

ATTAINMENT'S
EXPLORE
Geometry

STUDENT BOOK

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A Little about Professor Maryam and Professor Euclid

Fun Fact: The two professors exploring geometry with us are based on real-life mathematicians!



Maryam Mirzakhani (May 11, 1977—July 14, 2017) was an Iranian mathematician and professor at Stanford University. She was born in Tehran, Iran. As a schoolgirl, Maryam competed in many math competitions and won gold medals for her achievements. She continued her math studies in college, earning her a Bachelor of Science degree from Sharif University of Technology and then a PhD from Harvard University. Maryam's research and achievements as a professor won her many prestigious awards in the field of mathematics with a focus in geometry. In 2014, while teaching at Stanford University, she became the first woman and first Iranian to be awarded a Fields Medal, a special award given to mathematicians every four years for outstanding work in math.

Sadly, Maryam died from cancer in 2017. She is remembered as an exceptional geometer, among many other things.



Euclid (325 BCE—270 BCE) was an ancient Greek mathematician from Alexandria, Greece. He is known as the Father of Geometry! There aren't many specifics known about his life; however, his writing about geometry, titled *The Elements*, has been commonly used as a geometry textbook for the last 2,000 years! In addition to *The Elements*, he wrote several other books that are still used by geometers to this day.

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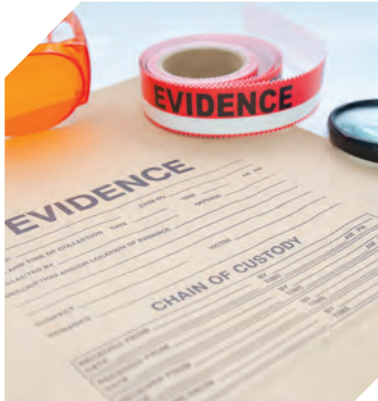
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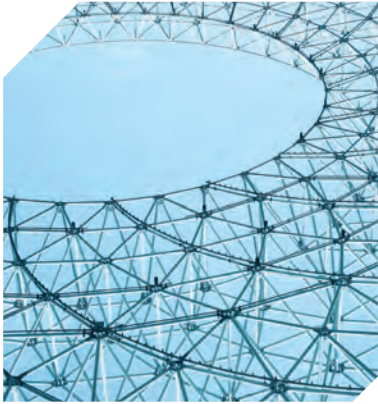
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CHAPTER

2

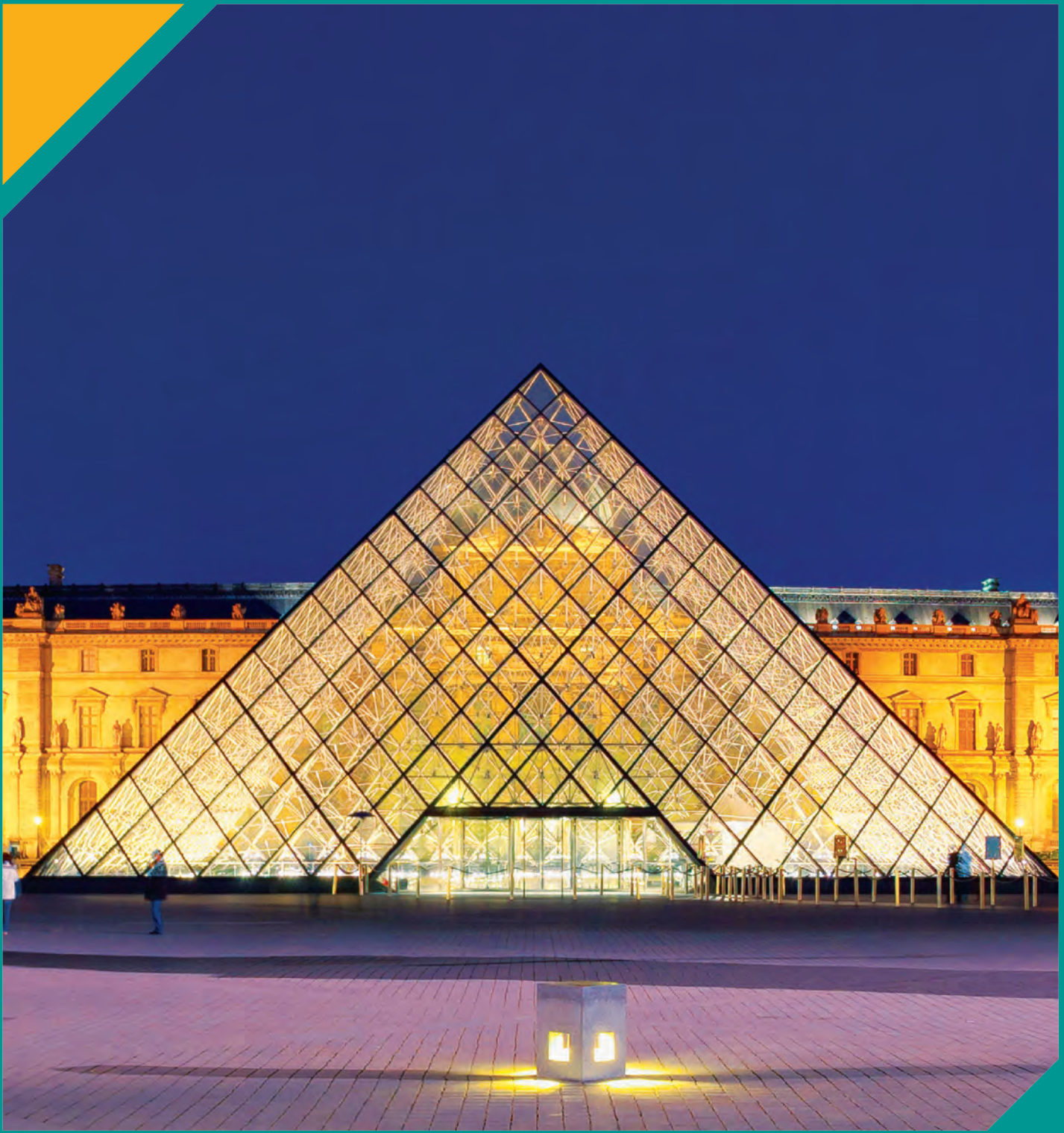
Angles and Triangles



CHAPTER 2

The BIG Picture

Angles are combined to create shapes.





What is Chapter 2 about?

Let's take a look!

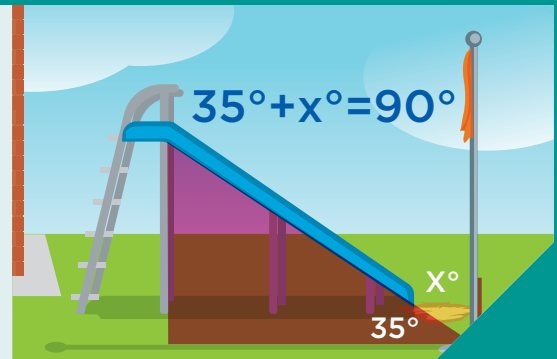
TOPIC 5

A ray starts at an endpoint or vertex. When two rays have the same vertex, they form an angle.



TOPIC 6

*Two angles that add up to 90° are complementary.
Two angles that add up to 180° are supplementary.*



TOPIC 7

Triangles are formed from the joining of three, nonlinear points.



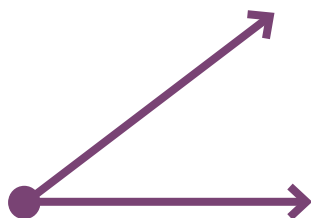
TOPIC 8

The angles of a right triangle with congruent sides will be $90^\circ/45^\circ/45^\circ$.



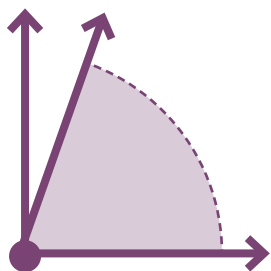
VOCABULARY

Angle



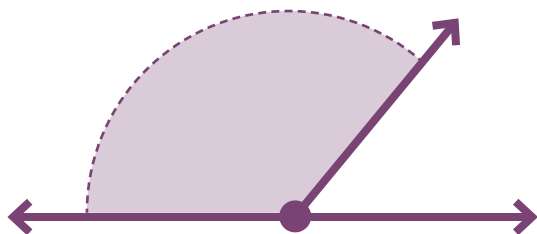
A figure created from two rays extending from the same endpoint.

Acute angle



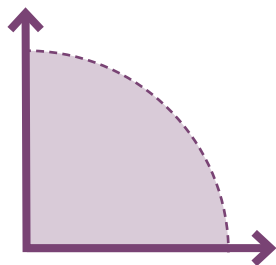
An angle with a measurement that is greater than 0° , but less than 90° .

Obtuse angle



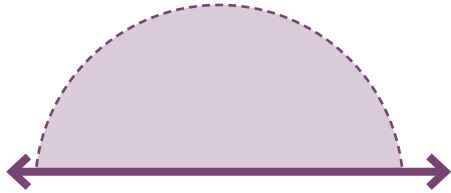
An angle with a measurement that is greater than 90° , but less than 180° .

Right angle



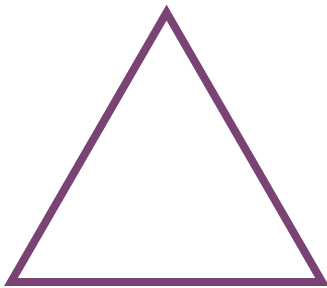
An angle with a measurement that is exactly 90° .

Straight angle



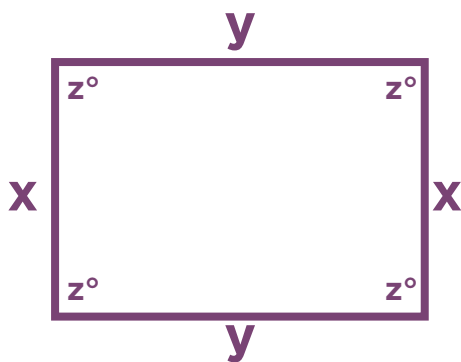
An angle with a measurement that is exactly 180° .

Triangle



Any three points on the same coordinate plane that are on the same line that are connected to create three sides and three vertices.

Congruent



When an angle, side, or shape is the same shape, size, or measurement.



After looking at all of these angles, I think we're ready to try triangle angles!

A triangle is any three, nonlinear points on a coordinate plane that are joined.

Every triangle has two key features:

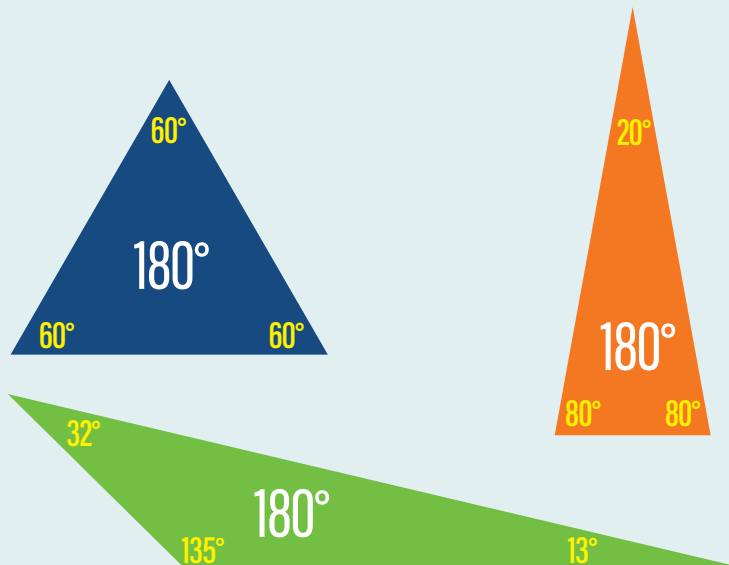
- It has three sides.
- It has three vertices.
- The sum of the inside angles of any triangle always equals 180° !

Isn't that cool?!



REMEMBER! Triangles are formed from the joining of three, nonlinear points.

Let's take a look at some different triangles and see how they compare.



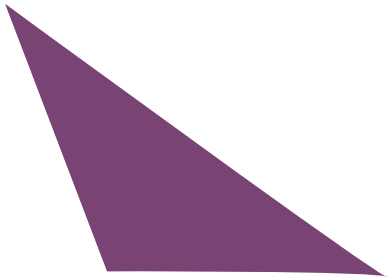


Fun Fact: The Triangle Sum Theorem says that the sum of the angles of any triangle always equals 180° .

Triangles, like angles, can also be identified by different angle measurements. However, **UNLIKE** angles, triangles can also be identified by the **congruence** of their sides.

Triangle Classification by Sides

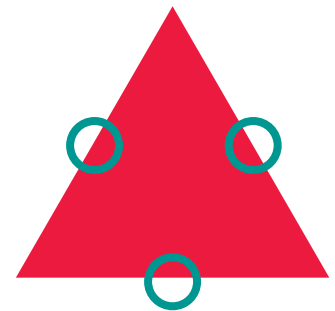
Scalene triangle
no sides congruent



Isosceles triangle
at least two sides congruent

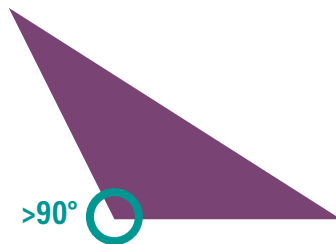


Equilateral triangle
all three sides congruent

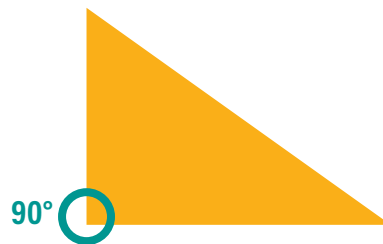


Triangle Classification by Angles

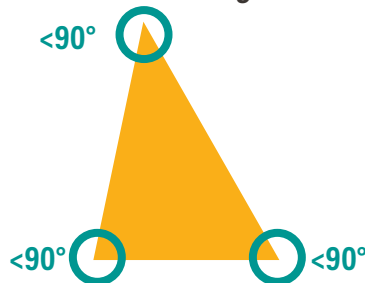
Obtuse triangle
one obtuse angle



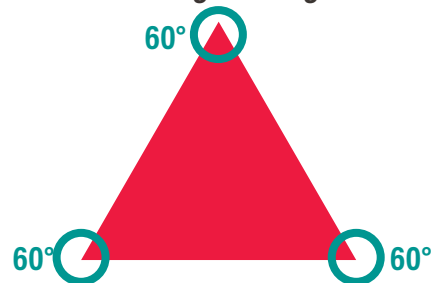
Right triangle
one right angle



Acute triangle
three acute angles



Equiangular triangle
all three angles congruent



More triangle types

Directions: Look at the figures and their measurements below. Then, use the word bank to write what type of triangle each shape might be. There may be more than one match for each triangle.

Scalene

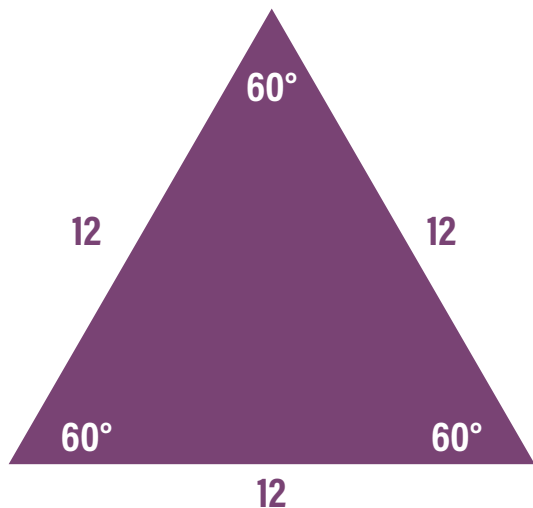
Obtuse

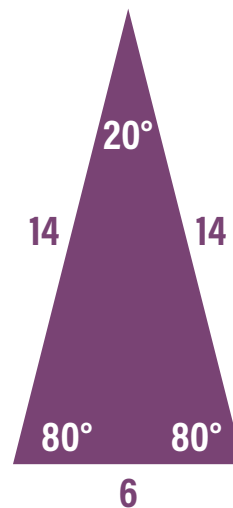
Acute

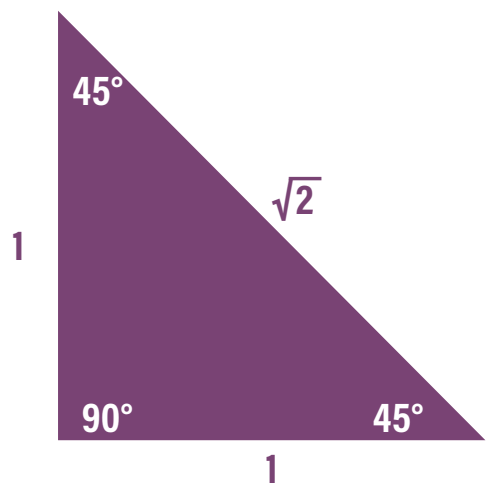
Isosceles

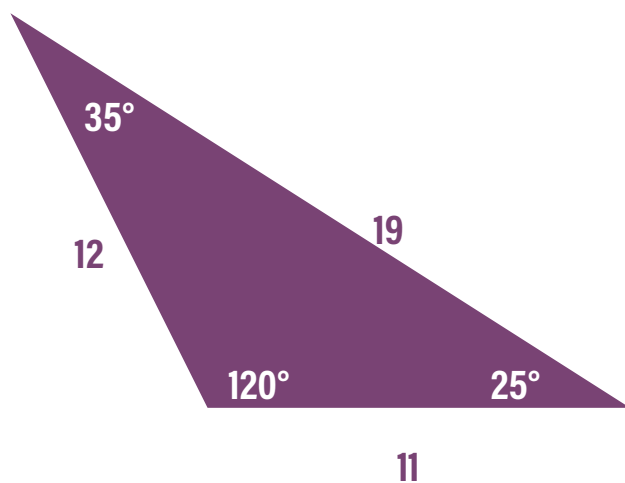
Right

Equilateral





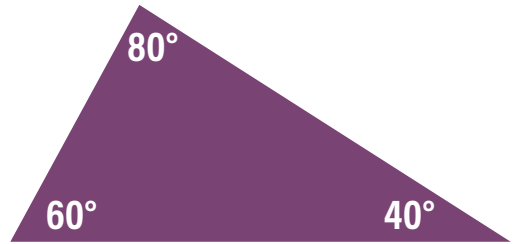




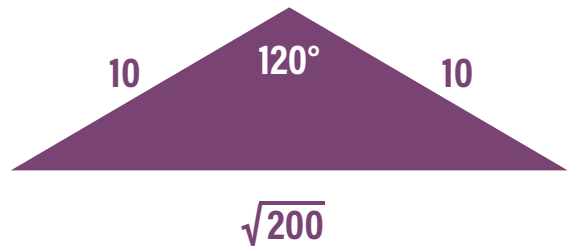
Try-angles!

Directions: Match the triangle type in the left column to the triangle in the right column. There may be more than one match for each triangle type or triangle.

Equilateral

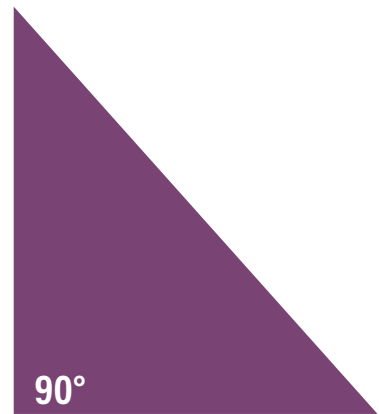


Scalene

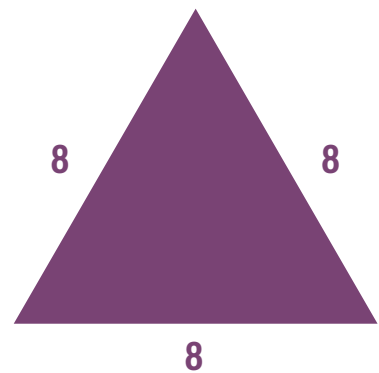


Isosceles

Obtuse



Acute



Right