CHAPTER

3

Solving equations

CHAPTER 3
The BIG Picture

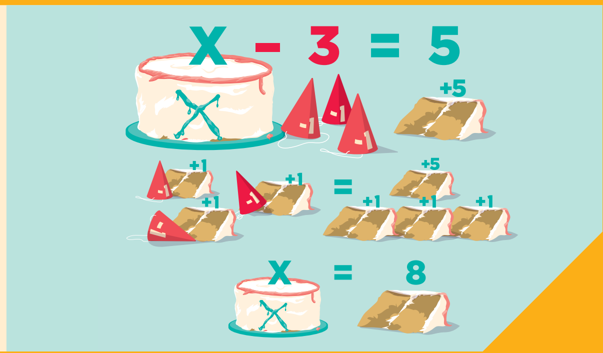
There are specific steps to take when solving an equation.

What is Chapter 3 about?

Let's take a look!

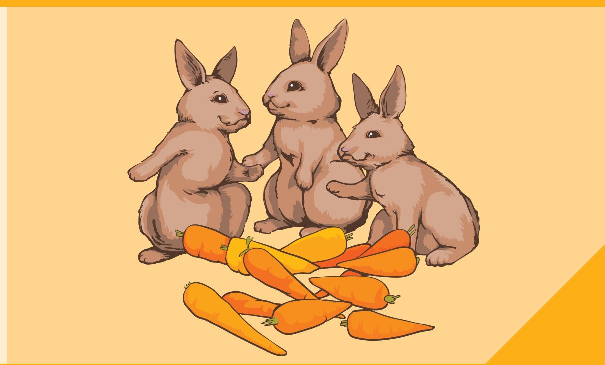
TOPIC 9

A positive or negative value is added to BOTH SIDES of the equation to make the integers add to zero.



TOPIC 10

The reciprocal is multiplied to each side of the equation to isolate the variable.

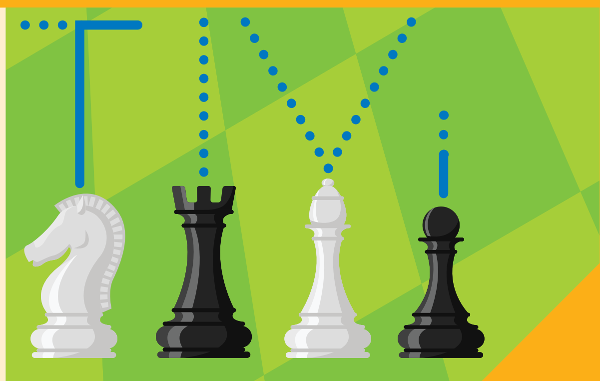


TOPIC 11

Insert the value of the variable into an equation and solve.

TOPIC 12

Solving requires operations to be used in the correct order to isolate the variable.



VOCABULARY

Equation

$$3 + x = 5$$

A mathematical statement in which two expressions are equal to each other. Uses an equal sign (=).

Variable

$$3x = 5$$

A quantity that may change within a mathematical problem. Typically, we use a single letter to represent a variable

Reciprocal

$$\frac{2}{3} \mid \frac{3}{2}$$

When you flip a number (the numerator becomes the denominator and vice versa), you have the number's reciprocal. It is also called the inverse.

Coefficient

$$3x = 5$$

The number or constant multiplied by a variable in a mathematical term. Example: 3x, the 3 is the coefficient

Substitution

$$2 + x = ?$$

$$x = 3$$

$$2 + 3 = 5$$

Inserting a known value in the place of a variable.

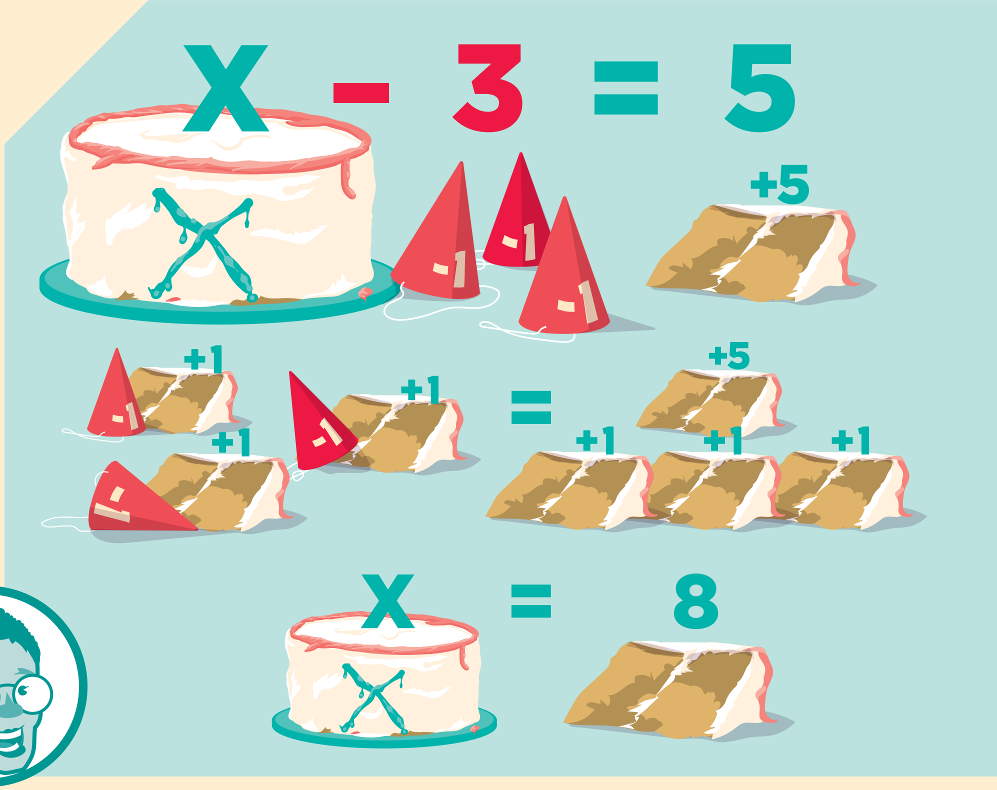
TOPIC 9 | Solve x + b = c

How do we solve an equation using addition?

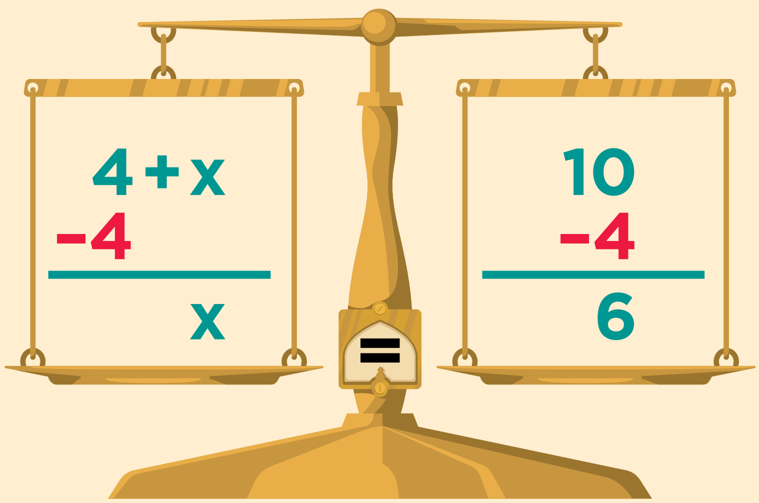
*An **equation** can be solved using the Additive Inverse Property. This means a + or – value is added to BOTH SIDES of the equation to make the integers add to zero on the left side of the equation and isolate the **variable** (x) term.*

The goal is to determine the value for the variable that will make the equation true all of the time.

Look at my cake! How many pieces did I start with if each guest had a piece and I have 5 pieces leftover? I started with 8 pieces of cake.



REMEMBER! *A positive or negative value is added to BOTH SIDES of the equation to make the integers add to zero*



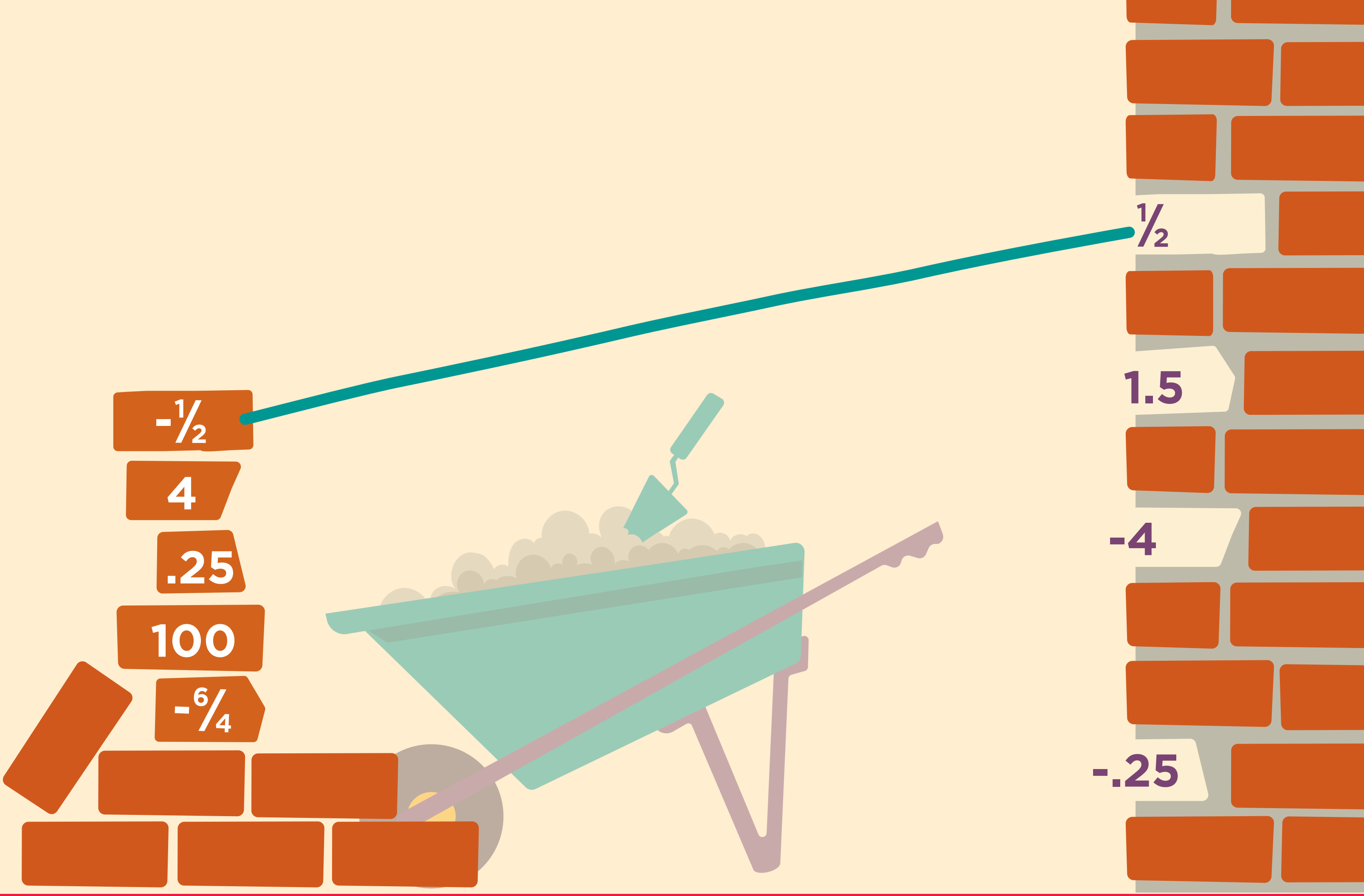
How we can we use the Additive Inverse Property to balance the equation: 4 + x = 10? We want to isolate x, so we subtract four from each side of the equation. This leaves x on the left side, and ten minus four (10 –4) on the right side. By simplifying, we find that x equals six (x = 6).

Fun Fact: The **Additive Inverse Property** means that any number added to its opposite will equal zero. (x) + (–x) = 0. By adding back what was removed, we get the whole amount.

The Additive Inverse Property doesn't just apply to whole numbers. Let's look at some fractions.

Fractions and their opposites work the same as whole numbers and their opposites. Whatever will make an expression equal to zero when added is the inverse. Sometimes, you have to convert a fraction into a decimal to see it. Let's try some more examples.

Match the numbers on the left with their inverses on the right in order to fill in the holes in the wall. An example has been done for you.



More solving x + b = c

Directions: *Solve each equation for the variable using the Additive Inverse Property. An example has been shown.*

$$x-2 = 5$$
$$x-\underbrace{2+2}_{=0} = 5+2$$
$$x = 5+2$$

$x =$ 7

x − 2 = 5

$$x+0 = 6$$
$$x+\underbrace{0-0}_{=0} = 6-0$$
$$x = 6-0$$

$x =$ ____

x + 0 = 6

$$x-3 = 1$$
$$x-\underbrace{3+3}_{=0} = 1+3$$
$$x = 1+3$$

$x =$ ____

x − 3 = 1

$$3+x = 4$$
$$\underbrace{3-3}_{=0}+x = 4-3$$
$$x = 4-3$$

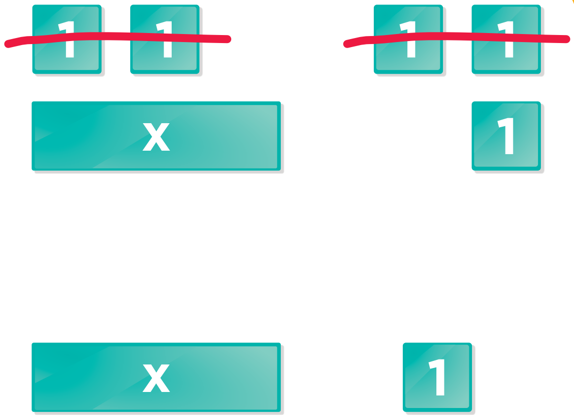
$x =$ ____

3 + x = 4

Tile it in!

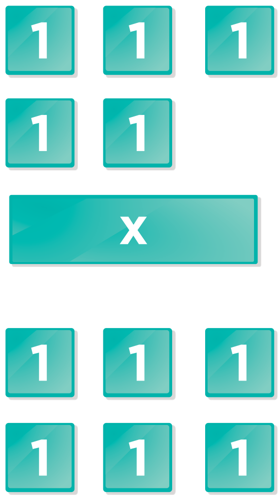
Directions: *Solve each equation for the variable using the Additive Inverse Property.*

REMEMBER! *A positive or negative value is added to BOTH SIDES of the equation to make the integers add to zero*

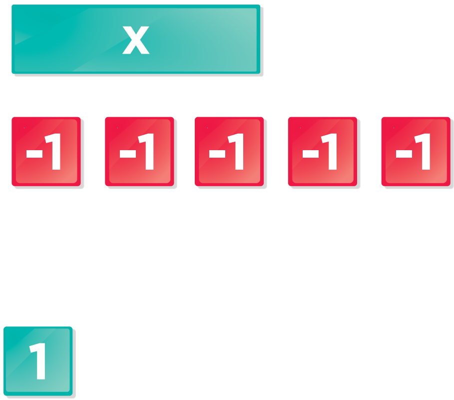


x = 1

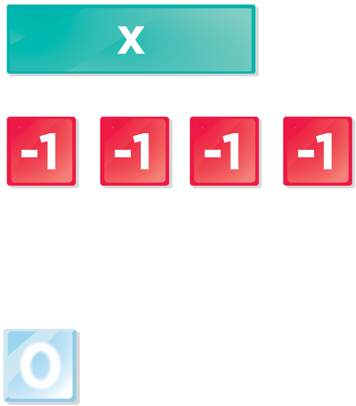
2 + x = 3



5 + x = 6



x – 5 = 1



x – 4 = 0

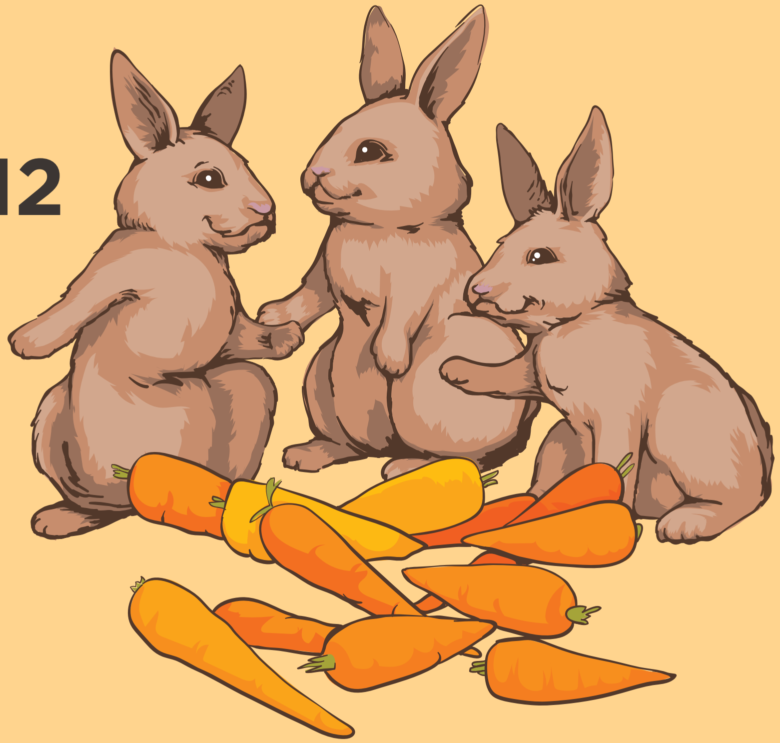
Solving ax = c

What do we do when an equation contains multiplication?

Any number multiplied to its inverse, or ***reciprocal****, will equal one. The reciprocal is multiplied to each side of the equation to make the integers multiply to isolate the variable (x) term. The goal is to determine the value for the variable that will make the equation true all of the time.*

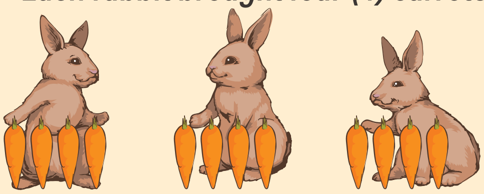
Three (3) rabbits each brought the same number (x) of carrots. They have twelve (12) carrots in total. How many did each bring?

$$3x=12$$



REMEMBER! *The reciprocal is multiplied to each side of the equation to isolate the variable.*

Each rabbit brought four (4) carrots!

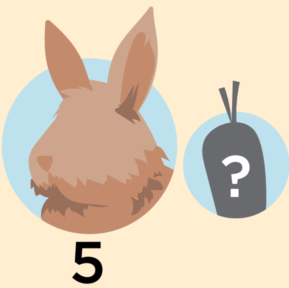


What if five (5) rabbits brought the same amount of carrots for a total of five (5) carrots? Five (5) rabbits times an unknown amount (x) of carrots equals five (5) carrots. 5x = 5

$$5x=5$$

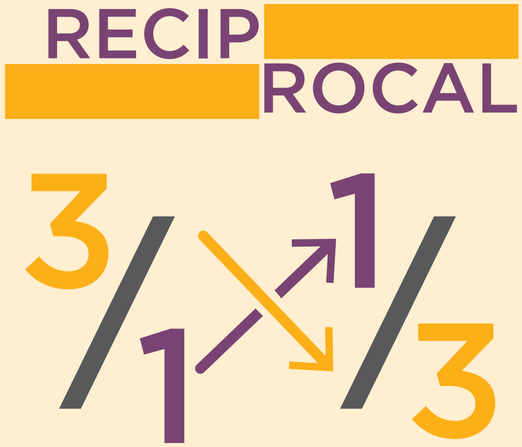
$$\frac{5}{1}\left(\frac{1}{5}\right)=\frac{5}{5}=1$$

$$5x\ \left(\tfrac{1}{5}\right) = 5\left(\tfrac{1}{5}\right) \qquad x = 1$$



Fun Fact: The **Multiplicative Inverse Property** means that any number multiplied to its inverse (reciprocal) will equal one. $(a)(1/a) = 1$ This property is very helpful in solving equations.

Finding the reciprocal of a number is easy. We just flip the numbers!



*Every whole number is that number OVER 1. For example, 10 is also $\frac{10}{1}$, 6 is $\frac{6}{1}$, and -159 is $\frac{-159}{1}$! Multiplying by the inverse is the same as dividing by the **coefficient**. When using tiles, we can't really show a fraction of a tile, but we CAN divide by the coefficient.*

Directions: *Match each number to its reciprocal.*

5	$-\frac{1}{6}$
10	$\frac{1}{5}$
-6	2
$\frac{1}{2}$	$\frac{1}{10}$

More solving ax = c

Directions: *Solve each equation for the variable using the Multiplicative Inverse Property.*

$$4x = 8$$

$$4x\left(\tfrac{1}{4}\right) = 8\left(\tfrac{1}{4}\right)$$

=1

$$x = 8\left(\tfrac{1}{4}\right)$$

x = 2

4x = 8

$$2x = -8$$

$$2x\left(\tfrac{1}{2}\right) = -8\left(\tfrac{1}{2}\right)$$

=1

$$x = -8\left(\tfrac{1}{2}\right)$$

x = ______

2x = -8

$$2x = 6$$

$$2x\left(\tfrac{1}{2}\right) = 6\left(\tfrac{1}{2}\right)$$

=1

$$x = 6\left(\tfrac{1}{2}\right)$$

x = ______

2x = 6

$$-2x = 8$$

$$-2x\left(-\tfrac{1}{2}\right) = 8\left(-\tfrac{1}{2}\right)$$

=1

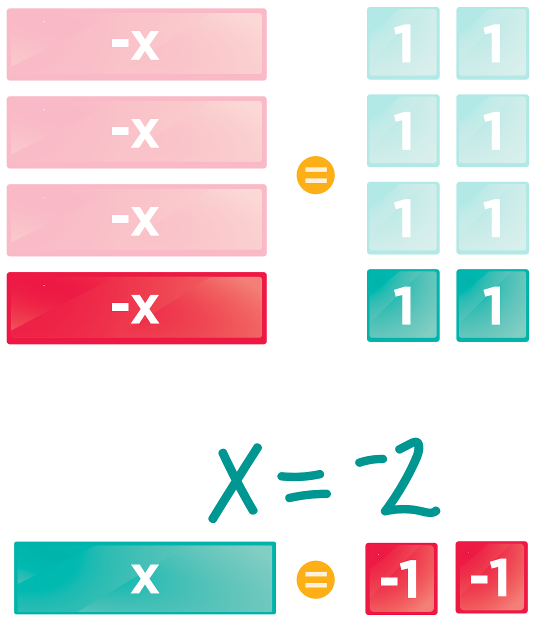
$$x = 8\left(-\tfrac{1}{2}\right)$$

x = ______

-2x = 8

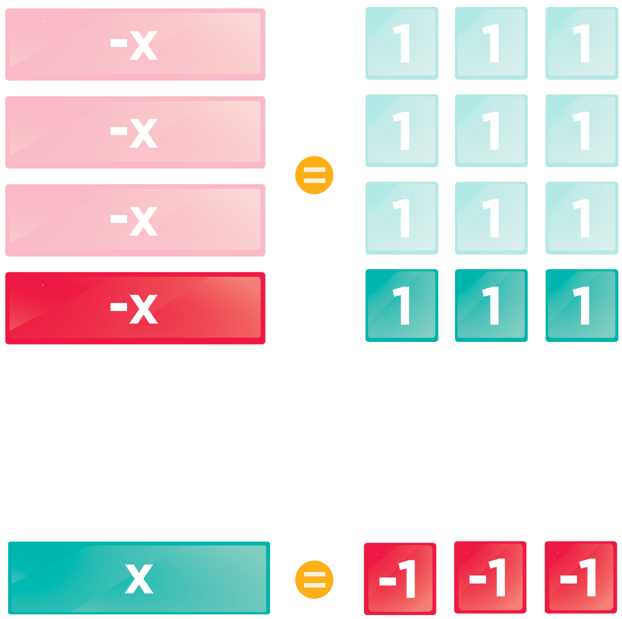
Tile it in!

Directions: *Solve each equation for the variable using the Multiplicative Inverse Property.*

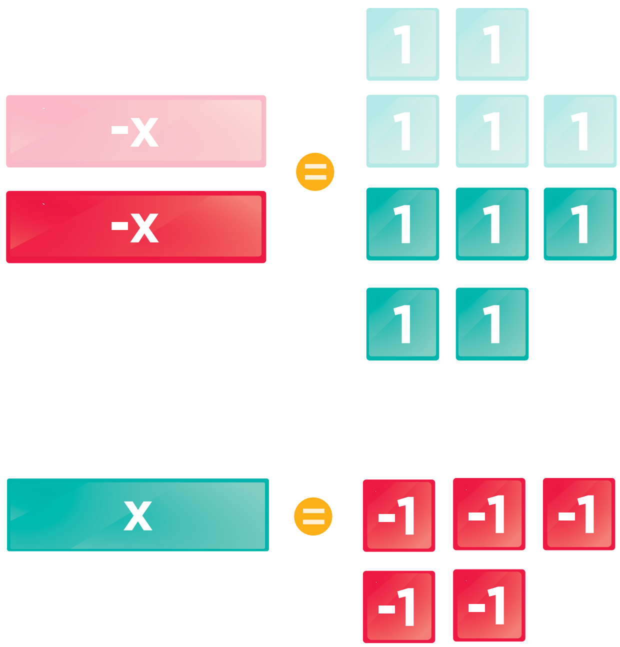


$x = -2$

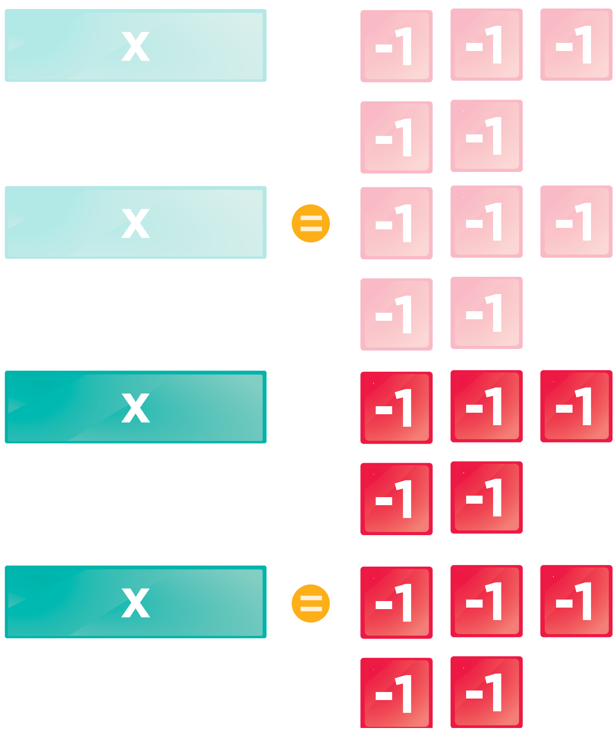
$-4x = 8$



$-4x = 12$



$-2x = 10$



$3x = -15$

TOPIC 11 | Simple substitution

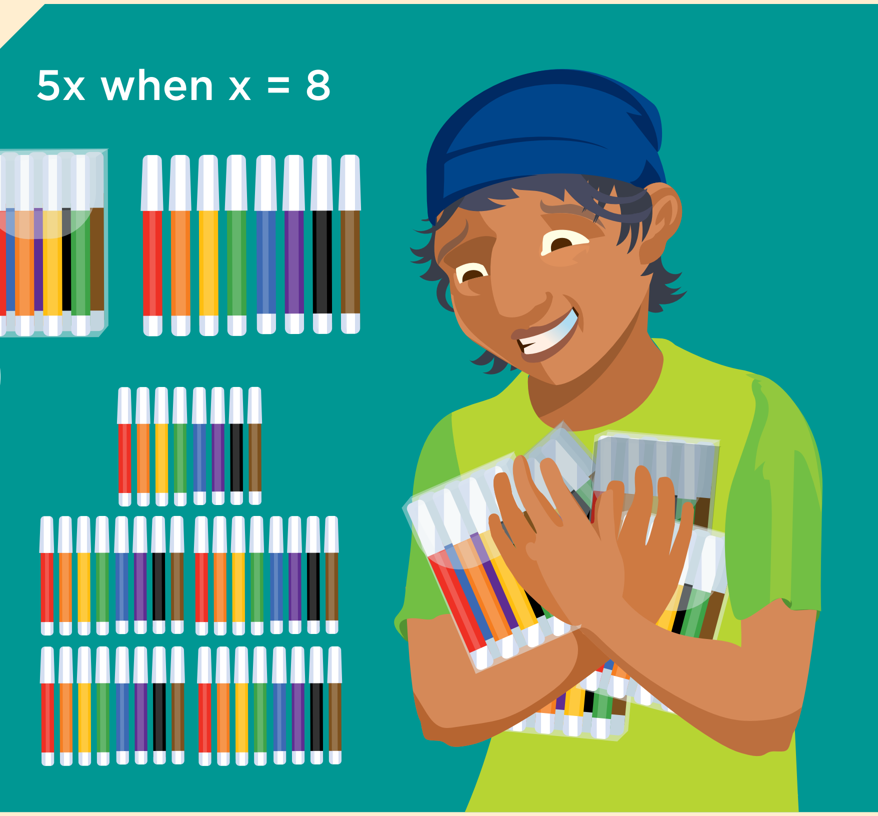
Once we know the value of x, we can solve for other integers.

*We call this **substitution**. We can insert the value of the variable into an equation and solve for any equations in the same set.*

Raj has five (5) packs of markers. Each pack holds eight (8) markers. How many markers does Raj have in total?

***REMEMBER!** Insert the value of the variable into an equation and solve.*

The total number of markers is five (5) packs times the number of each marker in a pack (x). 5x = ? The number of markers in each pack (x) is eight (8). x = 8. We substitute in the value for x and find that the total is 5(8) = 40!



We substitute words for values all of the time. A dozen eggs is equal to 12 eggs. A week is equal to 7 days. A couple of dogs is 2 dogs. How many cookies do we need for a bake sale if we need to bring three dozen?
The equations would be written: 3x = ? and x = 12. 3 (12) = 36 cookies.

3x when
x = 12

Fun Fact: The letter *x* is commonly used as a variable in algebra; however, *any* letter can be used as a variable (especially when there is more than one variable).

What do we do if there are more than one variable? Isn't that harder?

Not at all! We just do what we've learned to any extra variables. Substitute values for more than one variable and follow the order of operations to simplify the expression.

Look at the equations below and see if you can solve.

REMEMBER! *Insert the value of the variable into an equation and solve.*

$$2xy$$

$$x = 3 \qquad y = -1$$

$$2(\overset{x}{3})(\overset{y}{-1})$$

$$6(-1)$$

$$-6$$

2xy when x = 3 and y = -1

$$3x + y$$

$$x = 3 \qquad y = -1$$

$$3(\overset{x}{3})+(\overset{y}{-1})$$

$$9+(-1)$$

+-1= -1

$$9-1$$

$$8$$

3x + y when x = 3 and y = -1

More simple substitution

Directions: *Simplify the expression using the information about the variable.*

$x + 7; x = 2$

$(2) + 7$

____

X + 7 when x = 2

$4x; x = -1$

$4(-1)$

____

4x when x = -1

$x + 2; x = 4$

$(4) + 2$

____

X + 2 when x = 4

$10x; x = 2$

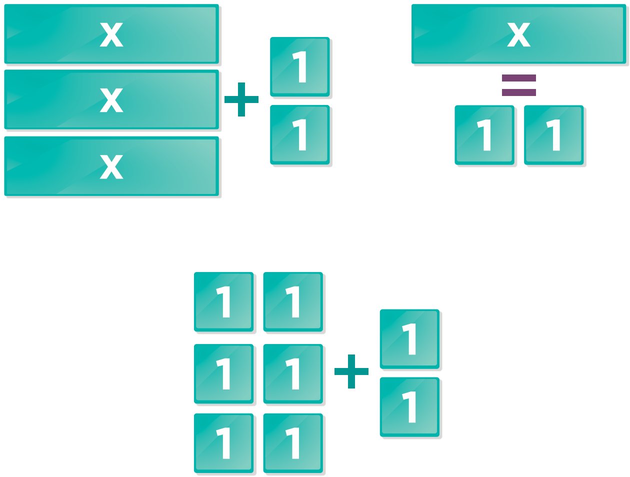
$10(2)$

____

10x when x = 2

Tile it in!

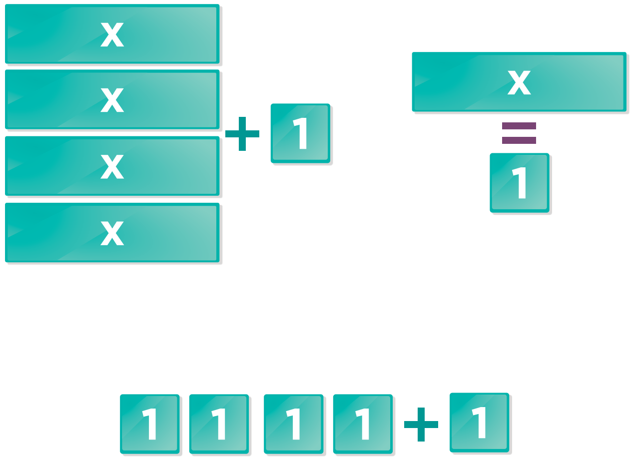
Directions: *Simplify the expression using the information about the variable. Remember to follow the order of operations (multiply first then add or subtract).*



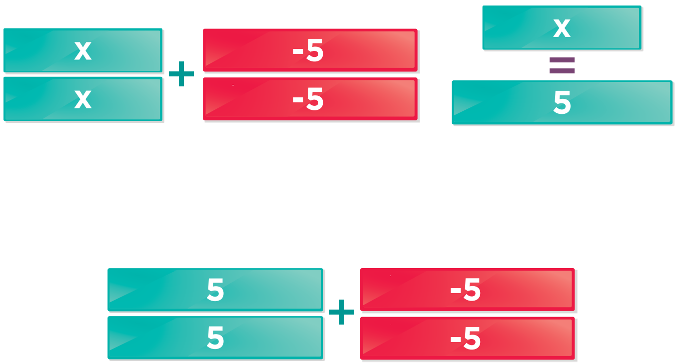
3(2)+2

3x + 2 when x = 2

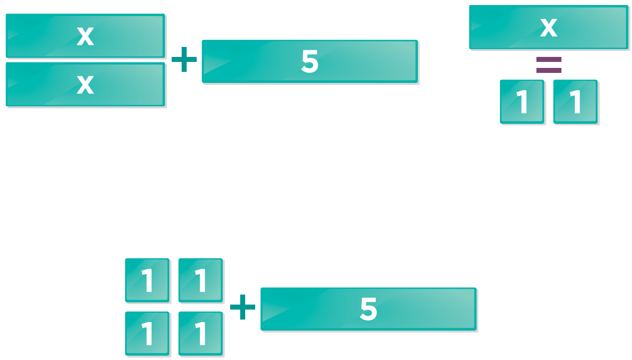
6+2=8



4x + 1 when x = 1



2x – 10 when x = 5



2x + 5 when x = 2

TOPIC 12 | Solving equations with variables on one side of the equal sign

Time to use what we've learned and solve some equations!

Solving equations means finding a value for a variable that will make the equation true. Solving requires operations to be used in the correct order to isolate the variable. We're trying to move the variable to one side of the equal sign and the known integers to the other.

Each piece in a game of chess has a specific type of operation, or move. A "legal move" is a step that follows the rules. Algebra also has certain steps, or "legal moves", that must be done in order to properly balance and solve an equation.

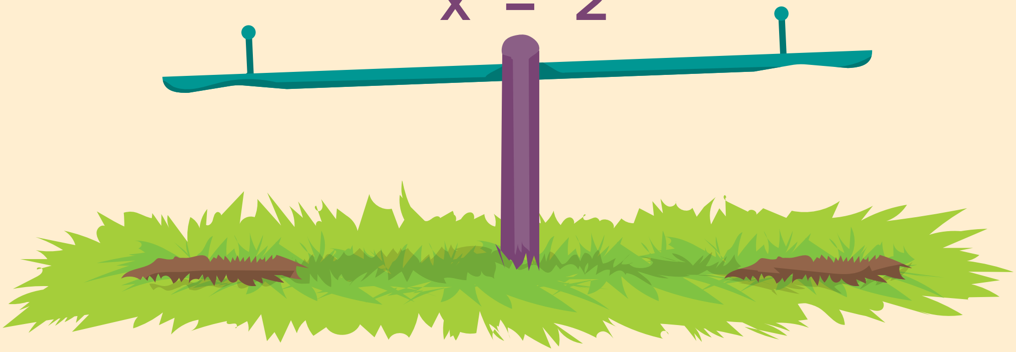
REMEMBER! *Solving requires operations to be used in the correct order to isolate the variable.*

$$3x + 2 = 8$$

$$3x + 2 - 2 = 8 - 2$$

$$\frac{\cancel{3}x}{\cancel{3}} = \frac{6}{3} = \frac{2}{1} = 2$$

$$x = 2$$



First, we need to simplify an equation as much as we can. Any operation we perform on one side of the equal sign needs to also be performed on the other side. We then multiply or divide to remove any ***coefficient*** *of the variable. With a single variable on one side of the equal sign and a value on the other, we've solved the equation!*

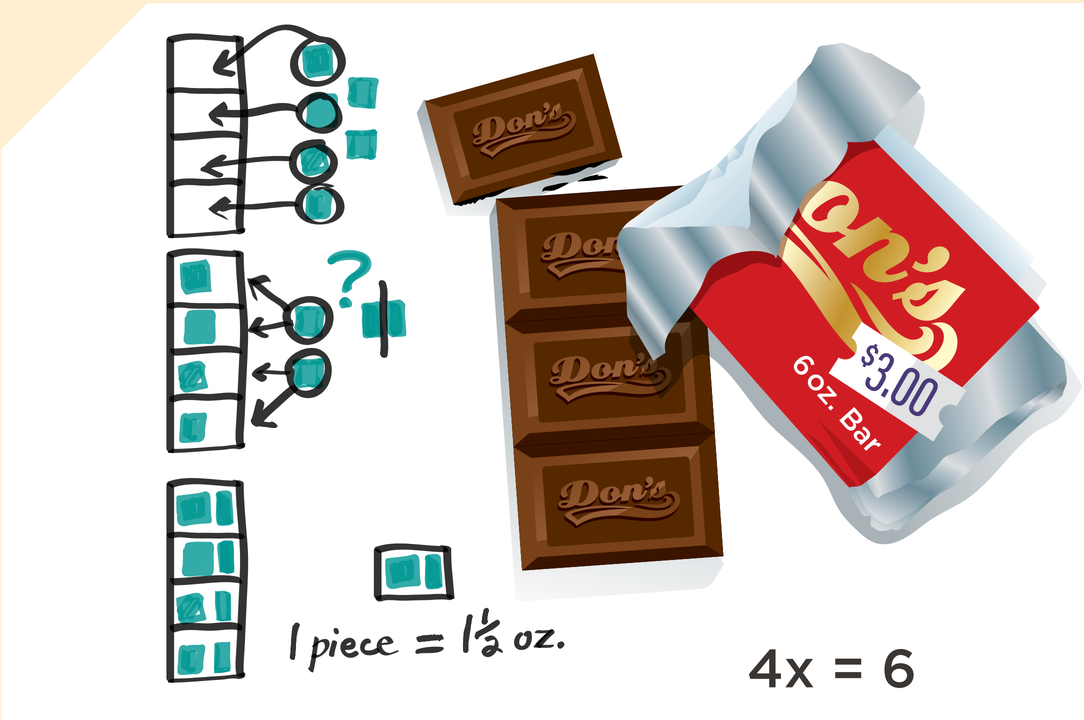
Fun Fact: When a pawn makes it all the way across the chessboard, it can be promoted to any position (except for the king). The pawn can become a knight, rook, bishop, or queen!

In the real world, answers don't always come to whole numbers.

Sometimes, there's a little bit left over or missing. For instance, how much does one piece of this candy bar weigh?

The equation is written as four (4) pieces weighing an unknown amount (x) that equal six (6) ounces. 4x = 6

Multiply each side by the reciprocal of four (4) to isolate the variable. Each piece 1.5 ounces!



REMEMBER! *Solving requires operations to be used in the correct order to isolate the variable.*

$$4x = \$3$$

$$4x\left(\tfrac{1}{4}\right) = 3\left(\tfrac{1}{4}\right)$$

$$\left(\tfrac{4}{1}\right)\left(\tfrac{1}{4}\right)x = \left(\tfrac{3}{1}\right)\left(\tfrac{1}{4}\right)$$

4/4=1x 3/4

$$x = \tfrac{3}{4}$$

Can you figure out how much each piece would cost? We know there are four (4) pieces. We also know that each bar costs three (3) dollars. Our equation is 4x = 3. Let's solve.

Each piece would cost 75¢ (or ¾ of a dollar).

More equation solving

Directions: *Use legal moves to isolate the variable on one side of the equation. Determine the value for the variable that will make the equation true.*

$2x - \cancel{1 + 1} = 8 + 1$

$\frac{\cancel{2}x}{\cancel{2}} = \frac{9}{2}$

$x = 9/2 = 4\frac{1}{2}$

2x – 1 = 8

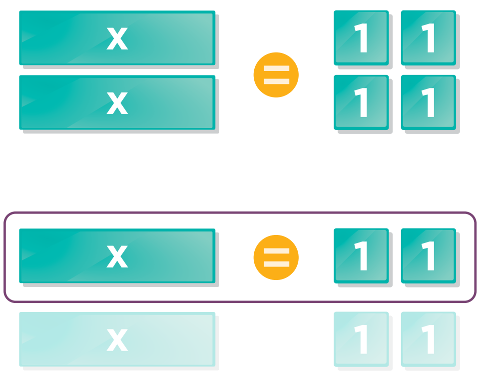
-4x – 2 = -3

2x + 3 = 4

-5x – 2 = -1

TILE IT IN!

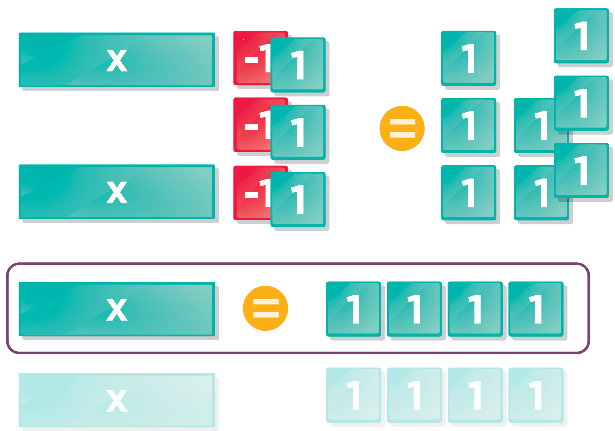
Directions: *Combine common terms to simplify the equation. Then, use legal moves to isolate the variable on one side of the equation. Determine the value for the variable that will make the equation true. -4 = -3x + 1x*



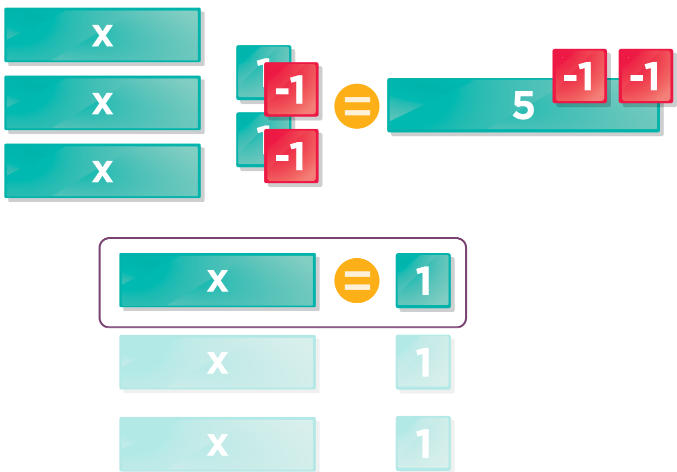
$$\frac{\cancel{2}x}{\cancel{2}} = \frac{4}{2}$$

2x = 4

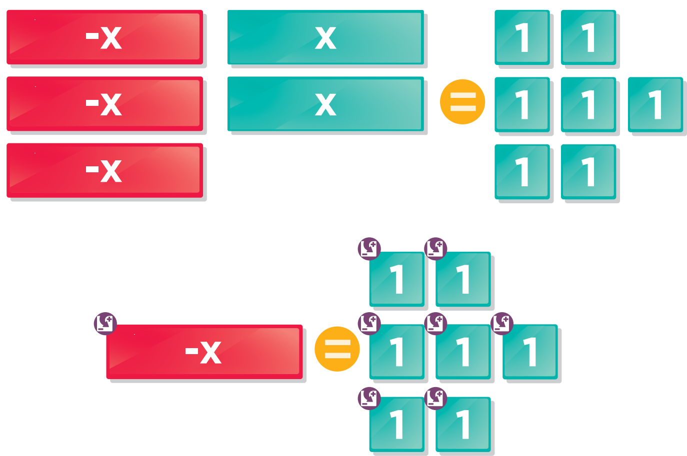
$x = 2$



x – 3 + x = 5



3x + 2 = 5



-3x + 2x = 7

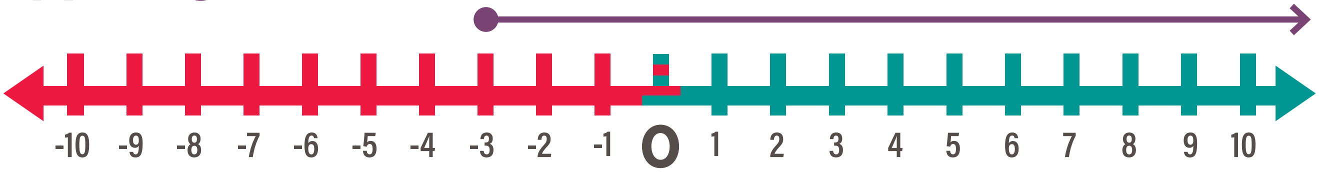
Explore inequalities

Sometimes there are a range of answers to an equation.

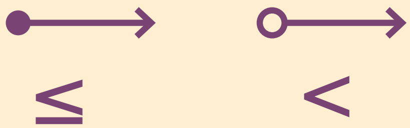
Using the rules we've learned to solve equations, we can find the range of the possible solutions to an inequality and show those possibilities using a number line!

Number lines require special symbols for including a value or excluding it. A filled-in circle on the boundary point ● is used if the point is included as part of the solution (≤ or ≥). We would say the number is greater than or equal (or less than or equal to) X. An open circle on the boundary point ○ is used if the point is not included as part of the solution (< or >). We would say the number is greater than (or less than) X.

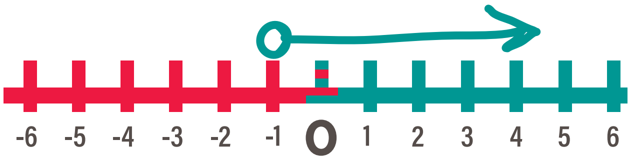
$X \geq -3$



REMEMBER! *Sometimes, there are a range of possible solutions to an equation.*



Directions: *Graph the equations on the number lines below.*



$x > -1$

$x \leq 3$

$x < 2$

$x \geq -5$

TOPIC 9

Directions: *Use the Additive Inverse Property to solve the equations below.*

$x - 3 = 6$	$x + 0 = 2$
$x + 1 = 3$	$3 + x = 0$

TOPIC 10

Directions: *Use the Multiplicative Inverse Property to solve the equations below.*

$4x = 4$	$2x = 8$
$3x = 12$	$6x = 3$

TOPIC 11

Directions: *Use substitution to solve each equation.*

Chapter CHECK-UP

x + 3 when x = 4

x + -7 when x = -3

x – 2 when x = 8

x - 0 when x = 0

TOPIC 12

Directions: *Solve for each variable.*

x + 3 + 2x = 9

8 – 8x + 8 = 0

CHALLENGE

Directions: *Graph the equation on the number line.*

x > 4