LESSON

4

Bigger and Bigger

Years 5 to 9

Enlarging Figures to Construct Polyhedra Nets

This lesson involves students using their MATHOMAT to enlarge regular polygons to produce nets of selected polyhedra, which are then used to construct three-dimensional models. The activities allow students to develop techniques for accurate construction.

In this lesson students will:

- enlarge two-dimensional figures;
- develop techniques for accurate construction;
- develop their knowledge of mathematical terminology;
- make three-dimensional models from nets; and
- (for older students) visualise, describe and compare the numbers of faces, edges and vertices of polyhedra.

Materials Required

For each student:

- a MATHOMAT;
- a fine-point pen and a sharp pencil;
- lightweight card, approximately A4 size;
- scissors;
- sticky tape;
- a copy of Handout 4.1, The Five Platonic Solids—alternatively this can be reproduced on an overhead projector transparency for display to the entire class; and
- (optional) a copy of Handout 4.2, How to Enlarge Regular Polygons—see the next page for details.

Additional materials:

- (optional) commercial or 'home-made' examples of regular polyhedra—e.g. straw models, mobiles, models constructed from three-dimensional construction toys such as Polydron or Geoshapes.

Lesson Summary

- Using the MATHOMAT to enlarge regular polygons;
- an introduction to polyhedra;
- using the MATHOMAT to create nets and construct polyhedra; and
- (optional) investigating properties of polyhedra.

For the Teacher

This lesson involves students using their MATHOMAT to enlarge regular polygons. Instructions for enlarging regular polygons are given in Handout 4.2, How to Enlarge Regular Polygons. The technique differs slightly for polygons with odd and even numbers of sides.

You may wish to use the How to Enlarge Regular Polygons sheet as a handout, but you would still need to demonstrate the procedure to students.

Younger students can be shown how to enlarge an equilateral triangle and a square. Older students can be shown how to enlarge an equilateral triangle only and can then be challenged to generalise the method to other regular polygons.

Developing the process for enlarging the figures can motivate a discussion of many properties of plane figures, using appropriate terminology. For example, terms such as mid-point, vertex, parallel, intersection of lines, similar, congruent and scaling factor would arise naturally at the appropriate year level, while properties of regular polygons can also be explored.

Students use their enlargements to draw nets of selected regular polyhedra onto lightweight card, which can then be used to construct three-dimensional models. You need to decide which polyhedra students will construct.

Depending on the year level, you can introduce or revise a suitable selection of the terminology relating to polyhedra listed below.

- A *polyhedron* is a three-dimensional figure bounded by polygonal regions—i.e. it has polygons as its faces.
- A *regular polyhedron* has identical regular polygons for all its faces.
- A polyhedron is *convex* if, whenever you join up two points on the surface of the solid with a straight line, every point on the line is inside or on the surface of the solid.

There are only five possible regular convex polyhedra—this was discovered by the ancient Greeks. These five polyhedra are known as the *Platonic solids*. These solids and their nets are illustrated on Handout 4.1, The Five Platonic Solids, which also indicates suitable lengths for the edges in order to produce a fairly rigid solid whose net fits onto an A4 sized card.

You can either give each student a copy of Handout 4.1, or reproduce it on an overhead projector transparency for display to the entire class.

For younger students, this activity can be motivated by using the solids constructed to make mobiles and other decorations.

For older students, this lesson can also provide opportunities to investigate and discuss properties of polyhedra, such as the number of vertices, edges and faces and their relationship. Year 9 students can verify that Euler's formula applies to these solids.

The lesson provides an opportunity for students to develop techniques for accurate construction. The challenge for students is to produce accurate nets and to construct their solids as neatly as possible.

It is important that the enlarged shape used as the template for the net is constructed accurately. Make sure that students use a sharp pencil and are careful with their construction and cutting.

Students should always use a fine-point pen and the edge of the MATHOMAT to lightly score all fold lines. This is a very important step in constructing neat solids with sharp edges. However, it takes practice to achieve score lines which are sufficiently firm but which do not rip the card. Students should practise this on scrap pieces of card before scoring their nets. The fact that the resulting edges are marked in pen does not detract from the finished solid.

This lesson will need to be continued over a number of class periods if all aspects are to be covered.

Lesson Outline

1. Enlarging regular polygons

A technique for enlarging regular polygons is described in Handout 4.2, How to Enlarge Regular Polygons.

Younger students can be shown how to enlarge an equilateral triangle and a square.

Older students can be shown how to enlarge an equilateral triangle only and then be challenged to generalise the method to other regular polygons.

Begin the lesson by demonstrating the chosen technique(s) to the class.

The technique depends on the fact that for regular polygons with an odd number of sides the lines joining the vertices to the mid-points of the opposite sides intersect in a point, while for those with an even number of sides the diagonals joining opposite pairs of vertices intersect in a point.

Use the activity to motivate a class discussion on various properties of triangles. For example, older students should notice that the lines joining the vertices to the mid-points of opposite sides intersect in a point (as expected).

The sides of the enlarged triangle are parallel to those of the original triangle. (In fact this feature can be used to produce the new triangle, using the parallel line guides on the MATHOMAT. This can be done by drawing parallel lines at equal distances from the three sides of the original triangle and using the intersections of these lines as the vertices of the new triangle.)

There are many examples of similar triangles and isosceles triangles contained in the construction of the enlarged triangle—with older students these can be used to construct an argument to justify the fact that the new triangle is also equilateral.

Challenge students to enlarge various regular polygons from their MATHOMAT. After students have had some practice, ask them to produce enlargements with specified side lengths (this is difficult to do accurately and reasonable approximations should be accepted).

2. An introduction to polyhedra

The enlargement procedures developed in part 1 of this lesson will be used by students to draw nets and construct three-dimensional models.

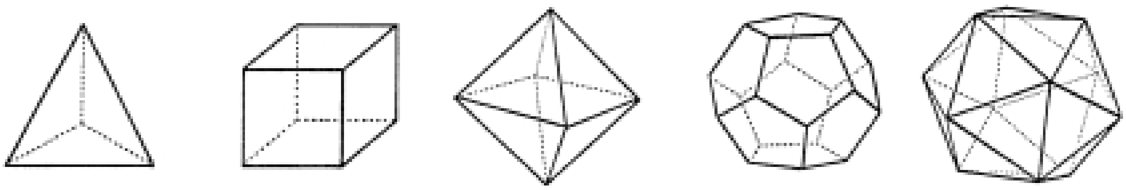
The degree of detail and the number of models constructed will depend on the year level and the time available to pursue these aspects.

Begin with a discussion of the models you have selected for the students to construct. Depending on the year level, students can be introduced to some of the terminology for regular polyhedra—see page 33 for some definitions.

Show students some examples of commercial or ‘home-made’ regular polyhedra (e.g. straw models, mobiles, models constructed form the three-dimensional construction toys such as Polydron or Geoshapes, models from card made in other classes, or even any box in the shape of a cube).

Explain to students that the Greeks had already discovered that there are only five possible regular convex polyhedra—these are illustrated below and on Handout 4.1. Their names, apart from the cube, are derived from the Greek words for the number of faces—e.g. ‘tetra’ for ‘four’ and ‘hedron’ for ‘face’.

Tetrahedron	Cube	Octahedron	Dodecahedron	Icosahedron
4 faces	6 faces	8 faces	12 faces	20 faces



These five polyhedra are known as the *Platonic solids* and have been the subject of interest to mankind for a very long time. For example, Kepler at first believed that, as part of God’s plan, the motion of the planets was related to the Platonic solids.

3. Using the MATHOMAT to create nets and construct polyhedra

The five Platonic solids all have faces which are either equilateral triangles, squares or regular pentagons.

Nets for each of the Platonic solids, together with approximate side lengths to allow comfortable construction from an A4 sheet of card, are shown on Handout 4.1, The Five Platonic Solids.

Depending on the year level and the time available, select one or more of the polyhedra for students to construct (or allow students to select their own). The first step in the construction of the nets is to use the MATHOMAT to draw and then enlarge the appropriate shape to a suitable size for constructing the solid. The enlarged shape should be drawn on a piece of card, which can then be cut out and used as a template to construct the net.

Before cutting out the net, students should lightly score their nets by using a fine-point pen and the edge of the MATHOMAT to lightly rule over each fold line on their net. This will result in neat, sharp edges on their solid.

After carefully cutting out their nets, students can use sticky tape to secure the faces.

For a better result (but one which takes a great deal more time and effort!) students can construct flaps on appropriate faces and use glue instead of sticky tape for the construction.

4. (Optional) investigating properties of polyhedra

A brief discussion of the polyhedra can include counting the faces and checking that—apart from the cube—their names represent the number of faces. Other terminology such as parallel edges, parallel faces, vertices, may also arise in the discussion.For younger students, this activity can be motivated by using the models constructed to make mobiles or other decorations.

For older students, this lesson can also provide opportunities to investigate and discuss properties such as the number of vertices, edges and faces and their relationship. Finding efficient ways of counting the numbers of edges and vertices is a challenging exercise which develops students' visualisation skills. Encouraging different students to explain their ways of counting can lead to an excellent discussion and a much better understanding of the solids.

Year 9 students can verify that Euler's formula applies to these solids.

[Euler's formula for simply connected solids states that

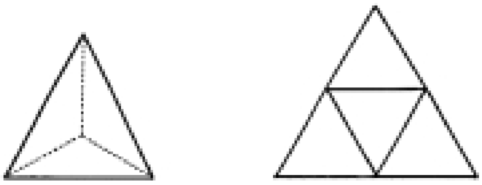
$$F + V - E = 2$$

where F is the number of faces, V the number of vertices and E the number of edges.]

The Five Platonic Solids

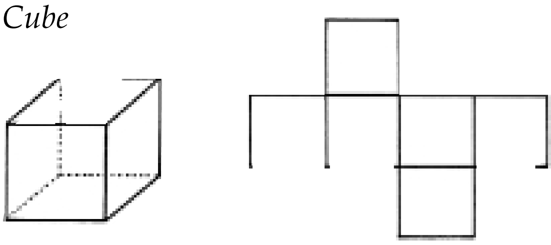
The five regular convex polyhedra (also known as the Platonic Solids) and their nets are drawn below. Next to each net is a suggested side length for each shape which produces a fairly rigid solid whose net fits onto an A4 sized card.

The Tetrahedron



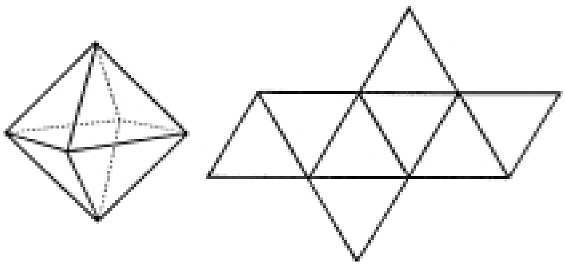
9 cm

The Cube



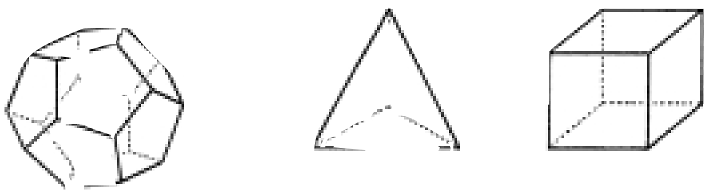
6 cm

The Octahedron



6 cm

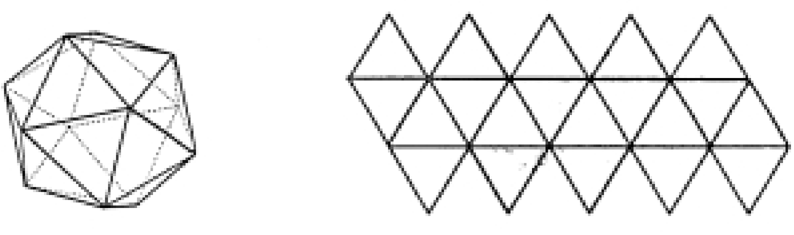
The Dodecahedron



3.5cm

(or 4.5 cm with two halves of the net on a sheet of A4 each, joined along line AB)

The Icosahedron



4.5 cm

How to Enlarge Regular Polygons

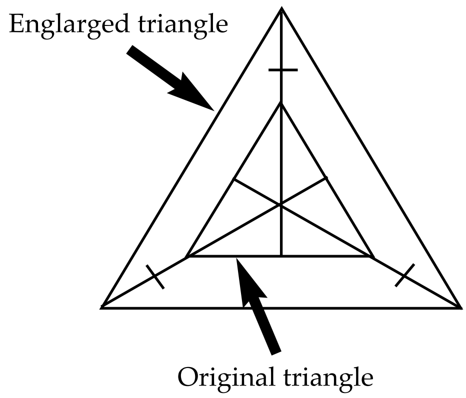
How to enlarge an equilateral triangle

Use your MATHOMAT to draw an equilateral triangle (shape 23) and mark the mid-points of the sides.

Join the mid-points of the sides to the vertices and continue your line beyond each vertex.

Mark off equal lengths beyond the vertex on each of these lines.

Join these points to make your new enlarged equilateral triangle.



This procedure can be used for all regular polygons with an odd number of sides.

How to enlarge a square

To enlarge a square, use shape 1 and repeat the same procedure as for the triangle, but join pairs of opposite vertices instead of mid-points and vertices—see diagram.

This procedure can be used for all regular polygons with an even number of sides.