangle. A 4-sided regular polygon forms a square. What sort of regular polygon forms a circle?

tasks 8-12

These 4 straight lines approximate a curve. Draw more lines to approximate it even better.



task 9 A

Define a circle as a set of points.

task 9 B

a. Sketch a circle freehand, marking the center c, plus points p_1 and p_2 on the circle.
b. How are points p_1 and p_2 related to point c?

task 10 A

Define an ellipse as a set of points.

task 10 B

a. Sketch an ellipse freehand, marking the foci f_1 and f_2, plus points p_1 and p_2 on the ellipse.
b. How are points p_1 and p_2 related to points f_1 and f_2?

task 11 A

Define a parabola as a set of points.

task 11 B

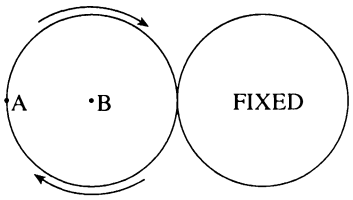
a. Sketch a parabola freehand, marking its focus f and fixed reference line l, plus points p_1 and p_2 on the parabola.
b. How are points p_1 and p_2 related to the focus f and line l?

task 12 A

Define a cardioid as a set of points.

task 12 B

A movable circle is rotated about a fixed circle as shown. What kind of figure is traced by point A on the circumference? Point B at the center?



task 13

A triangle has 3 equal interior angles. How big is each one? Explain.

task 14

Find the sum of the first 200 integers. Show your work.

task 15 A

Find 2 pairs of twin primes between 100 and 110.

task 15 B

Show that 143 is not a prime number.

task 16

To multiply numbers, _____ their logs.
To divide numbers, _____ their logs.

task 18

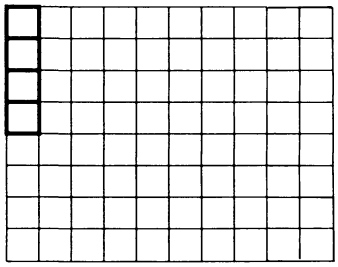
a. Explain how to make a Mobius strip from a 1 meter strip of paper.
b. If an ant travels in 1 direction along this strip, how far must it crawl before returning to its starting point?

task 19

Translate this 5-digit zip code. Show that its correction character is correct. (Recall that place values for the long bars are 7 4 2 1 0, and that the bar on each end is a frame bar.)

task 20

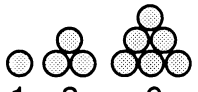
Here is 1 way to connect 4 tiles. Draw all the other ways. Each pattern must be unique.



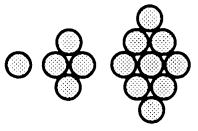
Task Objective (TO) discover patterns in the arrangement of pennies that form number sequences.

PENNY PLAY ○ **Math Lab ( )**

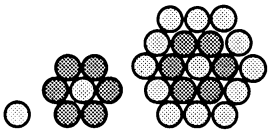
1. These penny patterns form number sequences. Write the first 10 terms of each sequence as a string of numbers.

a.  ... (The differences between neighboring terms also forms a number pattern.)

1, 3, 6,

b.  ... (These numbers are called *perfect squares*.)

1, 4, 9,

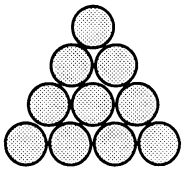
c.  ... (Build the next term like this: 1 "heads" surrounded by 6 tails, surrounded by 12 heads, surrounded by more tails...)

1, 7, 19,

2. Use the sequences you have just written to help you find the 10th term in each of these sequences:

a. 1, 1+2, 1+2+3, ...
b. 1, 1+3, 1+3+5, ...
c. 1, 1+7, 1+7+19, ...

3. Here is a penny puzzle: This triangle points up. Move just 3 pennies to make it point down.



Answers / Notes

1a. 1, 3, 6, 10, 15, 21, 28, 36, 45, 55. *(The difference between pairs of terms is 2, 3, 4, ...)*

1b. 1, 4, 9, 16, 25, 36, 49, 64, 81, 100. *(The difference between pairs of terms is 3, 5, 7, ...)*

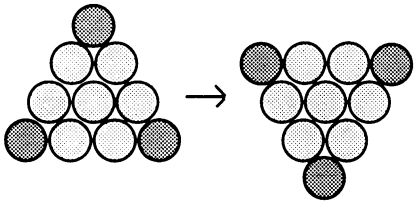
1c. 1, 7, 19, 37, 61, 91, 127, 169, 217, 271. *(The difference between pairs of terms is 6, 12, 18, ... This corresponds to the number of pennies added in each new outside ring.)*

2a. This sequence is the same as 1a. The tenth term, therefore, is 55.

2b. This sequence is the same as 1b. The tenth term, therefore, is 100.

2c. The 10th term in this sequence equals the sum of all 10 terms in 1c: 1 + 7 + 19 + 37 + 61 + 91 + 127 + 169 + 217 + 271 = 1000 *(Students who recognize problem 2c as a sequence of perfect cubes can arrive at the same answer by cubing the tenth term: $10^3 = 1000$.)*

3. *The key to solving this puzzle is recognizing the triangle as a 7-penny hexagon surrounded by 3 outside pennies. Move these 3 to point the triangle in any of a total of 6 different directions, including down.*

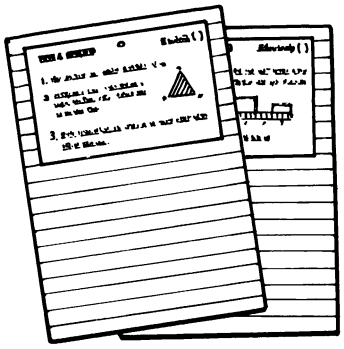


Materials

☐ A roll of pennies. Problem 1c requires 37 coins to build the 4th term. If you don't want to invest several dollars to supply the needs of your whole class, substitute index card punches. Color 1 side before punching so each side has a "head" and a "tail." The only requirement for using these is good eye-hand coordination. Use a straight pin to slide them into position.

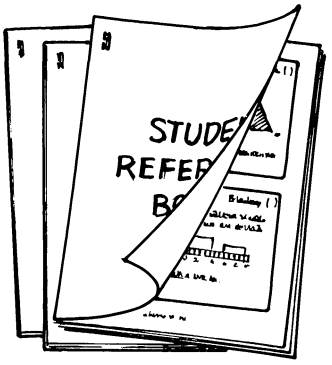
Task Cards Options

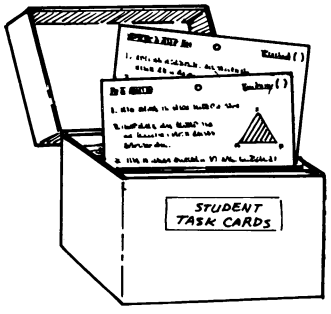
Here are 3 management options to consider before you photocopy:



1. Consumable Worksheets: Copy 1 complete set of task card pages. Cut out each card and fix it to a separate sheet of boldly lined paper. Duplicate a class set of each worksheet master you have made, 1 per student. Direct students to follow the task card instructions at the top of each page, then respond to questions in the lined space underneath.

2. Nonconsumable Reference Booklets: Copy and collate the 2-up task card pages in sequence. Make perhaps half as many sets as the students who will use them. Staple each set in the upper left corner, both front and back to prevent the outside pages from working loose. Tell students that these task card booklets are for reference only. They should use them as they would any textbook, responding to questions on their own papers, returning them unmarked and in good shape at the end of the module.





3. Nonconsumable Task Cards: Copy several sets of task card pages. Laminate them, if you wish, for extra durability, then cut out each card to display in your room. You might pin cards to bulletin boards; or punch out the holes and hang them from wall hooks (you can fashion hooks from paper clips and tape these to the wall); or fix cards to cereal boxes with paper fasteners, 4 to a box; or keep cards on designated reference tables. The important thing is to provide enough task card reference points about your classroom to avoid a jam of too many students at any one location. Two or 3 task card sets should accommodate everyone, since different students will use different cards at different times.