LESSON

2

Tiles, Tiles and More Tiles

Years 4 to 9

Patterns Using Transformations

This lesson fosters appreciation of the role of mathematics in art. It also provides students with the opportunity to demonstrate their creativity and gain personal satisfaction by drawing their own geometrical designs. Students can also develop their skills by transforming basic stencil shapes.

In this lesson students will:

- investigate the mathematics of decorative patterns;
- create decorative patterns using tessellations;
- identify and use transformations; and
- develop their knowledge of mathematical terminology.

Materials Required

For each student:

- a MATHOMAT;
- unlined paper (scrap paper will do);
- a sharp pencil;
- an eraser; and
- colouring materials such as felt-tip pens, pencils, crayons (optional).

Additional materials:

- examples of tessellations, including some by M. C. Escher.

Lesson Summary

- An introduction to tessellations, including examples of M. C. Escher's work;
- finding which MATHOMAT shapes tessellate using only one shape;
- producing tessellations using two or more shapes; and
- (optional extension) students produce their own Escher-style tessellations.

For the Teacher

A *tessellation* is a pattern which completely covers a surface or plane without any gaps or overlapping of the shapes used.

The challenge for students in this lesson is to systematically find which MATHOMAT shapes can be used—alone or in combination with other shapes—to tessellate the plane.

The lesson provides opportunities for students to discuss shapes (at an appropriate level) and to classify them according to their properties (such as whether or not they have all their sides of equal length, whether or not they possess parallel sides). With older students, some of the terminology of transformation geometry can also be introduced.

The extension activity, which can be completed at home, allows students to further express their creativity.

This lesson will need to be continued over a number of class periods if all aspects are to be covered.

In this lesson, encourage students to use a sharp pencil instead of a pen so that lines can be erased easily.

You will need to collect examples of tessellations (including some by M. C. Escher) to show the students. You may wish to produce one or more overhead transparencies of examples. Some examples which can be used are included in the Lesson Outline.

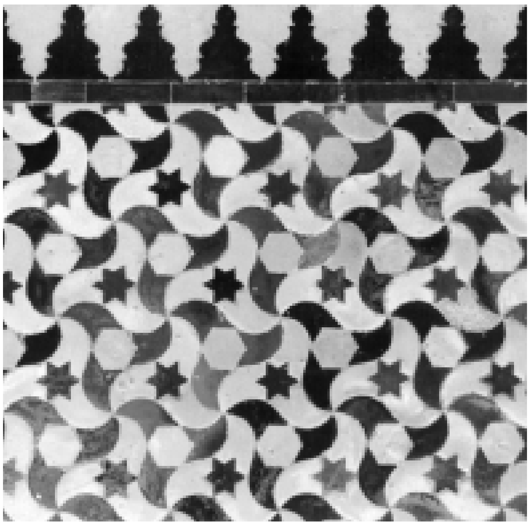
Lesson Outline

1. An introduction to tessellations—including viewing examples of M.C. Escher's work

Explain to students that a *tessellation* is a pattern which completely covers a surface or plane without any gaps or overlapping of the shapes used.

Some (boring!) tessellations include the types of tilings on most bathroom floors.

More exciting examples include the tilings produced by the Moors in places such as the Alhambra in Spain—an example of such a tiling is reproduced here.



The famous artist M. C Escher (1898–1972) derived much of his inspiration for space-filling designs from his first visit to the Alhambra in 1922. However, unlike the Moors, who were forbidden to use 'graven images' and who therefore only used geometric shapes for their tiling patterns, Escher attempted to completely cover the plane (i.e. create tessellations) with shapes which represented objects, such as animals or birds.

Most of Escher's work—which does not only consist of space filling designs—is mathematical. It is now widely available—for example as posters, in books, on coffee mugs, on the 'fifteen puzzle', on T-shirts, as jigsaw puzzles.

Two of Escher's space filling designs have been reproduced below, but you will probably want to collect your own examples to show your class.

2. Finding which MATHOMAT shapes tessellate using one shape at a time

Ask students to find which of the MATHOMAT shapes tessellate, using just one shape at a time.

Students can work individually or in groups to classify the shapes into two categories—those which do tessellate by themselves and those which don't.

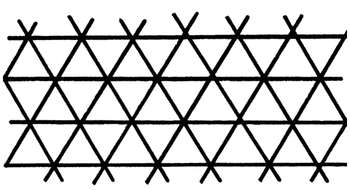
Working in groups allows this preliminary task to be completed more quickly and also encourages students to discuss their work with one another, thus helping them to develop terminology related to the shapes they are using.

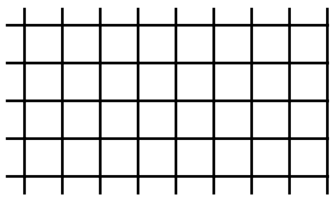
3. Sharing and discussion

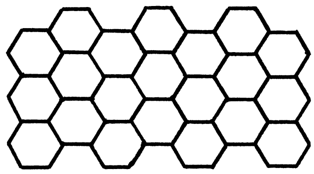
When most groups have completed the task, ask the class to share and discuss their solutions.

This discussion provides an excellent opportunity for students to revise terminology related to polygons—for example, the fact that a *regular* polygon is one which has all its sides of equal length and all its angles equal.

During the discussion, students will learn that there are only three *regular* polygons which can be used to completely cover the plane using just one shape at a time—the square, the equilateral triangle and the regular hexagon. These are reproduced below.



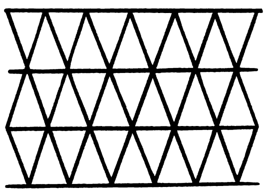




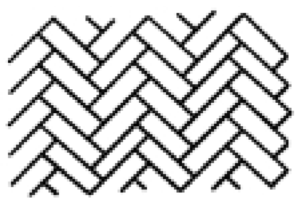
In addition, every convex quadrilateral and every triangle tessellates.

Some examples of the tiling patterns which can be formed using triangles and quadrilaterals on the MATHOMAT are reproduced below.

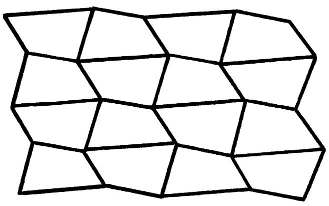
Isosceles Triangles



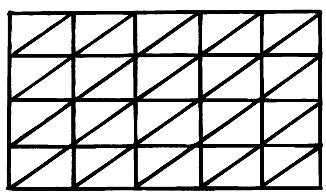
Rectangles



Quadrilaterals



Right Angled Triangles



4. (Optional extension for older students) introducing transformation geometry

For older students, this can be an appropriate time to introduce some terminology related to transformation geometry. All of the above tessellations (including the two examples by M.C. Escher) can be produced by starting with just one shape and *transforming* it by means of a combination of *translations, reflections*, and *rotations*.

Choose a tessellation using just one shape—it is best to use interesting ones like the examples by M. C. Escher for this activity. Ask students to select two copies of the shape, one near the centre of the design and one near the edge. Challenge students to find how to transform the shape near the centre into the one near the edge, using a combination of translations, reflections and rotations.

Students who have no previous experience with translations, reflections and rotations would need to do some preliminary activities before tackling this task. They might be also interested to know that certain translations and reflections combine to give the last remaining possible transformation which preserves length—namely the *glide reflection*. Most students are unlikely to have much experience of glide reflections, whereas they are usually familiar with translations, reflections and rotations.

5. Producing tessellations using two or more shapes

Homogeneous tessellations are ones which use two or more regular polygons to tessellate the plane in such a way that the pattern formed at each vertex is the same.

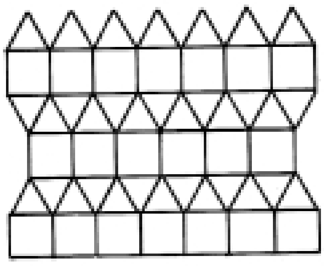
Challenge students to produce as many tessellations as they can using two or more MATHOMAT shapes in such a way that the pattern formed at each vertex is the same—i.e. challenge them to find as many homogeneous tessellations as possible.

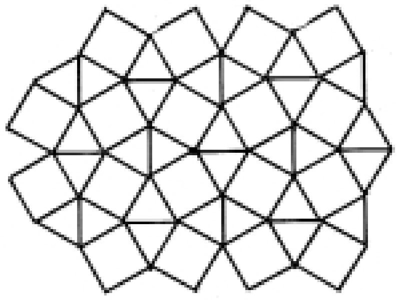
Altogether, there are eight homogeneous tessellations—they are reproduced opposite.

It is possible to produce the first six of these using MATHOMAT shapes 32 (equilateral triangle), 18 (square), 20 (regular hexagon) and 22 (regular octagon), which all have the same side length

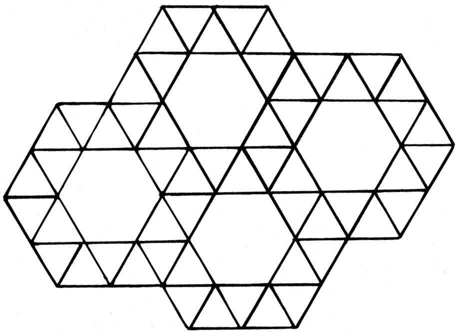
In order to produce the last two of these homogeneous tessellations, you need to construct a regular dodecagon using the half dodecagon shape 39 which has the same side length as the other regular polygons.

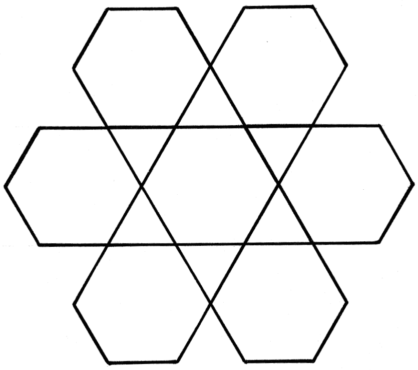
Squares and Triangles



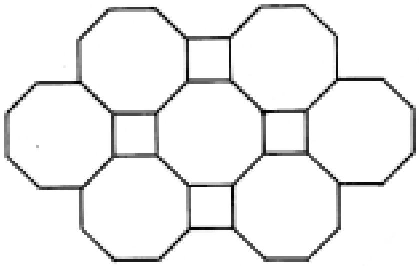


Hexagons and Triangles

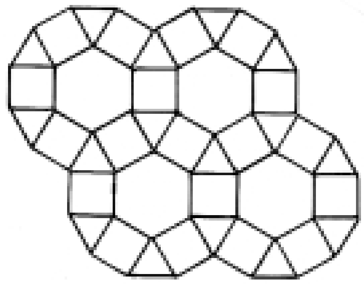




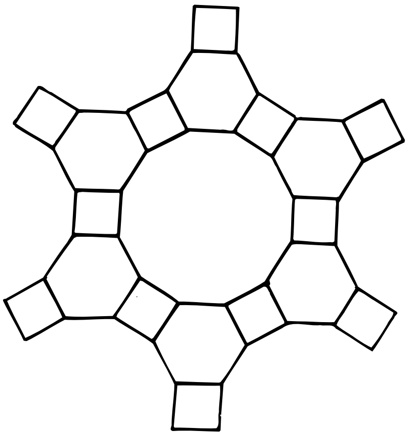
Octagons and Squares



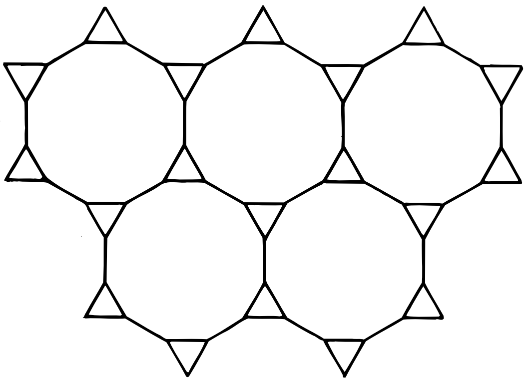
Hexagons, Squares and Triangles



Dodecagons, Hexagons, Squares and Triangles



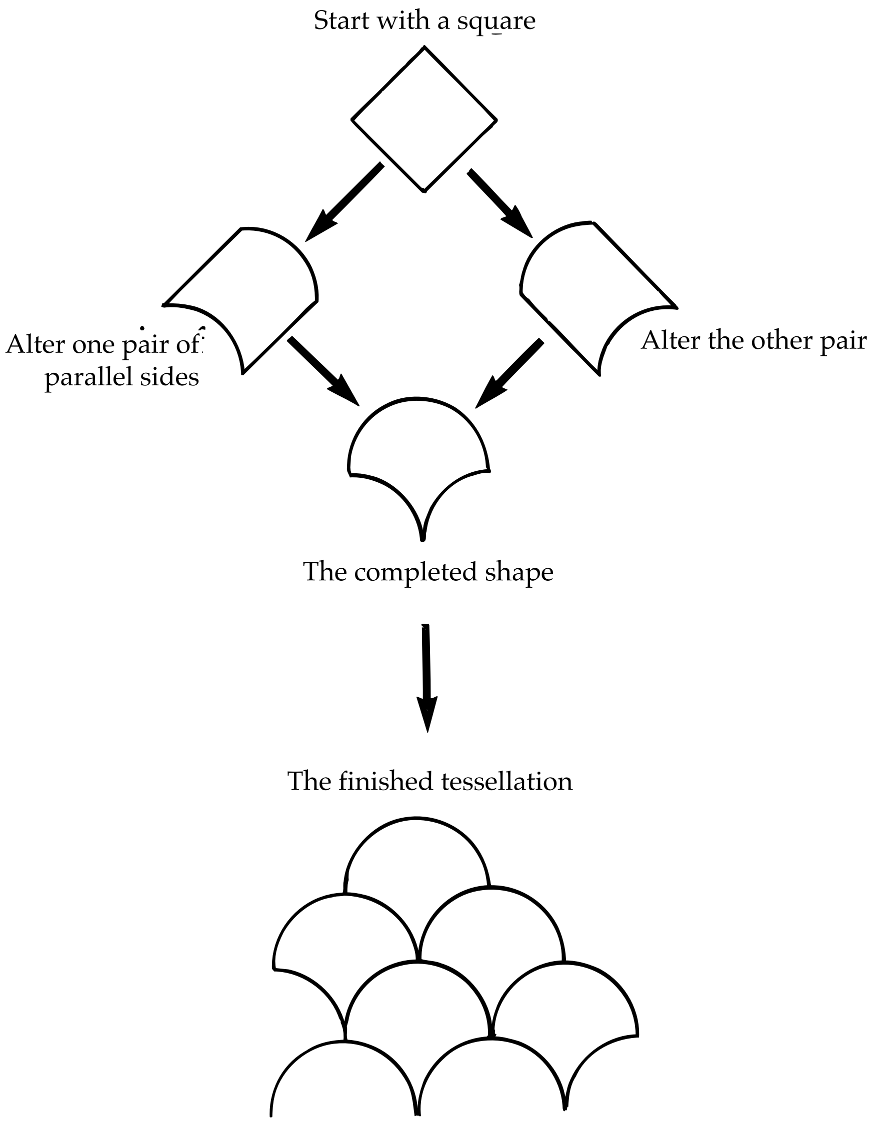
Dodecagons and Triangles



6. (Optional extension) students produce their own Escher-style tessellations.

The simplest technique for producing an Escher-style tessellation is to start with any tessellation which uses a single shape which has at least one pair of parallel sides of equal length—e.g. a square, a rhombus, a parallelogram, a regular hexagon, or a rectangle.

The technique is to change one side and then alter the parallel side in the same way, as shown in the example below. This procedure can be repeated for all pairs of parallel sides if desired.
Ask students to use this technique to design an Escher-type shape which tessellates. It takes some time and quite a bit of experimentation to produce an attractive shape which tessellates. Students may need to experiment with several different tessellations which use polygons with pairs of parallel sides and follow the technique described above before they find a satisfactory shape.



Once students have decided on their shape, the simplest way to produce a tessellation using their design is to follow the steps below.

Step 1: Trace one copy of the shape onto the original tessellation somewhere near the centre of the page, making sure it 'fits onto' the original shape. It is best to use a dark pen or a soft pencil so that the design will be clearly visible in Step 2 below.

Step 2: Place an unlined sheet of paper over the tessellation and trace the design onto the paper.

Step 3: Move the top sheet of paper until the traced design 'fits onto' one of the adjacent shapes on the original tessellation. Trace the shape again—this will give you a second (tessellating) copy of the design on the top sheet of paper.

Step 4: Repeat Step 3 until the tessellation is complete—you may need to 'move to a new place' on the sheet as parts of the tessellation are completed.

This method of producing the tessellation avoids the need to erase gridlines from the final copy.

Students may wish to colour their tessellation, taking care to produce a repeating pattern of colours. (While finding repeating patterns of colour for tessellations in general is itself a mathematical problem, older students can usually find a way to create a repeating pattern of colours for the particular case of their own tessellation.)

As it takes a long time to produce an attractive shape and even longer to produce the final design, students can be asked to complete this as a home-work task.

7. Other possible extensions

There are many possible extensions of this lesson. A Deakin University postgraduate student, Sally Wilson, recently developed and taught a four lesson unit of work based on tessellations to a Year 4 class.

Students began by using the MATHOMAT to make symmetrical patterns and moved through a variety of activities, culminating with the production of a class quilt which was presented at the school assembly. As part of their activities, students manipulated polygons on a magnetic board to form tessellations, drafted a design for their quilt block, made a polygon template, chose suitable fabric and accurately cut out the polygons for the quilt block, and finally sewed fabric polygons together to form the class quilt.
Materials used included the MATHOMAT, magnetic polygonal pattern blocks and a magnetic board, quilting and patchwork books and brickwork samples, example of tessellations produced by children of the same age (obtained from the internet—see references below), as well as a 'Big Book' produced by Sally for use with the class.

References

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