LESSON

3

Don’t Be Square!

Years 5 to 9

Parallel Lines and Isometric Grids

This lesson presents some activities for student exploration of the MATHOMAT drawing guides. Students construct an isometric grid and use it to represent three-dimensional shapes.

In this lesson students will:

- construct parallel lines;
- construct isometric grids;
- represent three-dimensional shapes on paper; and
- (optional) attempt to solve a non-routine problem which involves constructing and recording shapes.

Materials Required

For each student:

- a MATHOMAT;
- unlined paper (scrap paper will do);
- blocks or plastic interlocking cubes—about 16 for each student;
- fine-point pens or pencils—at least two colours;
- crayons or felt-tip pens for colouring;
- a copy of Worksheet 3.1, Making Shapes; and
- (optional for older students) a copy of Handout 3.1, How to Draw an Isometric Grid.

For each group of four students:

- a set of eight three-dimensional models constructed from interlocking blocks—you will need to construct these prior to the lesson (see the diagram in part 3 of the Lesson Outline).

Additional materials:

- an overhead projector transparency of Drawing Three-Dimensional Shapes—see Transparency 3.1.

Lesson Summary

- Drawing parallel lines and parallelograms; constructing an isometric grid;
- drawing representations of three-dimensional shapes on an isometric grid;
- constructing three-dimensional models from two-dimensional drawings; and
- (optional) attempting the Four-Cube Houses problem.

For the Teacher

An isometric grid allows three-dimensional shapes to be represented in two dimensions.

The MATHOMAT template contains guides for drawing parallel lines and isometric grids—you will need to familiarise yourself with their operation before the lesson.

The guides for drawing isometric lines are on both sides of the MATHOMAT, but only those on the right side are labelled. Both sets of isometric guides are inclined at 30 degrees to the rulers on the sides of the MATHOMAT.

The labelled guides for drawing parallel lines are on the right hand side of the MATHOMAT and can be used to produce lines 1 cm (or multiples of 1 cm) apart.

You will need to give the students time to find and practise using the guides on the MATHOMAT before they attempt to draw grids on which to represent three-dimensional shapes.

A photocopy master of detailed instructions for constructing an isometric grid using the MATHOMAT is provided—see Handout 3.1, How to Draw an Isometric Grid. You will need to demonstrate this procedure to the class. For older students, you may wish to reproduce the sheet as a handout.

Lesson Outline

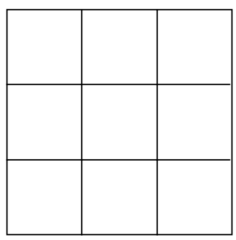
1. Drawing parallel lines

Start the lesson with a class discussion of the terms 'parallel' and 'parallelogram'. Demonstrate the use of the MATHOMAT parallel lines guide.

Ask students to use the guides to construct sets of parallel lines and parallelograms, and to make designs based on these.

Now ask students to construct a grid of horizontal and vertical lines 1 centimetre apart. (You might use the grids for Magic Squares or even allow the students to play Noughts and Crosses on them.)

Include a brief sharing and discussion time to allow students to display their work and to alert other students to some of the possibilities.



2. Constructing isometric grids

Discuss the features of an isometric grid with the class and compare it with the grid of horizontal and vertical lines drawn in the first part of this lesson. Use the step-by-step instructions in How to Draw an Isometric Grid (Handout 3.1) to carefully demonstrate to the class how the MATHOMAT *isometric lines* can be used to construct an isometric grid—some students will find this quite challenging. You may decide to distribute copies of Handout 3.1 to older students.

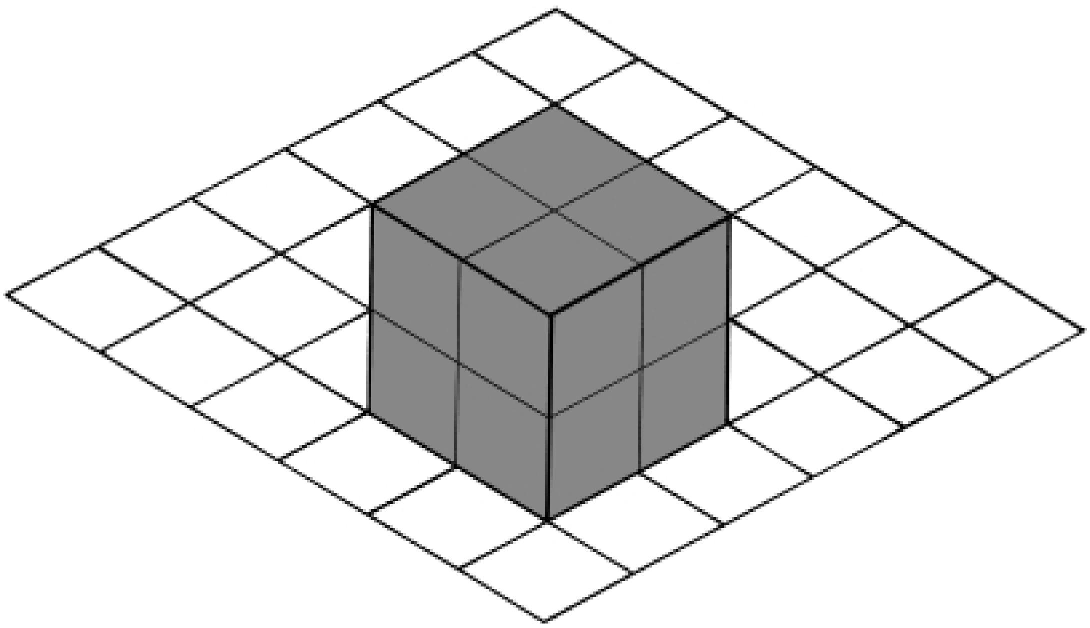
Ask students to construct their own isometric grid using this procedure.

If additional grids are required for the activities in later sections of this lesson, photocopies or commercially available isometric paper can be used.

3. Drawing three-dimensional shapes

An isometric grid can be used to represent a three-dimensional shape in two dimensions—the diagram shows a cube drawn on an isometric grid. The cube is viewed from above and three of its faces can be seen. Colouring and shading can be used to enhance isometrc drawings.

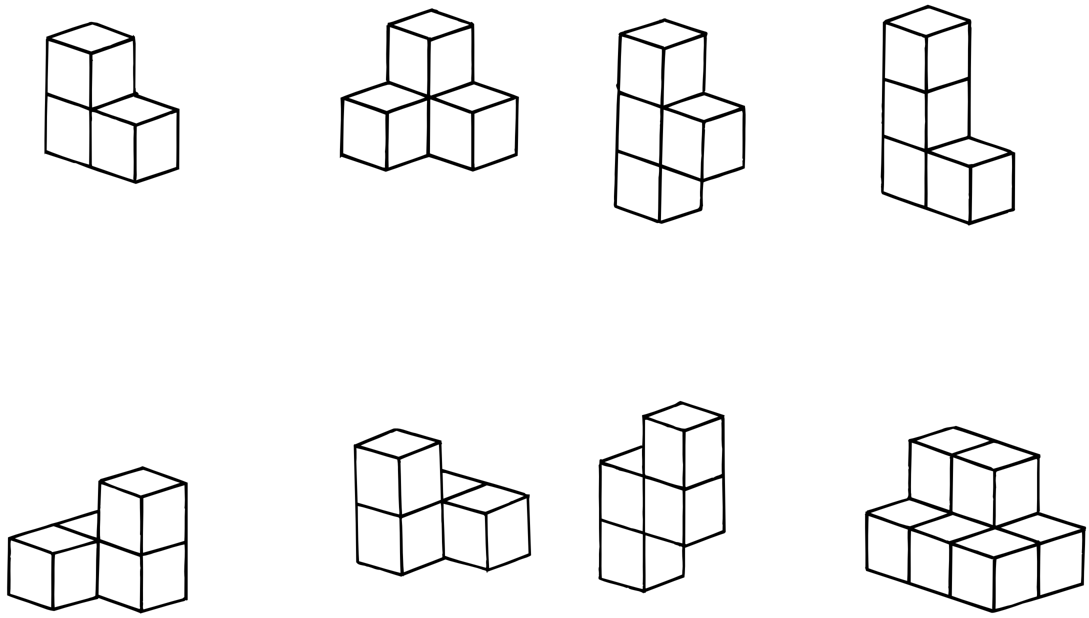
The representation of a cube on isometric grid paper



Discuss the representation of three-dimensional shapes in two dimensions—two examples are provided on Transparency 3.1, Drawing Three-Dimensional Shapes. This transparency also contains a blank isometric grid which you can use to draw extra shapes.

Divide the class into groups with about four students in each group. Give each group a set of the eight three-dimensional models that you have constructed from sets of interlocking cubes (see Materials Required—examples of the models are shown below).

Some shapes suitable for students to represent on isometric grids



Allow students to handle the models, pull them apart and reconstruct them—they will gain familiarity with each shape, which will assist them with its representation.

Ask each group to share the task of representing the set of eight models on their pieces of isometric paper—it is not necessary for each student to represent each model.

Some students may wish to construct and represent their own shapes. As an optional extension, students may use isometric grids to draw challenging shapes for each other to construct.

After most groups have completed their drawings, select students to display their drawings to the rest of the class.

Compare the different ways in which each shape is represented—there are several valid ways in which each may be represented.

4. Constructing shapes from drawings

Distribute Worksheet 3.1, Making Shapes, and ask students to construct the shapes shown. This will require them to determine the number of cubes required to construct each shape—an interesting task when not all cubes are visible.

5. (Optional) Four-Cube Houses

The problem Four-Cube Houses challenges students to find and construct all shapes that can be formed, under certain conditions, from four identical interlocking cubes. In order to keep track of which cubes have been constructed, it is useful to record these on paper. In this lesson, students can be asked to represent each shape on isometric grid paper.

In the delightful article Four-Cube Houses, Freudenthal (1980) describes how this problem was introduced to a third grade class in the Netherlands using the well-known Dutch children's TV character Paulus the Forest Midget.

> There is restlessness in the midget town. Some houses are more beautiful than others. Paulus is called in as a troubleshooter. He proposes to rebuild the town. The midgets will live pairwise in houses, each consisting of a drawing room, a kitchen, and two bedrooms. All rooms are to be (congruent) cubes, and each house will be built from four cubes, which touch each other along complete faces …. It would be a dull town if all the houses were the same shape. (Freudenthal,1980, p. 12)

How many different houses can the midgets build?

In order to solve this problem, students will need to agree on some 'ground rules', such as the prohibition of cantilever houses (i.e. ones with 'overhanging" rooms) as well as agreeing on what constitutes 'different'—a good rule for this is that mirror images are different, but houses which can be obtained from one another through a rotation are not. Also, we are only interested in the arrangement of rooms, not whether it is a bedroom or bathroom.

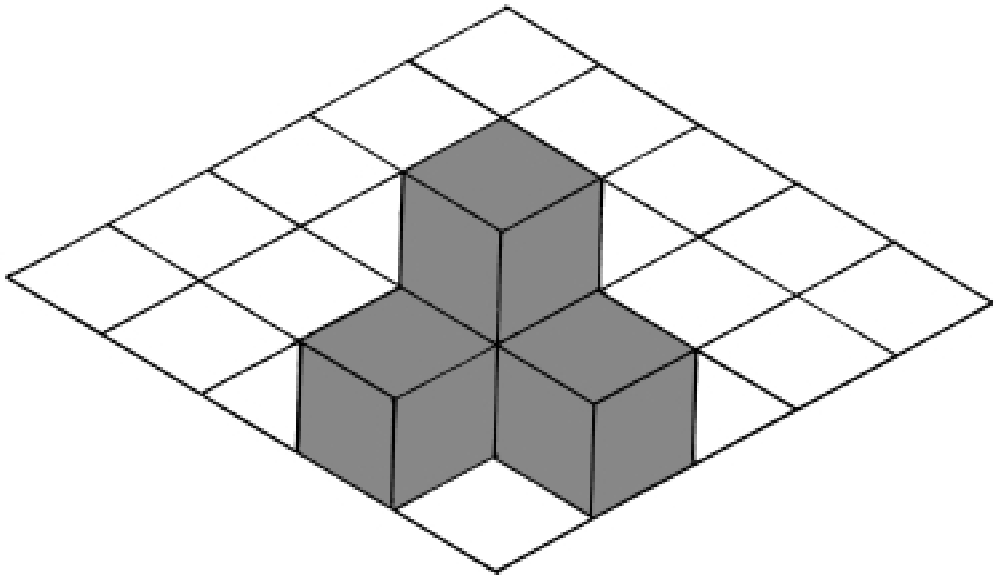
Under these rules there are twelve different houses—students may need the best part of a lesson to find them all and establish a convincing argument why there are no more possible. It is well worth challenging students to find such a convincing argument and in the process find a good way of breaking the problem into sub-problems. This can lead to an excellent class discussion of the different ways in which groups classified their houses.

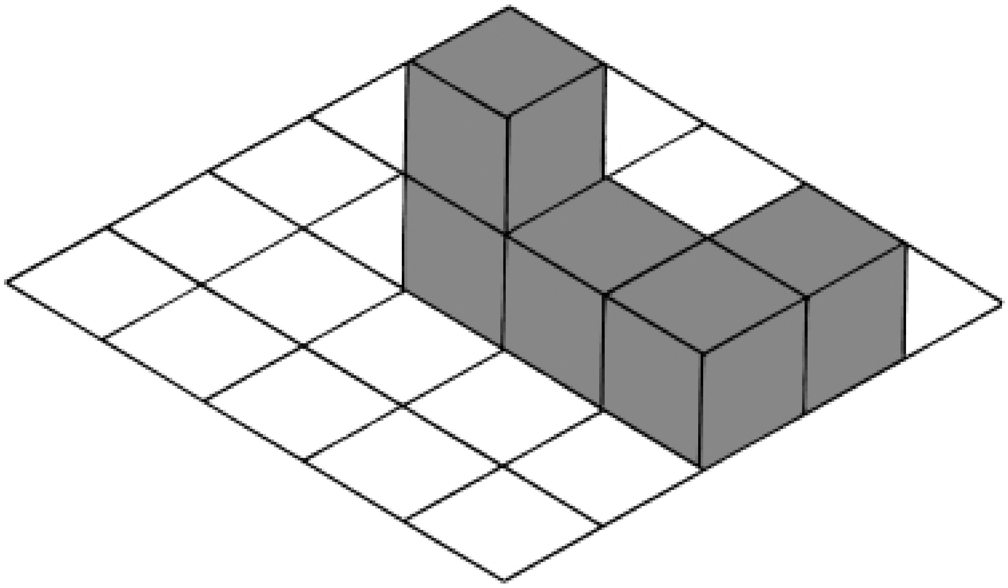
With older students, a good introduction to the problem is to set the context as a new ski village, which is being built using pre-fabricated houses transported from elsewhere.

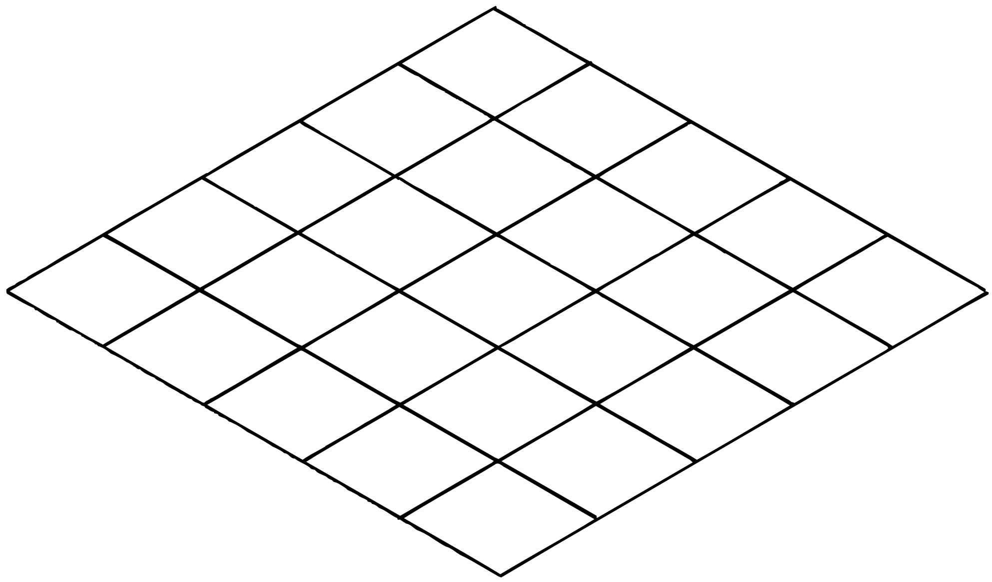
Reference

Freudenthal, H. (1980). Four-cube houses. *For the Learning of Mathematics, 1* (2), 12-13.

Drawing Three-Dimensional Shapes



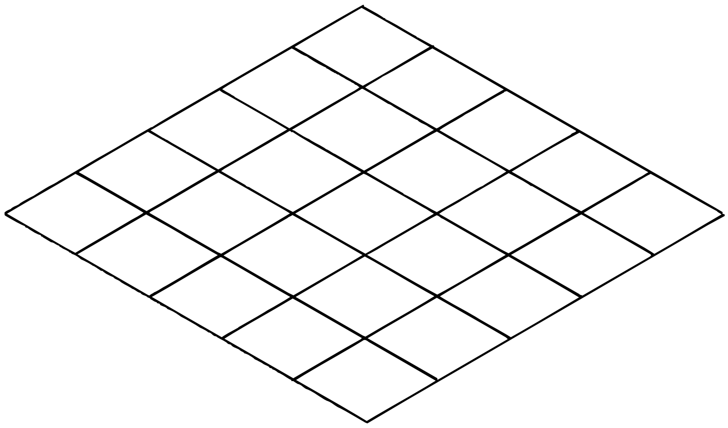




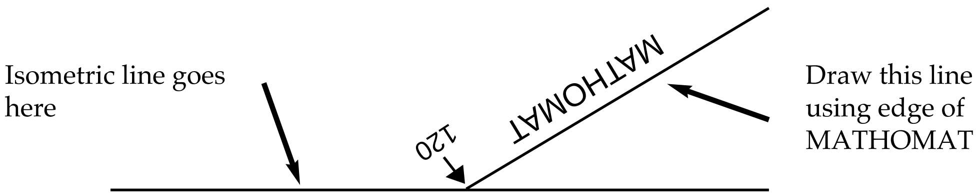
How to Draw an Isometric Grid

This sheet contains step-by-step instructions for drawing an isometric grid using the isometric line guides on MATHOMAT—a completed grid is shown in the following diagram.

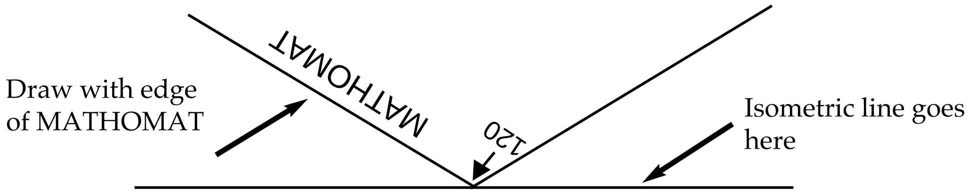
An isometric grid



1. Using unlined paper and a soft pencil, draw a horizontal line. This line, which forms the ‘base line’ for your grid, will later be erased.
2. Locate the *isometric line* near the top left corner of your MATHOMAT.
3. Superimpose the *isometric line* onto the horizontal line with the arrow head on the edge of the MATHOMAT oriented as shown in the next diagram.
4. Draw along the edge of your MATHOMAT as shown below.



5. Do the same on the other side using the isometric line from the bottom left hand corner of your MATHOMAT.



6. Now use the parallel lines on the right-hand side of the MATHOMAT to draw lines 1 cm apart to complete the grid.
7. Erase the horizontal base line to obtain the completed grid.

Making Shapes

Each diagram shows a solid three-dimensional shape on an isometric grid—only external edges of the shapes are shown. Beside each diagram write the number of cubes needed to make the shape and then make it.