# Attainment's <br> HANDS-ON MATH for Early Numeracy Skills 

Teacher's Guide
DON BASTIAN


## Attainment's <br> HANDS-ON MATH <br> for Early Numeracy Skills Teacher's Guide

This flash drive contains printable PDF files of the Student Book, Number Lines,
Peg Cutouts, Button Cutouts, Symbol Cards, and Progress Monitoring Forms. Peg Cutouts, Button Cutouts, Symbol Cards, and Progress Monitoring Forms. PDF reader software is required to view the PDFs. Acrobat ${ }^{\oplus}$ Readers ${ }^{\circledR}$ Software is included on the flashdrive.

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| Overview . | Lesson 6: Subitize within 3 | 22 |
| :---: | :---: | :---: |
| Materials | Lesson 7: Subitize within 6 | 23 |
| Lesson Structure | Lesson 8: Identify numbers within 12 on dice | 24 |
| How to Use | Lesson 9: Identify numbers within 18 on dice | 25 |
| Progress Monitoring Forms. | Lesson 10: Skip count by 2 s within 20. | 26 |
| Teaching Script: Rote Counting | Lesson 11: Skip count by 5 s within 20 | 27 |
| Teaching Script: Numeral Recognition | Sets . | . 29 |
| Counting and Numbers | Lesson 12: Create sets within 5 | 31 |
| Lesson 1: One-to-one correspondence within 5 | Lesson 13: Create sets within 10 | 32 |
| Lesson 2: One-to-one correspondence within 10 | Lesson 14: Create sets within 20 | 33 |
| Lesson 3: One-to-one correspondence within 20 | Lesson 15: Create two sets then add | 34 |
| Lesson 4: Counting within 10 by identifying numerals | Lesson 16: Add within 10 | 35 |
| Lesson 5: Counting within 20 by identifying numerals | Lesson 17: Add within 20 | 36 |

Lesson 18: Add with three addends . . . . . . . . . . . . . . . . . . . . . . . 37
Lesson 19: Subtract within 10 . . . . . . . . . . . . . . . . . . . . . . . . . . . 38
Lesson 20: Subtract within 20 . . . . . . . . . . . . . . . . . . . . . . . . . . 39
Lesson 21: Subtract two numbers within 20 . . . . . . . . . . . . . . . . . . 40

## Categories, Symbols, and Patterns . . . . . . . 41

Lesson 22: Sort two items with a cue redundancy . . . . . . . . . . . . . . . 43
Lesson 23: Sort two items with a cue constant . . . . . . . . . . . . . . . . . 44
Lesson 24: Sort two items with an irrelevant cue . . . . . . . . . . . . . . . 45
Lesson 25: Sort four items . . . . . . . . . . . . . . . . . . . . . . . . . . . . 46
Lesson 26: Identify the equals symbol . . . . . . . . . . . . . . . . . . . . . . 47
Lesson 27: Identify the less than symbol . . . . . . . . . . . . . . . . . . . . 48
Lesson 28: Identify the greater than symbol . . . . . . . . . . . . . . . . . . 49
Lesson 29: Identify numbers as less than, equal to, or greater than ..... 50
Lesson 30: Duplicate an ABAB pattern ..... 51
Lesson 31: Extend an $A B A B$ pattern ..... 52
Lesson 32: Complete an $A B A B$ pattern ..... 53
Lesson 33: Duplicate an $A A B B$ (or $A A B A A B$ ) pattern ..... 54
Lesson 34: Extend an $A A B B$ (or $A A B A A B$ ) pattern ..... 55
Resources ..... 57
Progress Monitoring Forms ..... 59
Scope and Sequence ..... 61
Lesson Activity Card Index ..... 62
Symbol Card Index ..... 64

## Overview

Hands-On Math (HOM) for Early Numeracy Skills is a curriculum for teaching early childhood students foundational numeracy skills using number lines. It is primarily for preschool students with an intellectual disability or autism. HOM for Early Numeracy Skills is based on two premises: (a) students learn well when their hands are purposefully engaged, and (b) math manipulatives help students understand abstract concepts. HOM for Early Numeracy Skills is an introduction to concepts included within school math standards. It is a skillsbased program focused on early numeracy concepts. The Teacher's Guide provides direction for teaching concepts with the hands-on manipulatives and the Student Book provides additional and extension practice with symbolic representation. Each student may use their own
consumable Student Workbook to document answers. All activities are designed to accommodate a wide variety of fine motor skills and are organized by lessons in the Teacher's Guide. Teachers may encourage students to share their answers by using the pegs or peg cards on top of the worksheet activities, or writing, circling, marking, or dotting onto the page. Sets of peg and button cards have been included at the end of each workbook that may be cut out to help generalize skills for Student Workbook activities or to provide an accommodation to the lessons in the Teacher's Guide.

The lessons are organized into three Skill Areas (Counting, Sets, and Categories, Symbols, and Patterns) which are then divided into three teaching directive types. The teaching directives progress in difficulty from concrete to representational to abstract understanding. Concrete learners are exposed to the math concepts incidentally and may participate in the lesson without a complete understanding. Representational learners are provided picture cues or other hints to help them complete the lesson. Abstract learners who successfully complete the lesson attain its objective by demonstrating
understanding of the underlying math concept. For further practice, students will complete worksheet activities in their own Student Workbook.

HOM for Early Numeracy Skills has been designed to support students' progression from concrete to representational to abstract understanding with supplemental and extension workbook activities for those individuals who have demonstrated proficiency with concepts or who may benefit from variety in skill practice.

## Concrete





## Materials

## Teacher's Guide

Provides step-by-step instructions for the 34 early numeracy lessons in three skills areas and supplemental resources.

## Flash drive

Provides a classroom license for printouts.

## Student Book

Provides supplemental activities for generalization and extension practice.

## 10 Student Workbooks

Each student may have their own, consumable workbook version of the Student Book.

## 4 bins with pegs

Includes 20 small yellow pegs, 10 small red pegs, 10 large yellow pegs, and 10 large red pegs.



## Number Line 0-10

The most commonly used number line.

## Number Line 11-20

Combines with the 0-10 number line for counting, operational, and pattern lessons.
pattern lessons
Laminated for writing with a dry-erase marker. Activity Cards have been labeled with corresponding lesson numbers from the Teacher's Guide for ease of reference.

## Number Book

Illustrated book that includes numbers $0-20$ with pegs and number line. Cardboard Shield

White blocker for covering illustrated cues on Activity Cards or Number Book.


## Symbol Cards

Nine, two-sided cards with symbols and describing words on the back.

## Peg Cards

Four cards with images of each size and color peg.

## 3 foam number dice

Large format foam dice showing numerals 0-5.

## 3 standard dice

Standard dot pattern commonly used in games.

## Dry-erase marker

Use to write answers on the Activity Cards.

## Yellow and red dauber

Use these with a Student Workbook and Number Line Tear-off sheets for students who may benefit from a supported answer format.

## Number Line Tear-off Pad

Use these for additional practice with problems.


## Peg and button cutouts

Use the Student Workbook pages or print additional copies from the flash drive and cut apart for sorting activities and generalization of skills.

## Number square cutouts

Use the Student Workbook pages or print additional copies from the flash drive and cut apart for sorting activities. These have the numeral on one side.
*Replacement and additional parts are available.


## Lesson Structure

All lesson descriptions in this guide have the same structure. They are divided into two sections. The top, color-coded by Skill Area, includes the lesson title, lesson number, materials needed, Lesson Objective, and an overview titled Narrative.
The bottom section is divided into Concrete, Representational, and Abstract lesson options. Each has a photo and instructions for teacher setup. HOM for Early Numeracy Skills follows the "two-minute rule" - your prep time is always under two minutes. Accordingly, you may teach multiple lessons, cycle through lesson options, and incorporate Student Workbook activities in a single math period. The student procedure may be read to the student as it is written, or as a guide to create your own script. Use the Teaching Scripts at the start of this guide to teach the pre-lesson concepts of rote counting and numeral recognition.
numeral recognition.

| Lesson 10: SkIp count by 2s within 20 |
| :--- |
| Lesson objective |

Lesson objective
Skip count by 2 s within 20 by placing pegs in a
number line.

Materlals

- 0 -10 and 11 -20 number lines
- Bin with small yellow pegs $\quad \begin{aligned} & \text { Student workbook } \\ & \text { pages } 30-32\end{aligned}$ - Bin with small yellow pe


Place a bin with small yellow pegs behind the 0-10 number ine (rand than 10
number is greater than 10). Place an Activity Card number is greater than 10). Place an Activity card
for skip counting by $2 s$ in front of the number line

## student procedure

Skip count by 2 s up to the number shown. Place pegs
from the bin in the number line, beginning with 2 , In from the bin in the number line, beginning with 2 , in
sequence as you count, saying the number of each peg as it's placed. Compare the completed number line with as it's piaced.
the picture.

Narrative
intraduces
itroduces skip counting. beginning with 2 s. and the use
of Activity cards. Use the cardboard shield to cover the or Activity cards. Use the cardboard shield to
pictorial representation for Abstract learners. Abstract

> Teacher setup Place a bin with
in
Teacher seup
Place a bin with small yellow pegs behind the 0-10
number line and the $11-20$ if the end number is greater number line (and the $11-20$ If the end number Is greater
than 10). Place an Activity Card for skip counting by 2 s in front of the number line and cover the picture with the cardboard shleld.
Student procedure
skip count by 2 s up to the number shown. Place pegs
skip count by 2 s up to the number shown. Place pegs
from the bin in the number line, beginning with 2 , in sequence as you count, saying, the number of each peg as its placed.
cardboard shleld.

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u
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## How to Use

Hands-On Math (HOM) for Early Numeracy Skills makes math concepts more explicit by representing them with pegs placed in number lines or into bins. These materials help students count, create sets and patterns, compare numbers, and sort and categorize. Not all math skills, however, are easily demonstrated with number lines; some HOM lessons may use different materials.
Two fundamental skills are the reach-grasp-retrieve-place sequence performed by taking a peg from the table or bin and placing it in the appropriate spot in a number line or group. This sequence can be difficult for some students with physical or attentional challenges. Consider using the Student Workbook, the Number Line Tear-off


Pad, or physical support to accommodate a variety of student needs. The second is the ability to consistently distinguish the color and size differences among the pegs. Students will need to recognize that the color and size of the pegs can represent different values.
The Concrete, Representational, and Abstract (CRA) options provide a high-to-low sequence of instructional support. You may follow the CRA sequence to introduce a lesson or select the option that best fits the student. When in doubt, start with Representational. Move to Concrete if the lesson is too challenging, and Abstract if it seems too easy. Your goal is for all students to become Abstract learners and progress to the Student Workbook activities.


Complete the Teacher Setup before introducing a lesson. Then, follow the explain-model-guide-observe-adjust process.

1. Explain by reading or paraphrasing the Student Procedure in the lesson description.
2. Model the procedure for the student slowly.
3. Guide the student through the lesson, giving prompts as needed.
4. Observe the student completing the task independently.
5. Adjust the lesson to present a unique problem to solve.

Lessons may be presented individually or in small groups of up to four students. You can teach multiple trials of a lesson by quickly adjusting the materials presented. For example, try using a variety of Activity Cards with a given lesson.
When a teacher determines a student is ready for practice generalizing or extending concepts, Student Workbook activities may be incorporated into math lessons. A student's fine and gross motor skills should be considered for activity setup. Workbook activities lend to a variety of student response modes as well as
concrete, representational, and abstract concept understanding. Pegs may be used on top of worksheets for Concrete learners and peg card cutouts may be used for Representational learners. If a student needs additional support, reference the Teaching Script examples at the front of the Teacher's Guide.
Hands-On Math for Early Numeracy Skills addresses a variety of conceptual understandings and ability levels so that all students may participate and increase awareness of fundamental math skills.


Identify numerals within 18 on dice

Cacuate the tota and whit the rumber on the ine.


## Progress Monitoring Forms

Use the Assessment Form to determine if students understand the concepts of HOM for Early Numeracy Skills. Assess with everyday classroom materials, like whiteboard and counters, or use the provided materials and cut-out cards.


Track student participation and progress in lessons with the Student Performance Data Sheet. It monitors the lesson option (Concrete, Representational, Abstract), individual or group instruction, and successful completion - plus a plan for future instruction. If a student is practicing the skill with a Student Workbook activity, make a note in the Notes column with the Student Workbook page number. Each lesson's corresponding Student Workbook pages are included with each lesson description in the Teacher's Guide.

Student Performance Data Sheet
student. Eleanore L. Teacher: Mrs. Kind
$C=$ Concrete $R=$ Representational $A=A b s t r a c t$

| Date | Lesson \# | Option |  |  | Setting |  | Completion |  | Notes | Plan |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | c | R | A | Indilvidual | Group | Yes | No |  | Repeat Lesson | Move to | Change Optlon |
| 12/10 | 2 | C |  |  |  | x | x |  | Well done! |  | 3 | $R$ |
| 12/11 | 3 |  | $R$ |  | $x$ |  |  | X | Repeated demonstration needed. | $x$ |  |  |
| 12/16 | 3 |  | $R$ |  | $x$ |  | x |  | Perfect. Repeat next month. | next month | 4 | A |
| 12/16 | 4 |  |  | A | $x$ |  |  | $x$ |  | $x$ |  |  |
| 12/22 | 4 |  |  | A | x |  | $x$ |  | Needed verbal instruction repeated. |  | 5 | A |
| 12/23 | 5 |  |  | A |  | x | $x$ |  | Good. One to one next time. | x |  |  |


| Introduction |
| :--- |
| Place the 0-10 number line on the table. |

Say, Today we are going to practice counting from 0 to 10 . First, I will say each number and point to it on the number line. Then it will be your turn to practice.

Model
Say, It is my turn to count from 0 to 10 Here is 1. Point to the number 1 on the number line and wait for the student to attend to the number. Once the student is attending, count aloud from 1 to 10 , and point to the appropriate numeral on the line as you count.

## Lead

Say, Now let's count together. Point to the number 1 on the number line and wait for the student to attend to the number. Once the student is attending, say, This is the number 1. The student should say "one" after you say "one."
Point to the number 2 and say, This is the number 2. The student should say "two" after you say "two."

Point to the number 3 and say, This is the number 3. The student should say "three" after you say "three."

Continue with this script for numbers 4-10. Feel free to use this script to extend the lesson to numbers 11-100 when appropriate.

## Test

Say, Now it is your turn to count from 1 to 10. Point to the numbers as you count.

If the student identifies the numbers from 1 to 10 , give specific praise. For example, Great job! You counted from 1 to 10.

If the student provides an incorrect response, say, Let me show you. Count aloud from 1 to 10 and point to each number on the number line as you progress. Say, Now it's your turn to count. Allow the student 5 seconds to begin their counting. If they do not, then provide a physical prompt to guide the student to point to each number as you count aloud.

## Hands-On Math for Early Numeracy: Teaching Script

## First Teaching Set: 0-Second Time Delay

Give the student a 0-10 number line and gather the number circles from 1-10. Say, For this activity, we will be checking to see how many numbers you can remember.

Round 1 (O-second delay): Say, When I hold up a number and say the number, touch that number on your number line. Write a number on a whiteboard while saying,
This is the number $\qquad$ Find the number $\qquad$ . Point to the number on the student's number line immediately as a prompt.

Give praise to the student if they touch the correct number without help. Make the praise specific to the action they completed.

If an incorrect response is given, correct the student with a physical prompt. Support the student's hand to locate the correct number. Then praise the student if they touch the correct number.

Repeat these steps for the numbers 1-10 in a random order. Repeat this process with the student three times for each number. Once the student provides consistently correct responses, progress to the Second Teaching Set.

## Second Teaching Set: 5-Second Time Delay

During this teaching set, the student has the chance to identify numerals independently within a 5 -second time frame. If a student does not respond within 5 seconds, then return to using a O -second time delay.

Give the student a 0-10 number line. Say, For this activity, we will be checking to see how many numbers you can remember. Write a number on the whiteboard while saying, This is the number $\qquad$ . Find the number $\qquad$ Wait student to respond independently. Instruct the student to wait if she or he is not sure of the answer. Say, If you do not know the answer, wait, and I will show you.

Praise the student if they touch the correct number without help. Make the praise specific to the action they completed.

If the student does not correctly respond, provide a physical prompt by supporting the student's hand to locate the correct number. Then praise the student. For example, Great job. This is the number 4. Then, provide the direction again, Find the number 4.

Repeat for the numbers 1-10 in a random order. Please note, 5 seconds is recommended, but you may adjust the length of time. This procedure may be used to extend the lesson to numbers 11-100 when appropriate.

## Counting and Numbers

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## Skill Area One: Counting and Numbers

Counting and Numbers lessons progress in difficulty from counting with one-to-one correspondence to skip counting. Each group of lessons follow a similar setup and procedure.

## Count with One-to-One Correspondence

For Concrete teaching instruction option, pegs begin in the number line. For the Representational instruction, the teacher lines up the pegs to be counted in front of the number line(s). With Abstract instruction, students respond to a verbal prompt from the teacher ("Count to 5") or roll a number die.

## Identify Numerals

Lessons 4, 5, 8, and 9 use the Activity Cards for all students. For Concrete learners, pegs matching the numeral are placed in front of the number line(s). Representational learners follow the pictures, while Abstract learners focus on the numeral (with the picture covered). A hand-written numeral can be substituted for the Activity Cards in the Abstract option.

## Subitize

In Lessons 6 and 7, pegs are prearranged on the Activity Cards in the Concrete option. Representational learners identify the number of yellow dots printed on the back of an Activity Card when it's turned over briefly by the teacher or a student, and Abstract students identify the number of pegs under a bin when it's lifted momentarily by the teacher. Students identify dot patterns on dice in the remaining two lessons.

## Skip Count

Pegs matching the picture are placed in front of the number line(s) for Concrete learners, Representational learners can follow the picture, and Abstract learners follow verbal instructions ("Skip count by 2 s to 10 "), since the picture is covered.

## Challenge Lessons

- Write numerals on small sticky notes, and have students place them over the matching words on the Activity Cards.
- Place a peg in any number line slot, and have students count backwards from that number. This activity prepares them for subtraction.
- Extend practice and challenge Abstract learners with Student Workbook activity pages.


## Lesson 1: One-fo-one correspondence within 5

Counting and Numbers

Lesson objective
Count up to 5 by placing pegs in the number line in sequence, beginning with the first slot.

## Concrete



Teacher setup
Place up to 5 small yellow pegs in the 0-10 number line.

## Student procedure

Count the pegs placed in the number line. Begin with the first peg, and touch and say the number of each peg as it's counted.

Materials

- 0-10 number line
- Bin with small yellow pegs
- Student Workbook pages 3-5

Representational


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## Teacher setup

Place up to 5 small yellow pegs in a row in front of the 0-10 number line.

## Student procedure

Place the pegs in the number line in sequence, beginning with the first slot. Count and say the number of each peg as it's placed.

## Narrative

Introduces one-to-one correspondence. Students place small pegs in number line slots in sequence, beginning with the first slot. The last occupied slot shows the number of pegs counted.

## Abstract



## Teacher setup

Place the bin with small yellow pegs behind the 0-10 number line.

## Student procedure

Follow the teacher's prompt to count to a number up to 5. Take the pegs from the bin, and place them in the number line in sequence, beginning with the first slot. Say the number of each peg as it's placed.

Lesson objective
Count up to 10 by placing pegs in the number line in sequence, beginning with the first slot.

Materials

- 0-10 number line
- Bin with small yellow pegs
- Student Workbook pages 6-8


## Narrative

Expands one-to-one correspondence counting to
10. Students place small pegs in number line slots in sequence, beginning with the first slot. The last occupied slot shows the number of pegs counted.

## Abstract



## Teacher setup

Place the bin with small yellow pegs behind the 0-10 number line.

## Student procedure

Follow the teacher's prompt to count to a number up to 10. Take the pegs from the bin, and place them in the number line in sequence, beginning with the first slot. Say the number of each peg as it's placed.

