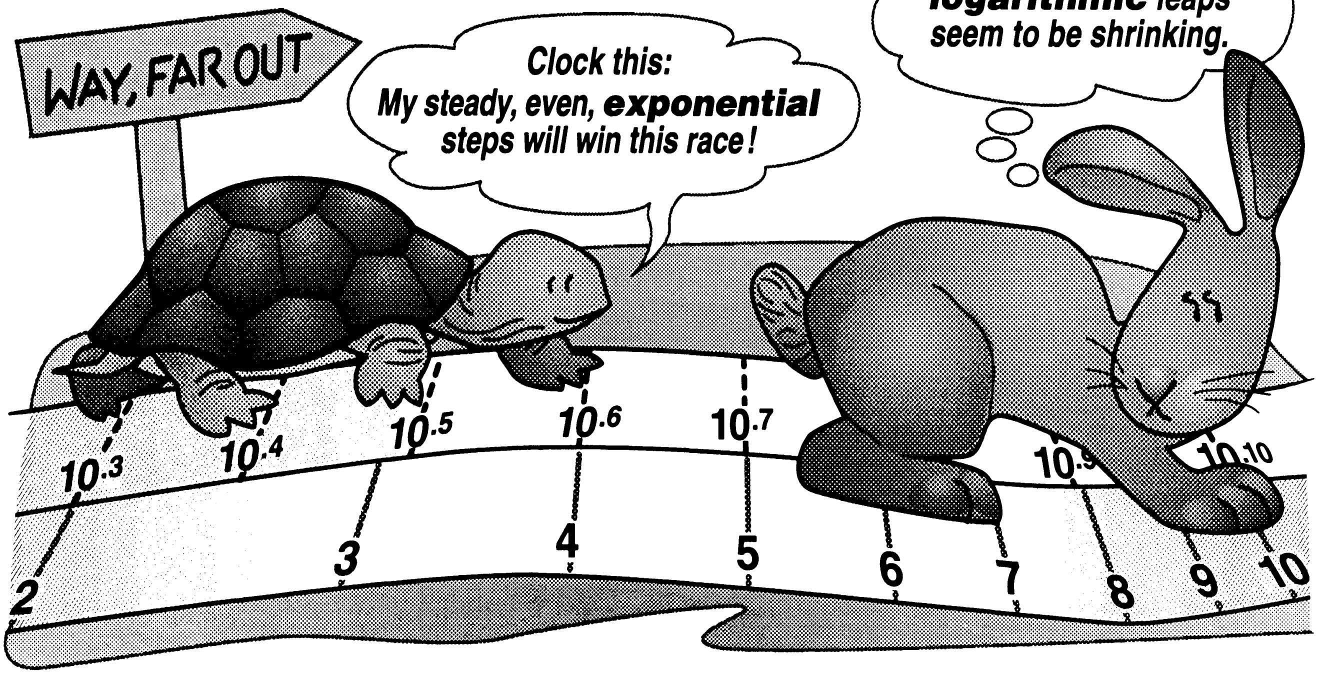
FAR OUT MATH!



SCIENCE WITH SIMPLE THINGS SERIES

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FAR OUT MATH!

Teach abstract, sometimes challenging standards-based concepts in science and math in creatively concrete ways. Students study orders of magnitude, exponents, logarithms and other fundamental concepts that enable scientists to launch observatories like GLAST into space, and to interpret the data received.

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Welcome, Dear Educator, ...

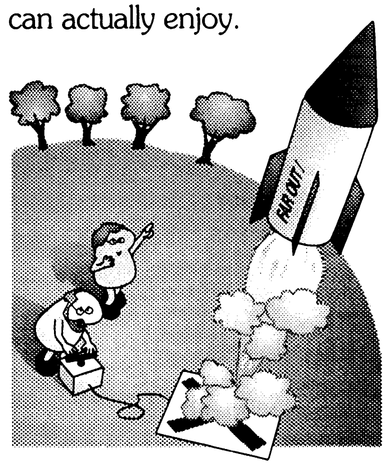
... to **FAR OUT MATH*!*** This book is the first of 3 activity units (published one per year) designed to inform students about NASA's 2006 space shot called GLAST *(now Fermi Gamma-ray Space Telescope).*

It's not hard to say what GLAST is, or to outline what it will do. GLAST stands for the Gamma-ray Large Area Space Telescope. It looks like a "flying" box with solar panel "wings." GLAST is designed to absorb, track and measure gamma ray photons across enormous energy ranges. It will open a window of observation and understanding into our Universe at the highest end of the electromagnetic spectrum, enabling us to better understand the dance of matter and energy at Nature's most extreme and energetic levels.

It's more difficult to bring GLAST down to earth, into the imagination and grasp of high school math and science students. How can TOPS create high-interest, hands-on activities that inform students about the GLAST mission, ***and*** teach to the standards in meaningful ways?

For our first year, we decided to launch into logarithms, an important mathematical tool that has helped us design space telescopes, and make sense of the huge ranges of data received back on earth. Those of us who are baby-boomers (or older) remember calculating with logarithms in high school, and carrying slide rules into our physics classes. Would our current generation of students, raised on scientific calculators, be well served and meaningfully educated by bringing back this bit of not-so-ancient history?

Do ***you*** really understand logarithms? Did you learn them in high school, then have to relearn them in college, then learn them again to teach second year algebra? Our experience at TOPS is that logarithms have been eminently forgettable. But not any longer! After extensive development and testing, we think we've come up with an accessible, creative program that you and your students won't easily forget, and can actually enjoy.



Whether you think we've achieved mission impossible (informing about GLAST and teaching stuff that future rocket scientists need to know), or that our first attempt is a fizzle, we'd love to hear from you.

Please direct your comments and suggestions to TOPS at the address on our copyright page, and send a copy to Lynn Cominsky, our program director.

Sincerely, Ron Marson

Ron Marson

FAR OUT MATH*!* ties in with ***National Math and Science Standards*** in the following areas. See page 6 for content details.

Math:

Understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions.... Appreciate that seemingly different mathematical systems may be essentially the same.... Make decisions about units and scales that are appropriate for problem situations involving measurement.... Develop a deeper understanding of very large and very small numbers and of various representations of them.

(National Council of Teachers of Mathematics)

Science:

Develop abilities necessary to do scientific inquiry.... A variety of technologies, such as hand tools, measuring instruments and calculators, should be an integral component of scientific investigations.... Mathematics plays an essential role in all aspects of an inquiry. For example, measurement is used for posing questions, formulas are used for developing explanations, and charts and graphs are used for communicating results.

(The National Science Education Standards)

Getting Ready

1. Review the scope and sequence outlined below, then page through the book to fill in conceptual details.

Lesson / Time		Main Idea(s)
A1	1h:15m	Slide rules, calibrated in a linear scale of numbers, add and subtract distance.
A2	1h:25m	Slide rules, calibrated in a logarithmic scale of exponents, multiply and divide distance.
B1	1h:30m	Logs are exponents *(log N = exp)*. Antilogs are numbers *(antilog exp = N = 10^{exp})*.
B2	1h:15m	Logarithm laws reduce the complexity of calculating ordinary numbers by 1 notch.
B3	:20m	To calculate with logarithms: *(1)* take logs; *(2)* apply laws; *(3)* take the antilog.
C1	:50m	All the log relationships you learned in base-10 also apply in base-2.
C2	:20m	Translate antilog equations into log equations by "pulling down the exponent."
C3	:40m	Solve exponential equations by applying base-ten or base-two logarithms.
D1	1h:0m	Significant figures are all figures you know for sure, plus the last estimated digit.
D2	1h:10m	Translate logs and antilogs using scientific notation as an intermediate step.
D3	1h:30m	A classic slide rule calculates the numbers, but *you* figure the decimal place.
E1	:40m	Graph N *vs* log N to generate a logarithmic scale. Convert it to a slide rule.
E2	:50m	Exponential equations and log equations are inverse functions.
E3	:50m	Log-log graphs transform higher order equations into straight lines.

2. Choose a lesson sequence that fits your time frame and curriculum goals. **STRAND A** lessons offer a gentle start that keeps all options open. Or, start with **STRAND B** to cut to the conceptual foundations of logarithms a bit faster. The ***Log Tape*** constructed in lesson ***B1*** is an important reference used in many lessons that follow. **STRAND C** reinforces the common logarithms presented in **STRAND B** from a base-2 perspective. **STRAND D** broadens into work with significant figures, scientific notation, and estimating the decimal place. **STRAND D** can stand alone, but the learning curve is steeper: it requires students to operate the ***Classic Slide Rule*** without having used earlier practice versions. **STRAND E** also stands alone as a 3-lesson unit on graphing, assuming your students already have a fundamental working knowledge of logarithms.

	Intermediate Options	Advanced Options
LESS TIME ↑	**A.**	**B** *or* **D** *or* **E.**
	A, B.	**B, C.**
	A, B, C.	**B, C, E.**
	A, B, C, E.	
MORE TIME ↓	**A, B, C, D, E.**	**B, C, D, E.**

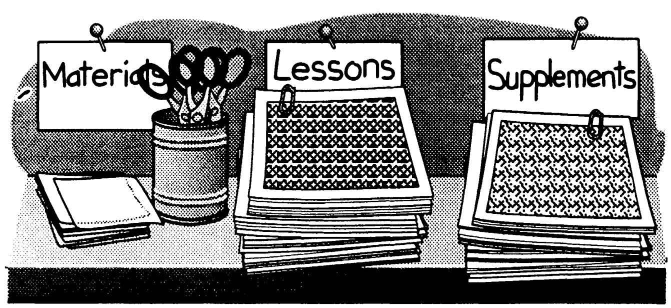
3. If you wish to use the optional ***Tracking Sheet*** for students (opposite), cross out any lessons you plan to skip.

4. Cut freezer bags to size and gather other materials. See teaching notes 11 for our simple ***materials list***.

5. Photocopy student materials for your first few activities. If you copy on both sides of the paper, only pages for lessons ***A1*** and ***A2*** need to be stapled together.

6. Jumbo-clip sets of duplicates, and organize a resource shelf or table. Students will each need a manila folder (or equivalent) to store their ***Tracking Sheet***, manipulatives, and completed lessons.

7. Direct your students to pick up a ***Tracking Sheet*** and a ***Plastic Hairlines*** supplement (hairlines are not needed if you will use strand B or E only), and begin work.



Suggested Teaching Strategies

- ✱ Allow students to move through each lesson at their own pace.
- ✱ Require check-points. Write your initials in each box (see opposite page), before starting a new lesson.
- ✱ Interrupt individualized activity at any point to present optional introductory materials from the lesson notes, or to clarify points of general interest.
- ✱ Post answer keys for self checking.
- ✱ Communicate behavior expectations.
- ✱ Explain grading procedures. Use any combination of the review/test questions (pp 8-9), plus questions of your own, to determine how well students have mastered key concepts.

Review / Test Questions

Photocopy these test questions. Cut out those you wish to use, and tape them onto white paper. Include questions of your own design, as well. Crowd them all onto a single page for students to answer on their own papers, or leave space for student responses after each question, as you wish. Duplicate a class set, and your custom-made test is ready to use. Use leftover questions as a class review in preparation for the final exam.

activity A1, A2

a. Explain how you would use a pair of rulers to show that 3 + 6 = 9. Provide a clearly labeled drawing to illustrate your answer.

b. What if the figures on your drawing represented base-10 exponents (logs) instead of ordinary numbers? Rewrite the addition problem in part a, showing how it becomes a multiplication problem.

activity A2

Slide rules add and subtract distances to multiply and divide numbers. How is this possible?

activity B1

Use your *Log Tape* to fill in the missing numbers:

a. 2 = 10^----- = antilog ______.
Thus, log ______ = ______.

b. 20 = 10^----- = antilog ______.
Thus, log ______ = ______.

activity B1

Use your *Log Tape* to fill in the missing numbers:

a. 3 = 10^----- = antilog ______.
Thus, log ______ = ______.

b. .03 = 10^----- = antilog ______.
Thus, log ______ = ______.

activity B1

The log of ***a number*** is ___________.
The antilog of __________ is ___________.

activity B2, B3

a. Fill in the missing number:
log 5 + log 2 = log ______.

b. Use your *Log Tape* to evaluate each log, and prove the equality.

activity B2, B3

a. Fill in the missing number:
log 5 - log 2 = log ______.

b. Use your *Log Tape* to evaluate each log, and prove the equality.

activity B2, B3

a. Fill in the missing number:
3 log 3 = log ______.

b. Use a *Log Tape* to evaluate each log, and prove the equality.

activity B2, B3

a. Fill in the missing number:
(log 25) + 2 = log _____.

b. Use your *Log Tape* to evaluate each log, and prove the equality.

activity C1, C2

The $antilog_2$ of ***an exponent*** is __________.
The log_2 of ___________ is __________.

activity C1, C2

Using your *Base-2 Slide Rule* as a reference, complete these equivalent equations:

$\log_2 64 + \log_2 2 = \log_2$ _____	___ x ___ = _____
	$\frac{4,096}{512}$ = _____
$5 \log_2 4 = \log_2$ _____	
	$\sqrt[3]{2,097,152}$ =

activity C1, C2

Complete this table:

2^{13} =		
	$\log_2 128$ =	
		$antilog_2$ 0 =
10^{-3} =		
	log 70 =	
		antilog 2.845 =

activity C3

Solve for x. Show your work: $4^x = 15$

activity C3

Solve for x in base-2, then again in base-10. Show your work: $8^x = 32$

activity D1

Read your *Log Ruler* as accurately as possible.
25 = 10^-------. Thus, log 25 = ______.
250 = 10^-------. Thus, log 250 = ______.
$10^{0.500}$ = _____. Thus, antilog 0.500 = _______.
$10^{2.500}$ = _____. Thus, antilog $\bar{2}$.500 = _____.

activity D2

a. Complete this table of logs and antilogs:

antilog	scientific notation	log
514		
22.8		
		4.069

b. Solve using logs. Show your work.
514 x 22.8 = ?

activity D3

Solve these problems on your slide rule. Report significant figures.

a. 1.234 x 12.34 = ?

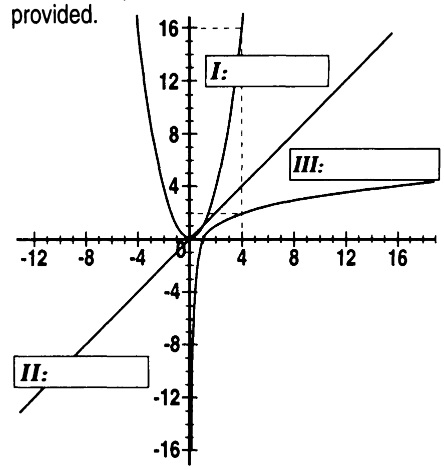
b. 2580. ÷ 1850. = ?

activity D3

Your *Classic Slide Rule* uses log distances to multiply and divide numbers, even though no logs are printed on it anywhere! How is this possible?

activity E1, E2

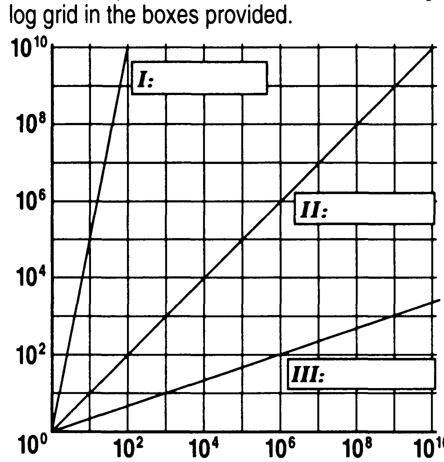
a. Label equations ***I***, ***II***, and ***III*** in the boxes provided.



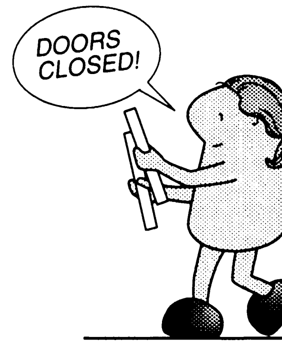
b. State the *inverse* of...
equation ***I***: ____________________
equation ***III***: ____________________

activity E3

a. Label equations ***I***, ***II***, and ***III*** for this log-log grid in the boxes provided.



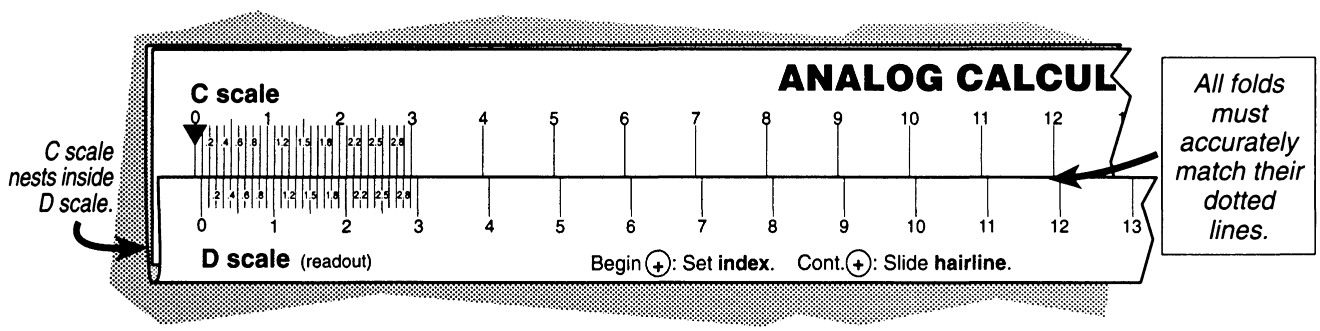
b. State the inverse of...
equation ***I***: ____________________
equation ***III***: ____________________



1. Get the ***Plastic Hairlines*** supplement and follow the directions. Then return here.

2. Get the ***Adding Slide Rule*** supplement and follow the directions. Return here again.

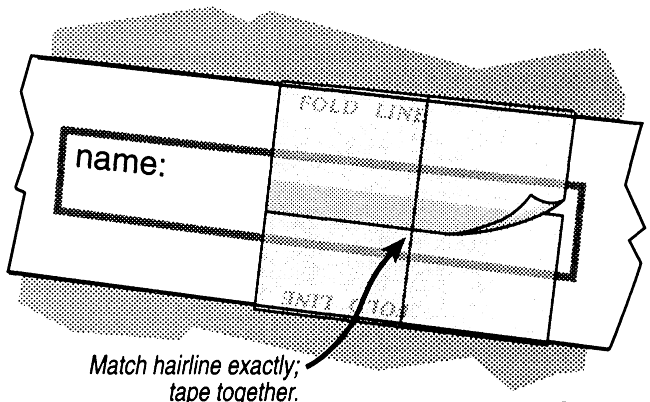
3. Fit the folded C scale completely *inside* (not over) the folded D scale, so both scales meet as shown. (If you did this right, you'll see a name box on the back.)



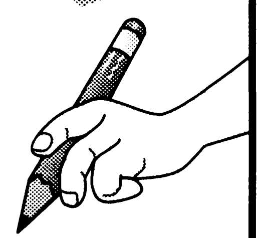
4. Write your name in the name box.

5. Wrap one of your hairlines across the name box (inked-side down), so the ink line *perfectly* overlaps. Pull firmly together and tape. (Don't tape the paper!)

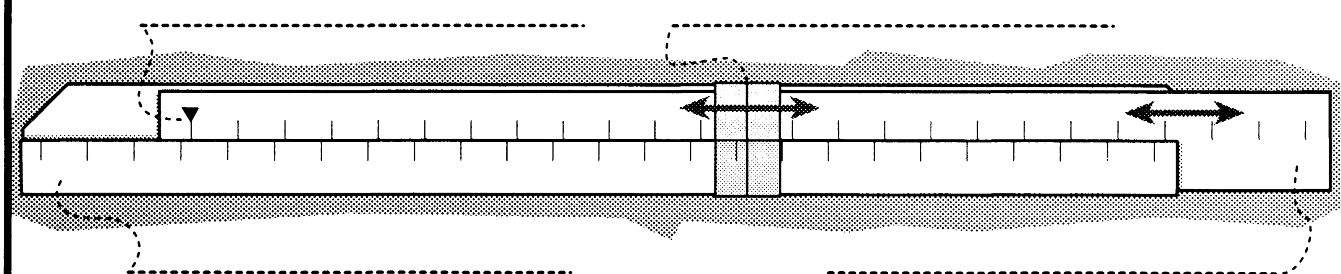
6. Check that the hairline fits snugly, yet still moves freely. (If too tight or too loose, peel the tape up and try again.)



TEACHER CHECK: ☐



7. Identify and label these slide rule parts on the dotted lines:
Hairline • C scale • Index • D scale



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OVERVIEW / OBJECTIVES

Students will construct *Adding Slide Rules*, scaled with linear calibrations like ordinary rulers. They will learn to move these scales (and the hairline) relative to each other in ways that add and subtract distance, thus calculating sums and differences.

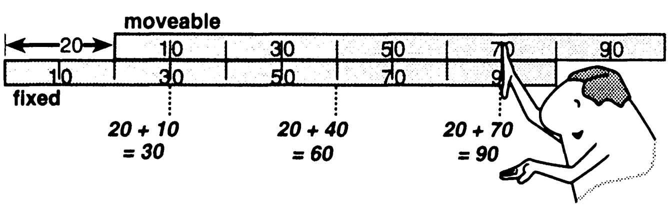
TIME: 1 hour and 15 minutes.

INTRODUCTION

★ Use this (and all), introductions as *optional* overviews. If you choose to present this one, please keep in mind that you are *not* trying to teach your students how to operate a slide rule. (They will pick this up quite naturally as they interact with these TOPS lessons.) Rather, your are demonstrating how sliding rulers add and subtract distance.

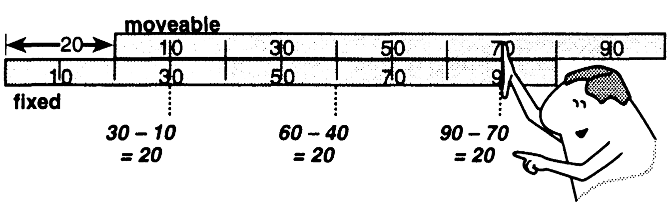
Snip sections of masking tape (5-10 cm long); roll them sticky-side-out. Use these to fix a meter stick to your board or classroom wall. Rest a second meter stick above the first. The bottom meter is ***fixed;*** the top meter ***moveable***.

Slide the zero calibration on the moveable meter 20 cm to the right of the fixed meter. Use your hand as an imaginary hairline to show how numbers on the top, plus 20, always equal numbers on the bottom.



Slide the top meter stick other distances (30 cm, 40 cm...) relative to the bottom to demonstrate other additives.

Then demonstrate the inverse process of subtraction. Start with your "hairline" on any fixed-scale distance, say 90 as illustrated below. Notice that the movable-scale distance, 70, subtracts to leave a difference of 20, indicated under the zero calibration. Notice how 20 is the common difference for *any* number pair related by the hairline.



Slide the top meter stick other distances relative to the bottom to demonstrate other subtraction differences.

★ Make an *Adding Slide Rule* yourself to show your students what they will be making. This will help orient them to how the *Plastic Hairlines* are used, and how the scales fit together. This will also give you, the teacher, an appreciation for the clarity and integrity of the directions you'll be asking students to follow. When they come to you for help, ask them to read the direction or problem that puzzles them aloud. This is often enough to clear confusion. Remember, our goal as educators is to foster learning independence.

NOTES (this page)

1-2. Point out that steps 1 and 2 are giant steps. Each involves the completion of a separate, supplementary sheet of instructions before students continue with step 3.

☞ ***See Plastic Hairlines supplement, pg 15.***

2-7. The double layer of plastic is folded *inside* the paper, as illustrated, not outside. The tape and paper clips serve to hold this plastic "sandwich" firmly together during trimming and cutting. Mature students who recognize that these layers must not shift may accomplish this task with a firm, careful grip, eliminating the need for clipping or taping altogether.

☞ ***See Adding Slide Rule supplement, pg 16.***

2. Demonstrate how to accurately pinch-fold a dotted line between thumb and finger along its *entire* length. When this is done accurately, the dots (or light dashes) don't end up on one side of the fold or the other, but precisely ***on*** the fold, where they belong. Caution against getting the fold started, then creasing it against a table top for the remainder of the line; usually the dashed line will stray to one side or the other.

Return to this page. ☜

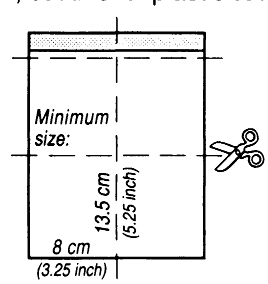
6. Inspect the slide rule edges, to check that all dotted lines remain *on* the folds. Direct students to refold (by pinching between thumb and finger) as necessary. Good folds make the slide rule a pleasure to use. Bad folds are reversible.

ANSWERS (this page)

7. *Index* — *Hairline*
D scale — *C scale*

MATERIALS

☐ A double layered rectangle of plastic cut from a transparent storage bag. Sandwich bags work, but thicker plastic cut from freezer bags has better handling qualities. Larger bags may be divided into halves or quarters for conservation. Each double layer should measure **8 cm x 13.5 cm** or larger, with at least 1 bonded edge holding both layers together. See page 15 for the paper template students will use with this plastic.



☐ Scissors that are sharp enough to cut through plastic bags with minimum drag.

☐ Paper clips.

☐ Tape. Masking tape and clear tape both work fine.

☐ A ballpoint pen that writes on plastic.

☐ Copier or notebook paper. Or recycle scrap paper printed on one side for conservation.

☐ No other materials will be needed for this unit except **manila folders** *(see page 7)*, and **string** *(see notes 29)*. Students will occasionally use **calculators**, and require **rulers** (any straightedge) in STRAND E. You may also need 2 **meter sticks** for optional lesson introductions.