tangram TANGO 2

Pricey Polygons—Money & Tangrams

In this issue we continue our exploration of the seven tangram shapes. Rachel McAnallen has designed an ingenious lesson which incorporates geometry and money sense into one activity, drawing on students' previous knowledge working with the tangrams. (Tangram Tango)

Topics Involved: Money sense, spatial problem-solving
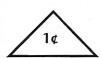*Type of Activity: Individual & small group*
Materials: One set of tangrams per student, pencils and notebooks, overhead projector

Rachel uses the same basic technique across the grade levels, adjusting the details for each age group.

Every student is given a set of tangrams "We are going to draw around each piece," explains Rachel. "Take out a sheet of paper and a pencil." Using a transparent set of tangrams and a washable marker, Rachel demonstrates on the overhead. She notes that learners in the earlier grades are still learning their tracing skills. "If it is easier to stand up, go ahead and do that," advises Rachel. "You are allowed to help each other—one of you can put your finger on the middle of the shape while the other person draws."

When the students have completed their tracings, Rachel calls their attention to the front of the room. She begins by assigning a value to the small triangle, taking into consideration the grade level of the learners. For example, with third through fifth graders, the price of the small triangle is a nickel. When she works with sixth graders and above, the same triangle is worth twenty-fiv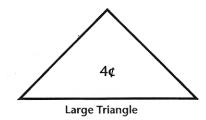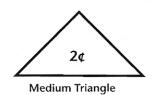e cents. With kindergarten and first grade students, she sets the price at a penny. "This triangle is worth one cent," she announces. Inside their tracings of smallest triangles, the students write 1¢.
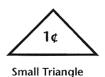
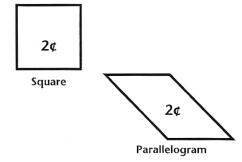"Let's look at something," says Rachel. On the overhead, she places the two little triangles together to form a square. "If each little triangle is worth one cent, then the square is worth how much?" "Two cents," students tell her. "Good," she replies. "Let's write that down." Inside the outline of the square they write 2¢.

Next, students are told to write in the values of the four remaining pieces. "You have enough information here to figure the rest of them

Large Triangle — 4¢

Medium Triangle — 2¢

Small Triangle — 1¢

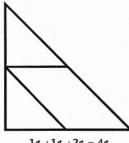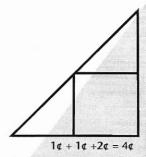1¢

Square — 2¢

Parallelogram — 2¢

out on your own," encourages Rachel. "When you think you have it, put your hands in your lap and give me the knowing smile." Walking around the room, she observes their work and offers guidance when necessary. Often she will see a student quickly assign a value of three cents to the next piece they pick up. "They haven't worked it out," Rachel explains. "They think they see a pattern—each piece is worth one penny more than the last one." For instance, one learner values the parallelogram at three cents, the medium triangle at four cents, and the large triangles as five cents. It is a common mistake. "Oh, you didn't work that out very well," comments Rachel. "Let's go back and figure it out."
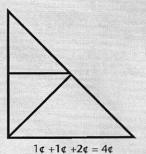
She gives the class plenty of time to work before she returns to the overhead projector. Using the two small triangles, she builds a medium triangle, and then a parallelogram. "The medium triangle is worth two cents, and so is the parallelogram," she says, writing the values inside the corresponding tracing. "How much is a big triangle worth?" "Four cents," students reply. Rachel writes 4¢ in each large triangle on the overhead.

Now she instructs, "Turn over your paper and use your tangram pieces to show me three ways that you can prove that the big triangle is worth four cents."

1¢ +1¢ +2¢ = 4¢   1¢ + 1¢ +2¢ = 4¢   1¢ +1¢ +2¢ = 4¢

"We're going to play a little game," announces Rachel. "I'm going to turn off the overhead projector and I'm going to write down three pieces of information. When I turn the projector back on, you will use your tangram pieces to build a shape that meets the conditions I've given you."
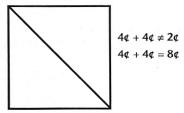
The students wait as Rachel writes down her three pieces of information. "Remember, once you have built the shape, put your hands in your lap and give me the knowing smile," she reminds them. "If the person beside you gets it first, you are allowed to look at their work if you need help. Okay—go." Flipping on the overhead projector, Rachel reveals the following three criteria:
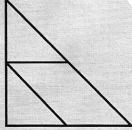
1: Make a square
2: Use 2 pieces
3: Worth 2¢

1¢ +1¢ =+2¢

In their rush to build the shape, some students grab the two large triangles and put them together to make a square using two pieces. "How much is your square worth?" Rachel asks one learner. He thinks for a moment. "Eight cents." "But I want it to be worth two cents," she reminds him.
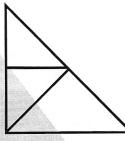
$$4¢ + 4¢ \neq 2¢$$
$$4¢ + 4¢ = 8¢$$

Once every student has successfully built the first shape, Rachel asks, "Are you ready for the next one?"
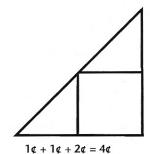
1. Make a triangle

2. Use 3 pieces

3. Worth 4¢

$$1¢ + 1¢ + 2¢ = 4¢ \qquad 1¢ + 1¢ + 2¢ = 4¢ \qquad 1¢ + 1¢ + 2¢ = 4¢$$
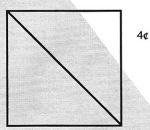
Though students may not realize it, they have already built this shape earlier in the lesson. "I want to keep stimulating their brains to use previous knowledge," comments Rachel.

"You've built this one before," she acknowledges once the students have completed the shape. "When you had to prove how much the large triangle was worth, you learned three different ways to build this shape. Look around—how did the person next to you make their triangle? Did they choose to build it differently?"

Next Rachel writes:
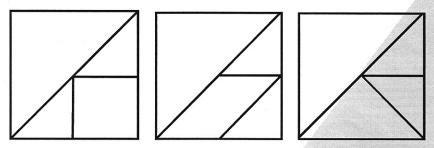
1. Make a square

2. Use 2 pieces

3. Worth 8¢

$$4¢ + 4¢ = 8¢$$

"Okay, now things are going to get a little harder," warns Rachel. "Here we go":

1. Make a square

2. Use 4 pieces

3. Worth 8¢

"It's not a race," she reminds the class. "Notice, the difference between this one and the last one is the amount of pieces—it is still a square worth eight cents, but you have to use four pieces. This is a tricky one!"

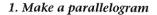With the next set of directions, Rachel makes just one change:

1. Make a triangle

2. Use 4 pieces

3. Worth 8¢

Rachel takes great delight in giving this particular sequence to older students. "When I'm working with middle school kids (using different values for the pieces), they make the square, and then they usually mess up their pieces. When I ask them to make the triangle, they are beside themselves!" She smiles, "Keep in mind, when I taught these same students the very first tangram activity—transforming a square into a triangle—they gave me an attitude." The students may have forgotten, but they discover that their teacher has a long memory. "Remember the trouble you gave me about "triangle, triangle, square?" she asks them wryly. "Well, maybe you ought to use it now." Flipping a switch, she reveals a square built from the previous directions on the overhead projector. "Watch," gloats Rachel. Using one hand, she rotates the large triangle, quickly transforming the square into a triangle!

"Who thinks they know what's next?" she asks.

"A parallelogram?" suggests one student.

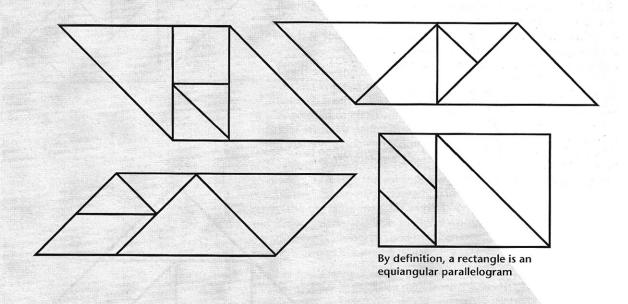Rachel flips on the projector to reveal:

1. Make a parallelogram

2. Use 4 pieces

3. Worth 8¢

"Even though they have done this transformation over and over again," says Rachel, "I want them to practice."

The overhead projector flips on and off and the students continue to build. "Now you are getting really good again!" Rachel praises. Now she begins to switch the criteria around, listing the value first, for instance, rather than the shape to build.

1. Worth 12¢

2. Use 5 pieces

3. Make a parallelogram

There are at least nine different ways to build the parallelogram, although some solutions aren't as obvious as others.

By definition, a rectangle is an equiangular parallelogram

"Now this one is harder," cautions Rachel. "There may be only one solution. Ready?"

1. Make a square

2. Worth 8¢

3. Use 5 pieces

Several students pick up their large triangles to build a square worth eight cents. "Ooh, you need to use five pieces," Rachel reminds them. When she notices that some learners are struggling, she has other students talk about how they solved the puzzle. "What are your problem solving strategies?" asks Rachel. Some students begin by putting the two large triangles together to make the outline of an 8¢ square, and then build on top of that template with five pieces to create a square of equal value.
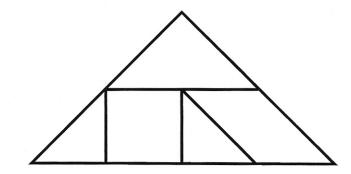
Another student explains, "I knew you couldn't use the two big triangles because it would be worth too much money." Still another learner worked out which five pieces were worth 8¢ . "Kids come up with a lot of different ways to solve the problem," notes Rachel. "Some approach it from the spatial aspect, others approach it from the monetary perspective."

"I looked at the person next to me for help," confesses one student. "That's perfectly okay," reassures Rachel. "You are allowed to look, remember?" She is careful to emphasize this point to teachers. "It is okay for a student to look at someone else's work. They still have to be able to make the transference from what they see and what is in front of them. If they are working in pairs, and their partner solves the puzzle first, the other student should be able to look at it and replicate it with their own pieces."

Rachel does have some firm rules about help. No one—be they a student or teacher—is allowed to touch another person's pieces. "Wait for the person to ask before you start giving them directions," she adds. "Do not start telling someone how to do something when they haven't asked you for help. Remember, there are lots of strategies you can use to solve these problems—don't interfere just because the person next to you is solving it differently."
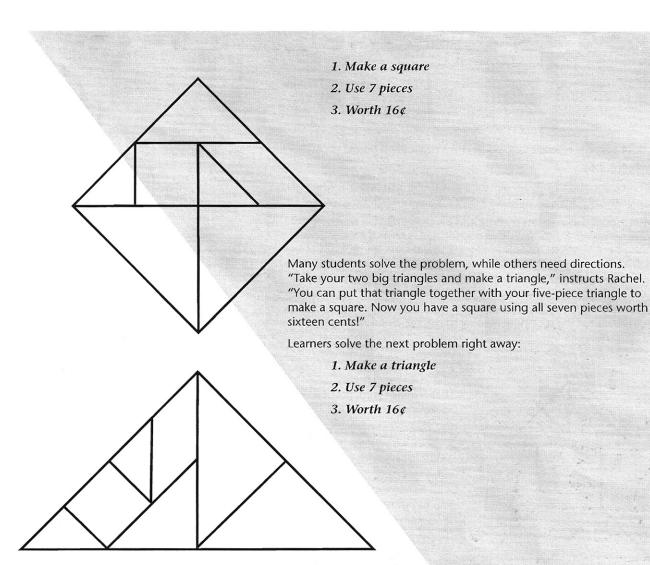
She gives them the next puzzle:

1. Make a triangle

2. Use 5 pieces

3. Worth 8¢
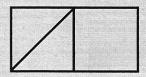

"Oh, you're all so good!" exclaims Rachel.

The next directions use all the tangram pieces:

1. Make a square

2. Use 7 pieces

3. Worth 16¢

Many students solve the problem, while others need directions. "Take your two big triangles and make a triangle," instructs Rachel. "You can put that triangle together with your five-piece triangle to make a square. Now you have a square using all seven pieces worth sixteen cents!"

Learners solve the next problem right away:

1. Make a triangle

2. Use 7 pieces

3. Worth 16¢

"It all keeps going back to those same basic skills," notes Rachel. "Triangle, triangle, square."

"Depending on the grade level, I do the same problems with new values," she explains. "When I works with 2nd graders and 3rd graders, I change the value of the small triangle to five cents, making the square, parallelogram, and medium triangle worth ten cents, and the large triangles worth twenty cents. With 4th graders and above, I make the little triangles worth twenty-five cents, then the square, parallelogram and medium triangle are fifty cents, and the big triangles are each a dollar."

Another fun approach is to begin with a composite shape, assign a value, and ask students to determine the worth of the seven tangram pieces. Using the square and two small triangles, Rachel builds a rectangle on the overhead.

"If this rectangle is worth one dollar, then how much do the pieces cost?" she asks.

Together, the students work out the values—the smallest triangles are worth 25¢, the square is worth 50¢.
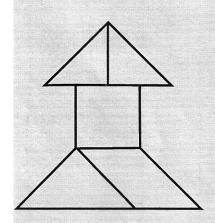
"What would the medium triangle and parallelogram be worth?" Rachel asks.

"Fifty cents."

"And what about the big triangles?"

"One dollar."

"So the value of the rectangle is equal to the value of a big triangle," notes Rachel.

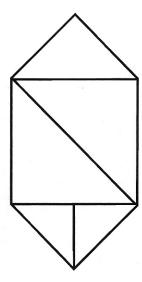She builds another shape that reminds students of a rocket ship.

"I've decided that this shape is worth four dollars," she announces, writing the value beside it. "Working with a partner, I would like you to figure out how much the pieces are worth."
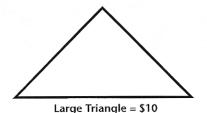
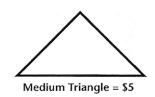After giving the groups time to work out the values, the students help Rachel fill them in on the overhead. "The square, the medium triangle and the parallelogram are all worth one dollar. The small triangles are worth fifty cents."
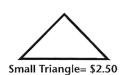
Next, Rachel builds a new shape:

"How much are pieces are worth if the value of the shape is thirty dollars?" she asks.

Large Triangle = $10

Medium Triangle = $5

Small Triangle= $2.50

Square = $5

Parallelogram = $5

"How about if the shape is worth fifteen dollars? And how much would the pieces be worth if the shape's value is twelve dollars?" She gives students time to work these values out. "Notice that the values I've assigned to the shape are all multiples of three," says Rachel. What if we made our shape worth ten dollars? What happens?"
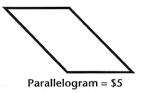
"It doesn't work out very well," says one student.

"That's right," Rachel tells her. "It isn't handy—it would work out to an amount of money that we don't have in this country."

Once students have the idea, they can work with their partner to design their own shapes and assign a monetary value. Some students want to make their shape worth a million dollars. "Sure!" responds Rachel. "You might want to write that as 1M instead of having to draw all those zeros. Just be sure to write the code on the top of your paper that 1M = one million."

"From here, the lesson becomes a directed activity, not a teaching activity," comments Rachel. "You must allow students to make mistakes along the way. This is where they can get very creative spatially."

Pick a Polygon!

Topics Involved: Spatial problem solving, vocabulary
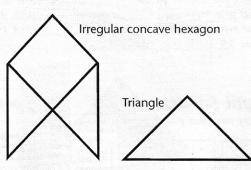*Type of Activity: Small groups of up to 4 students*

Materials: One set of tangrams per student, a deck of 24 of playing cards per group. Use a standard deck and remove all face cards, aces, eights, nines, and tens to create a deck of 24 cards—six cards numbered two through seven in each suite.

Directions: Shuffle the cards and lay the pile face down. The first player pulls the top card and lays it face up for everyone to see. Players must use the number of tangram pieces revealed on the card to build a polygon. In other words if a student draws the three of heart, players must use any three tangram shapes to build a polygon. When their polygon is completed, they must correctly name the shape they have created, for example:

Irregular concave hexagon

Triangle

Scoring: Award one point per side of the polygon a player has built. A person who builds a triangle would receive three points, a trapezoid would be worth four points, a pentagon worth five points and so on...
Other Scoring Ideas: Award one point for each letter of the polygon a player builds. Parallelogram = 13 points
Irregular Hexagon = 16

Use your imagination!