tangram TANGO 3

Travels with Tangrams

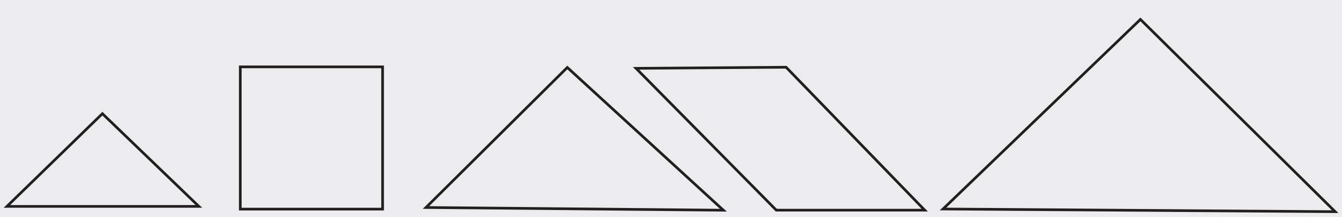
SAILING THROUGH FRACTIONS

Once again, Rachel McAnallen proves that tangrams are a wonderful tool in the mathematics classroom. In this issue, our resident math maniac presents a creative fractions lesson drawing inspiration from her favorite tangram books "***Tangram Geometry in Metric***" by Juanita Brownlee, and "***It's a Tangram World***" by Lee Jenkins & Peggy McLean. *

Information

Topics Involved:	*fractions, spatial problem solving*
Materials:	*a notebook and pencil, 1 set of tangrams per student.*
	(Note to teachers:When working in small groups, we recommend giving each student in the group a different colored set of tangrams to avoid confusion!)
Type of activity:	*small groups*
Grade level:	*1st and up, dependent on the fractional value the instructor chooses to assign.*

The opening activities of Rachel's lesson are based on the same framework used in the previous issue's lesson "Money & Tangrams". Students begin by tracing around each tangram shape, but instead of a monetary value, Rachel assigns a fractional value to one piece. "Let's give the small triangle a value of one half," she suggests.
On the overhead, she writes = ½ next to the tracing of the smallest triangle.
With this single piece of information, students work out the values for the remaining shapes.
Still working from the premise that smallest triangle piece is worth ½, Rachel directs the class to build shapes

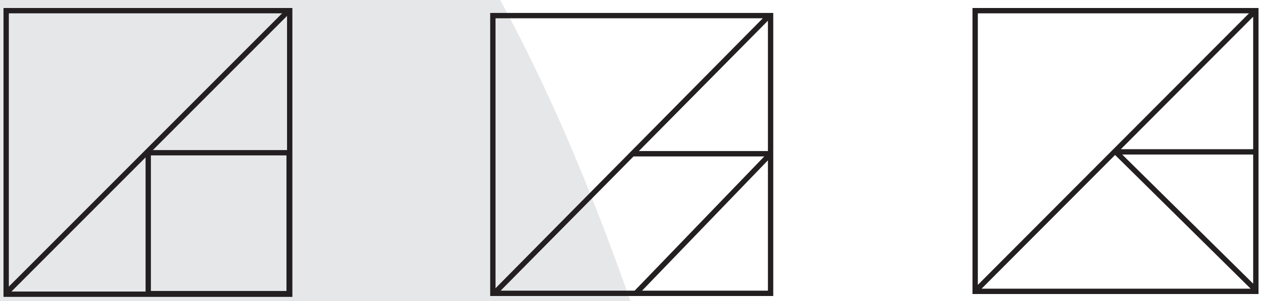


If the small triangle = ½, then…
square = 1, medium triangle = 1, parallelogram = 1, large triangle = 2

of differing size and value based on three criteria which she reveals on the overhead projector.
The first example is a familiar one. "Here is a warm-up," Rachel tells them. "You should recognize this one!"

1: Build a square
2: Use 4 pieces
3: Worth 4

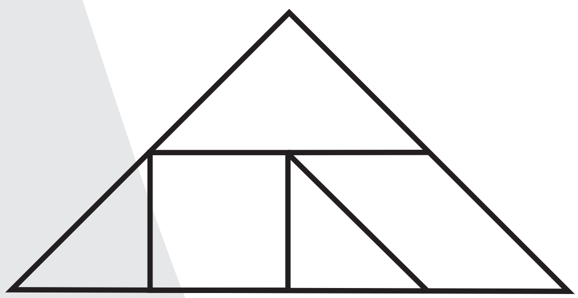
Quickly recalling their experience with this same puzzle from earlier activities, the students sketch the three possible solutions in their notebooks and write out the equations to prove their answers are correct.



$$1/2 + 1/2 + 1 + 2 = 4$$

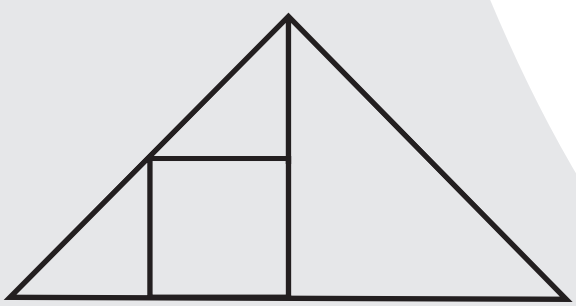
Next, the class builds a series of triangles. In many cases, they discover, the directions allow for more than one solution to the puzzle.

1: Build a triangle
2: Use 5 pieces.
3: Worth 4



"Here's another good one," says Rachel. She writes:

1: Build a triangle
2: Using four pieces
3: Worth 4 1/2



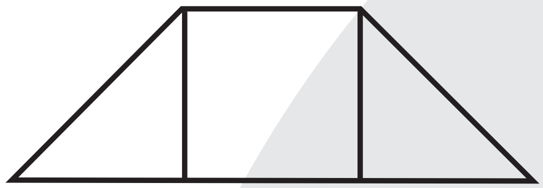
Tangram Trapezoids

After the class builds several triangle combinations, Rachel announces, "Now we're going to work with quadrilaterals—four sided polygons. The first set of quadrilaterals that we're going to look at are trapezoids."

For the purpose of the lesson, the students will use the classical definition of a trapezoid, though Rachel explains there is some debate over the matter. “The mathematical world is arguing about the definition of a trapezoid right now,” she admits. “Some mathematicians want to define a trapezoid as a quadrilateral having one pair of opposite sides parallel—which would mean that parallelograms, rectangles, squares and rhombuses would all be a type of trapezoid. The strict definition of a trapezoid is much more specific—it says a trapezoid is a quadrilateral having one and only one pair of parallel sides.”

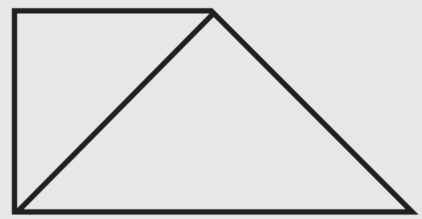
Following this strict definition, the students build trapezoids according to the criteria Rachel reveals on the overhead, still assuming that the value of the triangle is ½.

1: Build a trapezoid
2: Use 3 pieces
3: Worth 2



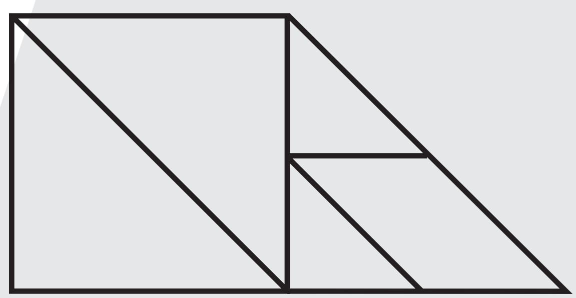
Isosceles trapezoid (non-parallel sides are congruent)

1: Build a trapezoid
2: Use 2 pieces
3: Worth 3



Non-isosceles trapezoid (non-parallel sides are incongruent)

1: Build a trapezoid
2: Use 5 pieces
3: Worth 6



As the activity continues, Rachel adds more quadrilaterals to the possible criteria, alternating between trapezoids, parallelograms, rectangles, and squares.

BIY! (Build-It-Yourself)

Next, students are instructed to build a specific shape of their own design, deciding for themselves how many pieces to use. Working in small groups learners create and trace their own triangles, squares, trapezoids, rectangles and parallelograms, and eventually other polygons.

Once they have made the shape, they must determine it’s total

value if the smallest triangle is worth ½.
"Make a trapezoid," directs Rachel. "You can make any trapezoid you want, using however many pieces. Is it an isosceles or non-isosceles trapezoid?" she asks. "Tell the people in your group. Draw around your pieces in your notebook and write down how much your trapezoid is worth."

Next, Rachel has each group of students to choose a new value for the smallest triangle. "What do you want the littlest triangle to be worth?" she says to a table of four students. "One-fourth," they decide. "Okay—now, how much are the shapes you just built worth if the triangle is one-fourth?" she asks them. Another group chooses a value of 1 ½ for the smallest triangle. One table selects 1, but Rachel vetoes this choice. "No, that's too easy," she tells them. "We're working on fractions, not whole numbers." Still another group chooses ⅓. "You can have one-third," agrees Rachel, "but keep in mind that it might be difficult to work out." Difficult or not, it is important to allow students to try. Later, Rachel asks the students, "What are the easy fractions to work with? What are the hard fractions to work with? Why?"

Who Is One Today? Revisited

Calling students attention to the overhead projector screen, Rachel reveals the outline of a rectangle. "In the last activity, you built a big shape and had to work out it's value," she explains. "Now you are going to do the opposite. I am going to give you the big shape and tell you how much it is worth. Then you must figure out what pieces go into it and their fractional values."

Next to the outline of the shape, she writes: = 1

= 1

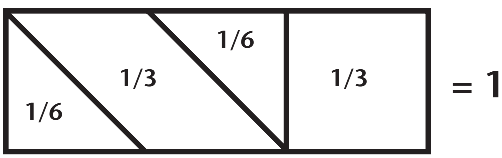
"This rectangle is one today," Rachel announces. "Take out your square, two small triangles, and the parallelogram. Using those four pieces, I want you to build this rectangle and trace around it on your paper."

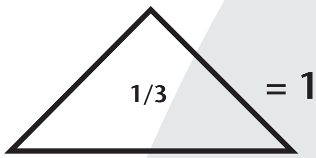
For this activity, Rachel has students work small groups. "I want you to talk to your partners." she explains. "In your group you must decide the value of the seven tangram pieces if this shape is one. Draw the pieces in your notebooks and fill in the values for each piece. You must prove your work."

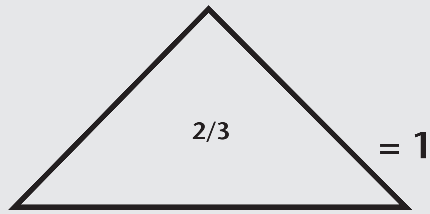
Teachers should be prepared for arguments over the first problem. For example, students might disagree on how much the little triangle is worth. Groups are encouraged to solve disputes on their own.

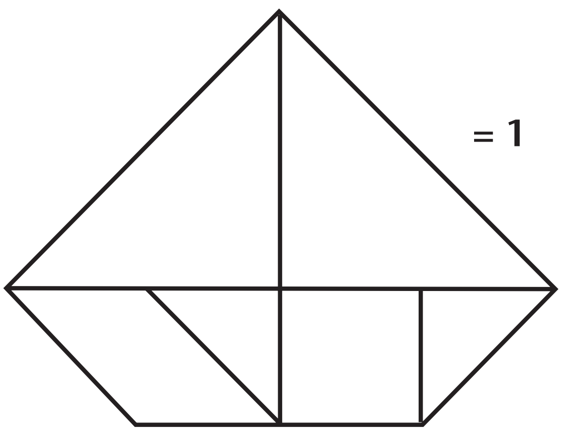
Rachel sets the standards for group work right away. "You must check with everyone in the group before you call me over," she explains. "You must solve the problem on your own, and you must all agree upon the answer before hand. No one is allowed to ask me a question unless all the people in your group have the same question. She cautions, "Only one person can talk to me at a time. If you all talk at once, I will turn and walk away."

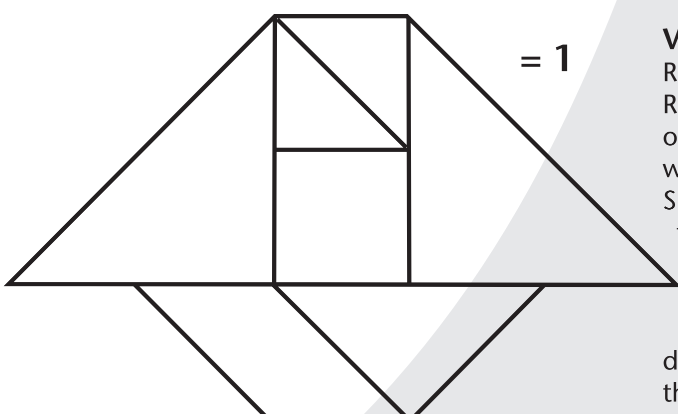
One by one, each group solves the first problem. When she is called over by a table of students to check their work, Rachel asks the group spokesperson, "Have you conferred with your partners?" "Yes," he replies. "Okay, show me what you all agreed upon."











Sailing Through Fractions!

The next outline Rachel reveals is a sailboat.

Using the square, two small triangles, and two large triangles, students replicate the shape in their notebooks. Another option is to assign one person the task of tracing the ship for the entire group. As with the rectangle, the students work out the values for the seven tangram shapes if the value of the sailboat is 1. "Talk it over, and when you agree, wave to me furiously," Rachel reminds the class.

She reveals still another larger sailboat shape for the next puzzle. "We're sailing through fractions," she jokes.

As she goes around the room checking the students' work, Rachel asks, "Do I need to show the answers up on the overhead?" "No!" they reply confidently.

Variations

Returning to the outline of the rectangle on the overhead projector, Rachel writes in a new value. "What if this rectangle was worth one one-half?" she suggests. "What would the tangram pieces be worth?

She makes a chart representing possible values for the rectangle from ½ through 4, with columns for the values of each tangram piece. "We have a small triangle, square, parallelogram, medium triangle, big triangle," she says, writing. Looking down the chart, Rachel stops at the row for 1. "We already did this one, didn't we?" she asks. "Let's write these in as a group." Together, they fill in the fractional values for each piece if the rectangle is worth 1. "The small triangle is worth one-sixth, the square, the parallelogram, and the medium triangle are all worth one-third, and the big triangle is worth two-thirds," says Rachel, writing.

With one row completed, the individual groups complete the rest of the chart on their own.

If the Rectangle =	…then the small triangle =	…then the square =	…then the medium triangle =	…then the parallelogram =	…then the big triangle =
1/2					
1	1/6	1/3	1/3	1/3	2/3
1 ½					
2					
2 ½					
3					
3 ½					
4					

Extended Activities

Challenge groups to give their own values to the pieces and make up a chart for the their classmates to solve. "What if we gave the small triangle a value that wasn't so simple?" queries Rachel. "What if we wanted to use fractions that we can't really show with the pieces?"

For example:

If the small triangle is…	…then the small triangle =	…then the medium triangle =	…then the parallelogram =	…then the square =	…then the big triangle =	…then the 2 medium triangles =	…then the 2 big triangles =	…then the medium triangle + big triangle =
1/2	½	1	1	1	2	2	4	3
1/4								
1/8								
1/16								
3/8	3/8	¾	¾	¾	1 ½	1 ½	3	2 ¼
5/8	5/8	1 ¼	1 ¼	1 ¼	2 ½	2 ½	5	3 ¾
7/8								
1 1/2								

Rachel advises teachers, "Don't be afraid if groups come up with fractions that are impossible to show with the tangram pieces, but are mathematically possible."

There are many more possibilities for using the tangrams with fractions, decimals and percentages. "Learners can build a shape and give it a decimal or percentage value," suggests Rachel. "From there, they can figure out how much the seven pieces are worth. Encourage your students to be creative!"

Ω