tangram TANGO 4

The BIG Picture

This is the fourth installment in the Tangram Tango series!

Our resident mathematician, Rachel McAnallen, has packed her newest lesson with even more tangram pieces, fractions, decimals, and mathematical patterns.

Finding the Pattern in the Pieces

Topics Involved: *problem-solving, fractions, decimals, writing generalizations*
Materials: *2 sets of tangrams per student or 2 sets between partners, pencils & notebooks*
Grade Level: *Type of Activity: individual or partners*

Note to Teachers: We strongly recommend students be well-versed in the previous tangram lessons before they are introduced to the following activity.

Starting Big

"Put your two big triangles together and make the square." It is a familiar direction to the students in Rachel McAnallen's tangram classes—yet no matter how many times they build the square, the same old shape always leads them to something new. In this case, Rachel informs them, they'll be going in reverse. "In the last lesson, we assigned values to the individual tangram pieces," she explains. "The small triangle was worth one-fourth, or one-half, and so on. In everything we did, we started small and worked up. This time we are going to start big and work backwards."

She compares it to working with fractions. "Sometimes when we work with fractions, we want to find an equivalent fraction in which the numbers are bigger. For example, we might have one-half and try to find an equivalent fraction—four-eighths, or five-tenths, or fifty-one hundredths. We work up, even though it's equivalent," Rachel observes. "Other times, we do the opposite. We might begin with a fraction of nine-twelfths, and work backwards to find an equivalent fraction, like three-fourths. Within fractions we work back and forth. This isn't exactly the same, but it is similar," she concludes.

The students prepare their notebooks for the first problem by tracing around the large square they have built, and then drawing the outline of the different tangram pieces.

Working from the overhead projector, Rachel sets up the first example. Out to the side of the square they have built, she writes:

Let the square be $8 or eight dollars

"We know the value of the big shape—now what we have to do is find the value of the other pieces," Rachel explains. "If the big square we've built is worth eight dollars, then how much is the large triangle worth?" she asks.

"Four dollars," answers one learner.

"Okay, let's write that in," she instructs. On the overhead, she writes $4 inside the outline of the big triangle.

"Now, I want you to build a big triangle using the square and two little triangles," Rachel tells the class. "If you build it right on top of your big triangle, you can see right away that the little square is worth how much?"

"Two dollars."

"And the little triangle is worth?"

"One dollar."

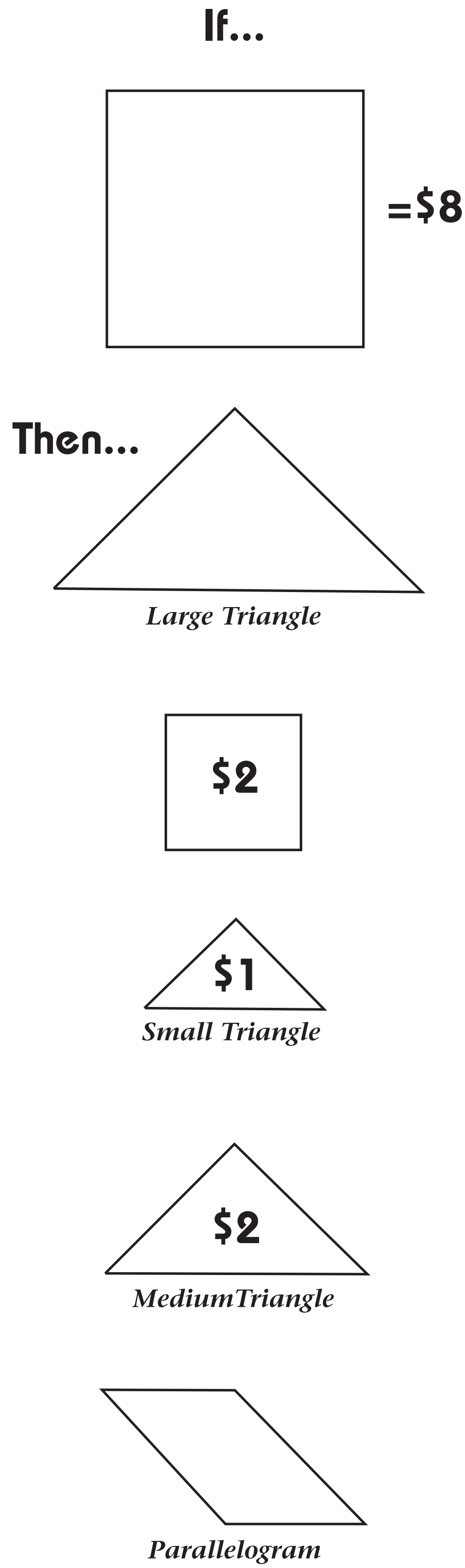
"That's correct," says Rachel. "Since we know the little triangle is worth one dollar, we can also work out the value of medium triangle and the parallelogram."

"Two dollars," one student volunteers.

"What if we give this same square a different value?" Rachel asks. "What if we let the square be six dollars? What are our tangram pieces worth now?

Let the large square be $6

Large Triangle (LT) = $3
Medium Triangle (MT)= $1.50
Square (S) = $1.50
Parallelogram (P) = $1.50
Small Triangle (ST) = $.75



Still working with the square built from the two large triangles, Rachel selects a more difficult value for the shape. On the overhead, she writes,

Let the square equal five dollars

The class quickly works out that the value of the big triangle is $2.50, and the value of the square is $1.25.

"What about the little triangle?" inquires Rachel. There is a pause as they mull over the problem. When someone suggests a value sixty-two and a half cents, another student protests, "You can't have half a cent."

"Oh, sure you can," says Rachel. "Whenever your parents buy gasoline for the car, the price is in fractional parts. The next time you are going past a gas station, look at the prices on the sign—it is never an even number. Five-tenths of a penny is the same as half a penny."

On the overhead, Rachel shows the class different ways to write the value of the small triangle.

$.625

62.5 ¢

$62\frac{1}{2}$ ¢

"Or, if we were talking about gas prices," she adds, "we would write the fraction out in tenths:"

$62\frac{5}{10}$ ¢

For the next part of the activity, students are paired with a partner. "I want you to do three or four more problems using this same square. Assign different values to the big shape and then work out how much each tangram piece is worth. I want you to look for the pattern."

Rachel finds that middle school students work better with a partner, observing, "Learners from 5^{th} to 8^{th} grade tend to get into a competitive mode when they are in a small group of four." She also recommends pairing students with different skills together. "I try to place a spatially skilled learner with a student who is arithmetically skilled—in this activity, the spatially skilled student will often teach their partner some tricks of the trade in spatial problem solving."

After students have worked on some of their own problems, Rachel calls their attention back to the overhead. "Do we have to continue to build the shape all the time or are you beginning to see a pattern?" she asks.

Simply Mathematics!

"When mathematicians see a pattern, we like to simplify the problem," explains Rachel. "We are going to simplify what we've discovered about this square, no matter what it's value. If we call the area of the square we've made "a" for area, we can now make what is known as a generalization."

"Basically, what we're doing is calling the area of the square a one," explains Rachel.

$a = 1$

"If 'a' equals one, then the big triangle would be one-half of 'a'." She writes:

$\frac{1}{2}(a)$ or $\frac{a}{2}$

"The square is one-fourth of 'a'.

$\frac{1}{4}(a)$ or $\frac{a}{4}$

Then the little triangle is $\frac{1}{8}$ a.

$\frac{1}{8}(a)$ or $\frac{a}{8}$

"Now we have our generalization," says Rachel.

The students build a new shape.

"We have an irregular hexagon," says Rachel. "Let's make this shape worth twelve dollars. Find the value of the other pieces."

Looking at the shape, one student solves the problem by visualizing putting the two small triangles together with the medium triangle. "It makes three big triangles!" she exclaims.

"Oh cool!" says Rachel. "So what are the values for all the pieces?"

"The big triangles would be four dollars each," says the student. "Then the medium triangle is worth two dollars, and the small triangle would be worth one dollar."

"That's right," Rachel confirms. "What values could I give to the irregular hexagon so that the values of the other pieces would work out to an even dollar amount?"

"How about $6?" suggests one learner.

"Here we go," says Rachel, writing:

Let the IH (irregular hexagon) be $6

LT (large triangle) = $2

MT (medium triangle) = $1

S (square) = $1

P (parallelogram) = $1

ST (small triangle) = $.50

Next, they try $3:

Let the IH be $3

LT = $1

MT = $.50

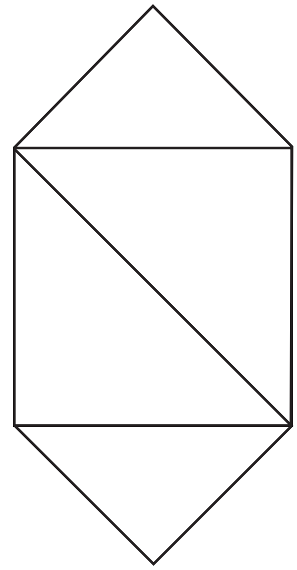
S = $.50

P = $.50

ST = $.25

After students have worked on the shape with a few other values, Rachel suggests, "Let's make the irregular hexagon worth one dollar."

Let the IH be $1



Irregular Hexagon

"This is a really tough one," she notes. "See what you can come up with." For this example, students may use a calculator to find the values of the pieces, rounding off their answers to the nearest thousandth.

LT = $.333

MT = $.167

S = $.167

P = $.167

ST = .083

"Now, we can generalize this shape," says Rachel. "If the irregular hexagon is 'a', then what is the area of the other shapes?"

If IH = a then...

$LT = \frac{1}{3}(a) \text{ or } \frac{a}{3}$

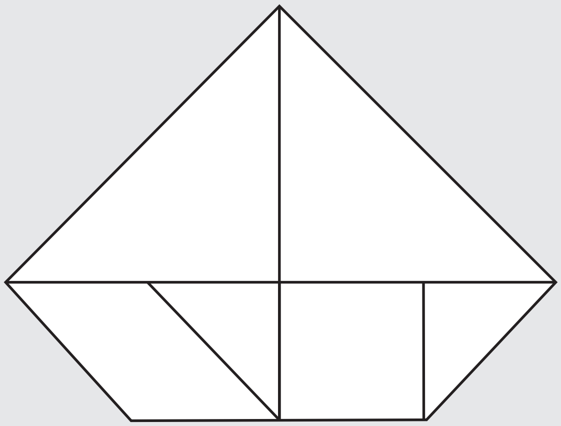
$MT = \frac{1}{6}(a) \text{ or } \frac{a}{6}$

$S = \frac{1}{6}(a) \text{ or } \frac{a}{6}$

$P = \frac{1}{6}(a) \text{ or } \frac{a}{6}$

$ST = \frac{1}{12}(a) \text{ or } \frac{a}{12}$

Using six tangram pieces, Rachel builds an irregular pentagon shape that resembles a boat.



"My boat is expensive," she tells the students. "It costs fourteen hundred dollars!" She writes:

Let the boat be $1400

LT = $400

MT = $200

S = $200

P = $200

ST = $100

Some students discover that the easiest way to determine the value of the pieces is to look at the smallest unit. The boat is made up of fourteen small triangles, which would mean their value is $100 each, for a total of $1400.

Working with their partner and a calculator, students give new values to the boat. "What are the nice values for the boat?" Rachel asks, as she walks around the room. "Pay attention to which values work out evenly."

When students have had sufficient time to do a few problems with their partner, they write out the generalization for the irregular pentagon shape as a class.

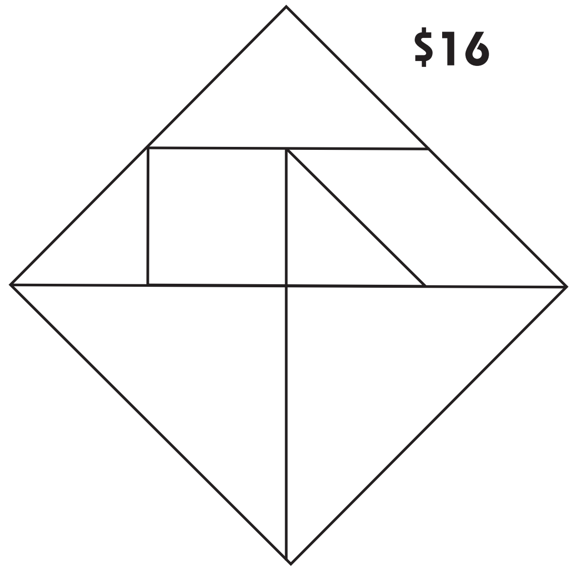
Let the boat be "a"

$LT = \frac{2}{7}(a)\frac{2a}{7}$

$MT = \frac{1}{7}(a)$ or $\frac{a}{7}$

$ST = \frac{1}{14}(a)$ or $\frac{a}{14}$

On the overhead projector, Rachel builds another square using all seven tangram pieces. "I'm going to give this square a value of sixteen dollars," she announces.



$16

Let the square be $16

LT = $4

MT = $2

S = $2

P = $2

ST = $1

Once again, students are given time create new values for the shape and problem solve with their partners before the class works out the generalization for the shape.

Let the 7-piece square be "a"

$LT = \frac{1}{4}(a)$ or $\frac{a}{4}$

$MT = \frac{1}{8}(a)$ or $\frac{a}{8}$

$S = \frac{1}{8}(a)$ or $\frac{a}{8}$

$P = \frac{1}{8}(a)$ or $\frac{a}{8}$

$ST = \frac{1}{16}(a)$ or $\frac{a}{16}$

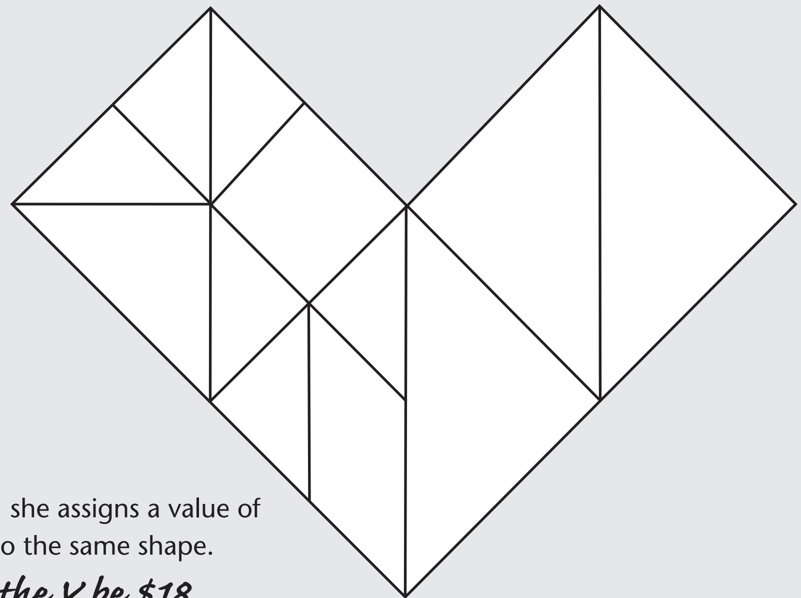
Double Tangrams!

For the next problem, Rachel builds a shape using pieces from two sets of tangrams.

Stepping back to admire her work, Rachel notices, "We have another irregular hexagon."

"It looks like a V," says one student.

"Let's make the V shape worth twenty-four dollars," says Rachel, writing:



Let the V be $24

LT = $4

MT = $2

S = $2

P = $2

ST = $1

Next, she assigns a value of $18 to the same shape.

Let the V be $18

LT = $3

MT = $1.50

S = $1.50

P = $1.50

ST = $.75

Then, they work out the generalization:

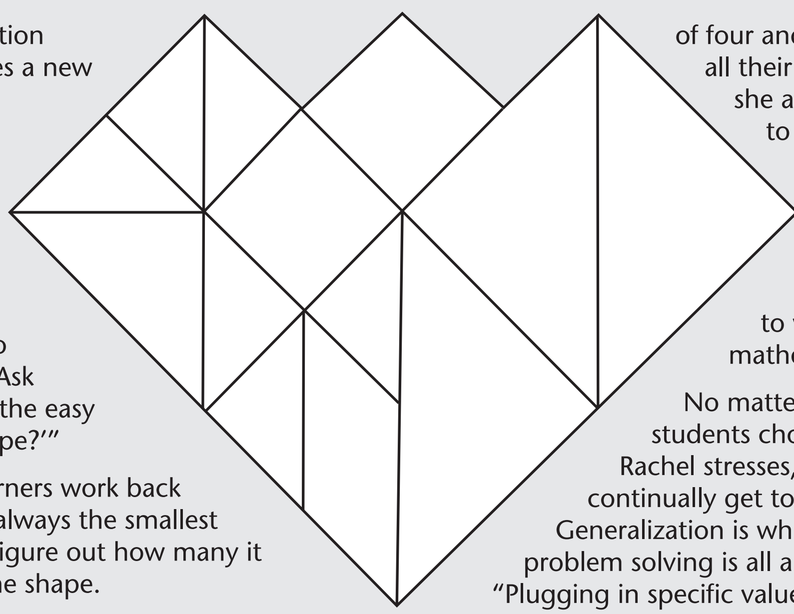
Let the V be a

LT = $\frac{1}{6}(a)$ or $\frac{a}{6}$

MT = $\frac{1}{12}(a)$ or $\frac{a}{12}$

ST = $\frac{1}{24}(a)$ or $\frac{a}{24}$

With a quick modification to the V, Rachel creates a new shape:



She has the students pair off again. "Working with your partner, I want you to give new values to the shape, and then generalize to a." instructs Rachel. "Ask yourselves, 'What are the easy values to give this shape?'"

Around the room, learners work back to the smallest unit—always the smallest triangle—in order to figure out how many it would take to build the shape.

"Once they have reached this point in their problem solving, students are able add more pieces, subtract more pieces, and generalize for the shapes they build," explains Rachel.

After students have worked with two sets of tangrams for a while, they may want to keep going. "Be prepared for students who want to work as a group of four and build shapes using all their sets of tangrams,*" she advises. "I tell students to go for it—they still must assign the shape a value and work out the generalization. The more pieces they have to work with, the more mathematics is involved."

No matter how many sets the students choose to work with, Rachel stresses, "I want learners to continually get to the generalization. Generalization is what higher math and problem solving is all about," she explains. "Plugging in specific values to the shapes is arithmetic. The simplification comes in the generalization of the pattern—and that is where you find the beauty of the mathematics. Ω

** We recommend students working together each use a different colored set of tangrams.*