LESSON

6

The Area is Right!

Years 4 to 8

Estimating and Measuring Areas

In this lesson students compare the areas of the individual MATHOMAT shapes, firstly by estimation and then by measurement. Students initially work with informal units and older students convert their answers into formal units.

MATHOMAT shapes are used to investigate packing problems—fitting objects such as soft-drink cans into containers.

In this lesson students will:

- use estimation to rank MATHOMAT shapes according to area;
- measure the area of the shapes by drawing them on grid paper and counting squares; and
- (optional extension) investigate the packing of soft-drink cans into containers of different shapes.

Materials Required

For each student:

- a MATHOMAT;
- unlined paper (scrap paper will do);
- fine-point pens or pencils; and
- grid paper of various sizes (photocopy masters for 5 mm and 2 mm grid paper are provided at the end of this lesson —see Handouts 6.2 and 6.3).

For each group of four students:

- a copy of The Area is Right! Entry Form (photocopy master provided—see Worksheet 6.1); and
- a copy of the Area of MATHOMAT Shapes sheet (a photo copy master is provided as Handout 6.1, although making a copy of this sheet for each group is optional—see later sections for details).

Lesson Summary

- Ranking MATHOMAT shapes by estimating their areas;
- measuring areas by tracing shapes on grid paper and counting squares; and
- (optional extension) investigating the packing of selected shapes into larger shapes.

For the Teacher

In this lesson students compare the areas of the individual MATHOMAT shapes.

This is done firstly by estimation and then by measurement.

In order to measure the areas in informal units, each shape is drawn on grid paper of appropriate size and the number of squares it covers is counted. Older students can convert their answers into formal units (cm^2 or mm^2).

Photocopy masters for both 5 mm and 2 mm grid paper are provided (see Handouts 6.2 and 6.3). It is only feasible to use 2 mm grids if the task of measuring is shared amongst a group of students and the students are able to take short cuts by calculating the area of rectangular portions within shapes. Depending on the year level of the students, you might use both the 5 mm and the 2 mm grids, or only use one of these, or only use the 2 mm grid when it is impossible to decide between the area of two shapes using the 5 mm grid.

A competitive game The Area is Right!, loosely based on a television quiz-show with a similar title, is used to add interest to the lesson.

To play the game, divide the class into groups with about four students in each group. Ask each group to use estimation to select and rank in order the ten MATHOMAT shapes with largest area. They trace and label these shapes in order on The Area is Right! Entry Form (Worksheet 6.1) and enter the shape number in the second column of the table provided. They should then also 'submit' their entries in a form suitable for display—e.g. write their rank ordering on the chalkboard. After a brief class discussion of similarities and differences between the 'entries', each group measures the areas of their selected shapes using grid paper and the class scores the entries to find the winning group.

The Area of MATHOMAT Shapes sheet (Handout 6.1) is a table of the actual areas of the MATHOMAT shapes which can assist you when the class is scoring the entries. (The ten largest MATHOMAT shapes, in descending order of area, are shapes 29, 3, 15, 4, 1, 9, 33, 21, 36, 10).

You may wish to organise the scoring of entries in a different way from that given in the Lesson Outline, in which case you may wish to give each group a copy of the Area of MATHOMAT Shapes sheet. Note that areas given in the 'Area in 5 mm squares' and 'Area in 2 mm squares' columns are approximations and the class will need to come to an agreement as to the degree of accuracy expected in order for area measurements to be regarded as 'correct'.

As an extension activity, the MATHOMAT shapes are used to investigate a realistic problem involving fitting soft-drink cans into containers.

Lesson Outline

1. Using estimation to rank MATHOMAT shapes according to area

Start the lesson with a brief class discussion on area.

Ask students to select the MATHOMAT shape which they think has the largest area and discuss reasons for their choices. Repeat this for the shape with the smallest area.

Explain to the students that their estimates of relative areas will be the basis for the game The Area is Right! Explain how to play the game, including the way in which the entry form is to be used.

Organise students into small groups with about four students in each group and give each group an entry form (Worksheet 6.1).

Ask each group to select and rank in order the ten MATHOMAT shapes with largest area, using estimation only—at this stage, counting squares is not allowed. As each group agrees on the selected shapes and their order ask them to trace and label these shapes, in order, on The Area is Right! Entry Form and enter the shape number in the second column of the table provided.

Each group must then also 'submit' their entries in a form suitable for display—e.g. by writing their rank ordering on the chalkboard.

As each group submits their entries, give them copies of the appropriate grid paper and ask them to start on part 2 of the lesson, measuring areas.

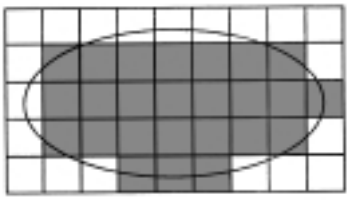
When all groups have submitted their entries, conduct a brief class discussion, comparing and contrasting the 'entries'.

2. Measuring areas

Ask each group to share the task of measuring the areas of their ten shapes among the members of the group—there is no need for each student to measure the area of each shape.

The areas of the MATHOMAT shapes can be measured by carefully drawing them onto a grid and counting squares (photocopy masters of 5 mm and 2 mm square grids are provided—see Handouts 6.2 and 6.3). For example, the area of the ellipse (shape 4) is 25 squares on the 5 mm grid paper as shown in the next diagram. Of course this is an approximation obtained by 'balancing out' incomplete squares which are shaded with those which are not.

Measuring the area of shape 4



Ask students to record their results in the appropriate columns of the table on their group's The Area is Right! Entry Form.

Depending of the year level and the time available, the last two columns can be used for students to convert the areas to square centimetres or square millimetres.

4. Scoring the entries

Use a class discussion, based on the students' results from measuring the areas, to decide the ten MATHOMAT shapes with largest area and their correct ranked order. As some of these shapes are similar in area this will generate lively discussion and the class will need to come to an agreement as to the degree of accuracy expected in order for area measurements to be regarded as 'correct'.

Give each group a score based on the number of shapes they have in 'correct' rank order 'from the top'— e.g. if a group has the first six shapes in correct order, but their seventh shape is incorrect, they get a score of 6. The group with the highest score is declared the winner.

5. Optional modifications

The game may be modified in a number of ways. For example, the students may select and rank the ten smallest MATHOMAT shapes. Alternatively, the selection of sets of shapes for ranking could be based on geometric properties of shapes, such as:

- polygons;
- triangles;
- shapes with at least two side parallel; or
- ellipses and circles.

6. (Optional extension) a packing problem

Organise the students into groups and present them with the following problem.

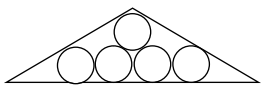
Packing soft drink cans

Your task is to design an unusual and attractive container for soft drink cans. However, it is also important to consider how efficiently the cans are packed into the container.

You are trying to decide between a number of different shapes to use as the base of the container. These are represented on the MATHOMAT by shapes 8, 20, 24 and 23. On the same scale, the soft-drink cans are represented by MATHOMAT shape 14.

Investigate fitting shape 14 into the other shapes and decide which shape you would choose for the container, giving reasons.

For example, the cans might be packed into the triangle as shown below.



Extension for older students: Estimate the percentage of waste space for different shapes, using your earlier area measurements.

After the groups have tackled the problem, conduct a class discussion. Ask the groups to share the results of their investigations and give reasons for their choice of shape for the container.

As an appropriate follow-up of this problem, you may wish to discuss broader issues associated with packaging, particularly as exemplified by the packaging of articles that may be purchased in super-markets.

The Area is Right! Entry Form

GROUP SCORE

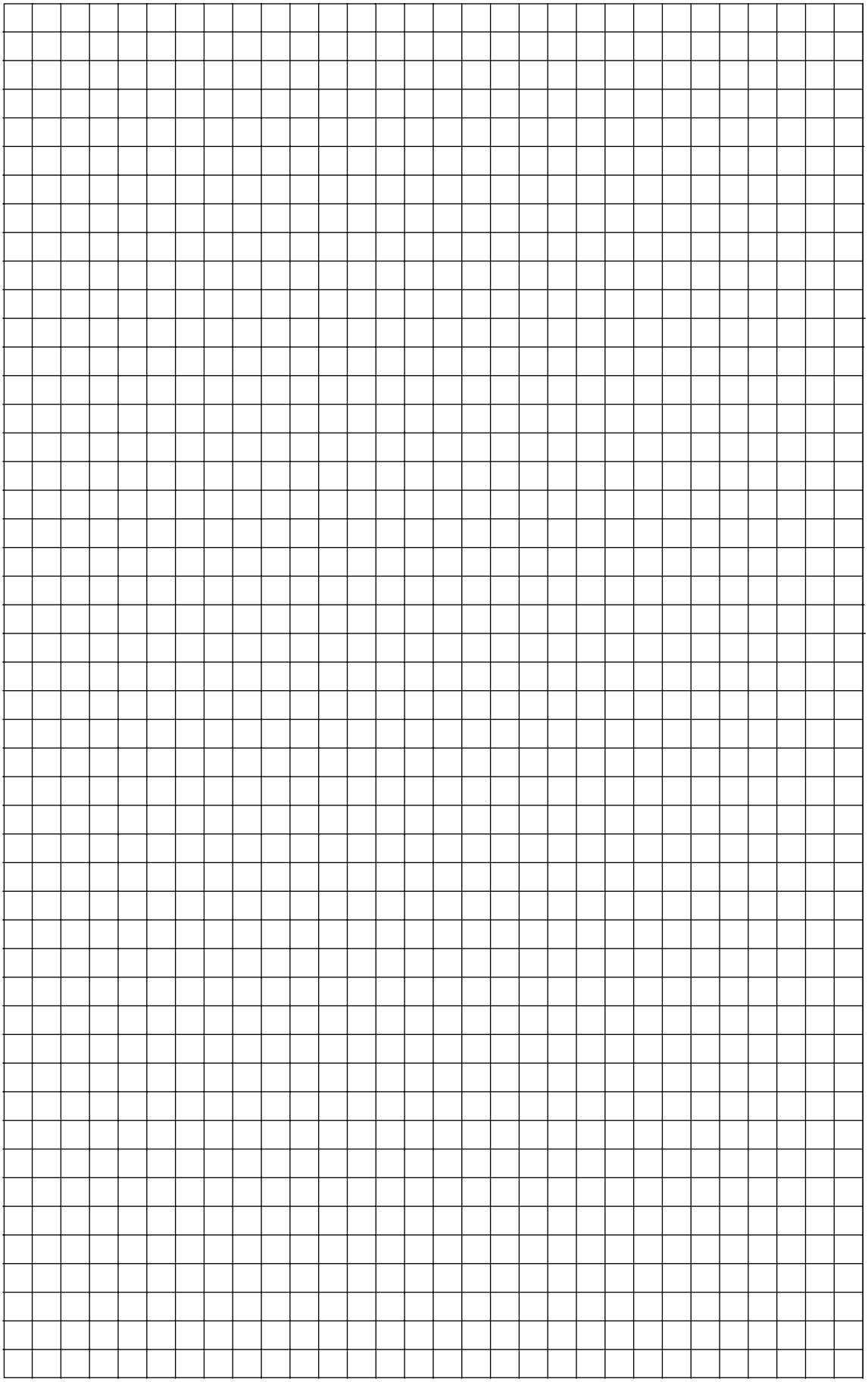
In the space below, trace and lable the ten MATHOMAT shapes you estimate have the largest area. Start with the largest shape and then enter the others in order—as you do this, record each shape's number in the second column of the table.

Ranking by area	Shape Number	Area in 5 mm squares	Area in 2 mm squares	Area in cm^2	Area in mm^2
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					

Area of MATHOMAT Shapes

Shape number	Area in 2 mm squares	Area in 5 mm squares	Area in cm^2
1	156	25	6.25
2	123	20	4.91
3	314	50	12.57
4	157	25	6.27
5	109	17	4.36
6	47	26	1.89
7	66	11	2.65
8	79	13	3.14
9	146	23	5.85
10	134	21	5.35
11	94	15	3.75
12	39	6	1.58
13	10	2	0.39
14	5	1	0.20
15	177	28	7.07
16	49	8	1.95
17	14	2	0.58
18	56	9	2.25
19	28	5	1.13
20	65	10	2.60
21	139	22	5.56
22	120	19	4.78
23	73	12	2.92
24	73	12	2.92
25	43	7	1.72
26	25	4	1.00
27	98	16	3.91
28	79	13	3.14
29	491	79	19.63
30	20	3	0.79
32	11	2	0.43
33	146	23	5.83
35	44	7	1.77
36	135	22	5.41
37	81	13	3.23
38	24	4	0.97
40	43	7	1.73

Grid Sheet 5 mm



Grid Sheet 2 mm