

# tangram

# TANGO

# 1

## Parallelograms, Triangles & Squares! OH MY!

In this lesson Rachel McAnallen introduces learners to the fundamentals of working with the tangram pieces by relying on their sense of sight, sound, and touch. Through a succession of tactile activities students gain confidence with the shapes and a familiarity with some of the basic vocabulary of geometry.

### Information

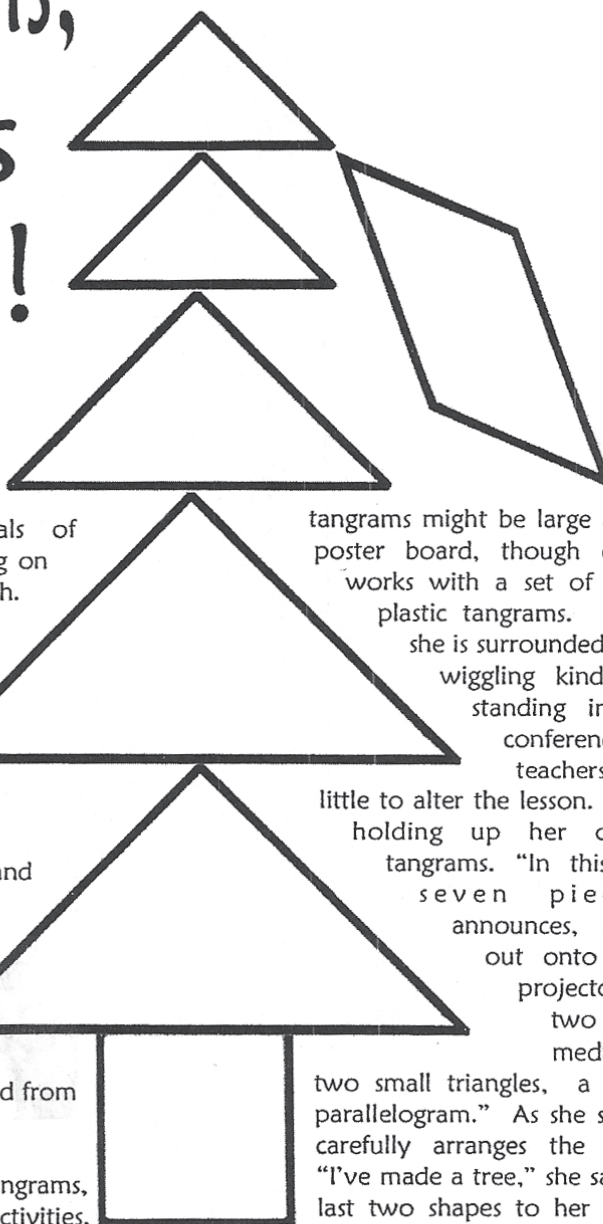
**Topics involved:** spatial problem-solving, vocabulary, definitions of polygons, plane and transformational geometries

**Materials:** 1 tangram set per student, wooden craft (Popsicle) sticks, paper & pencil

**Type of activity:** small group or individual

**Grade Level:** Certain sections can be applied from kindergarten on up!

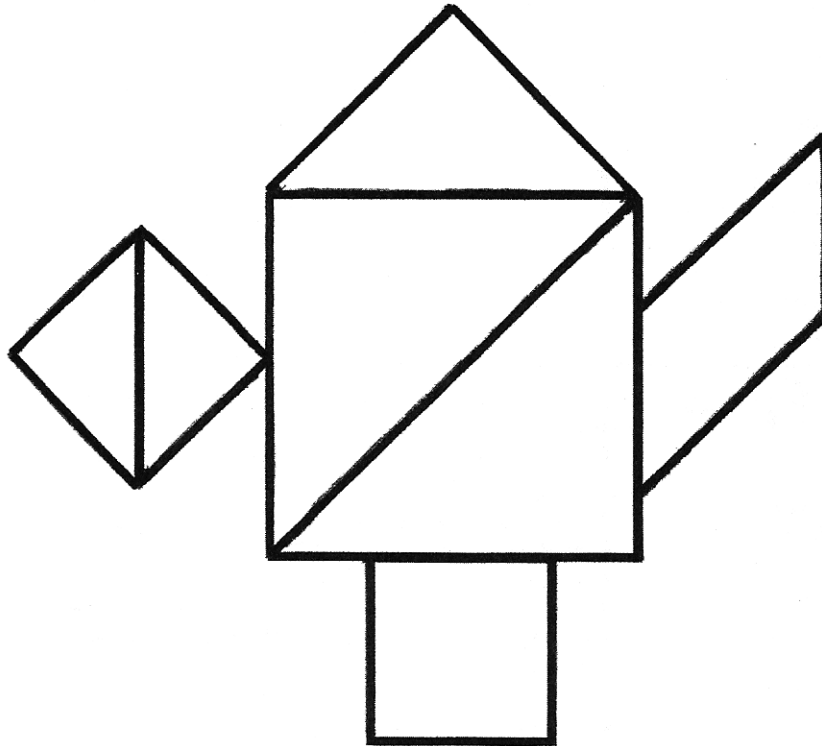
When it comes to tangrams, Ms Math teaches the same essential activities, no matter what the age of her audience. If she's working on the floor with younger students, the



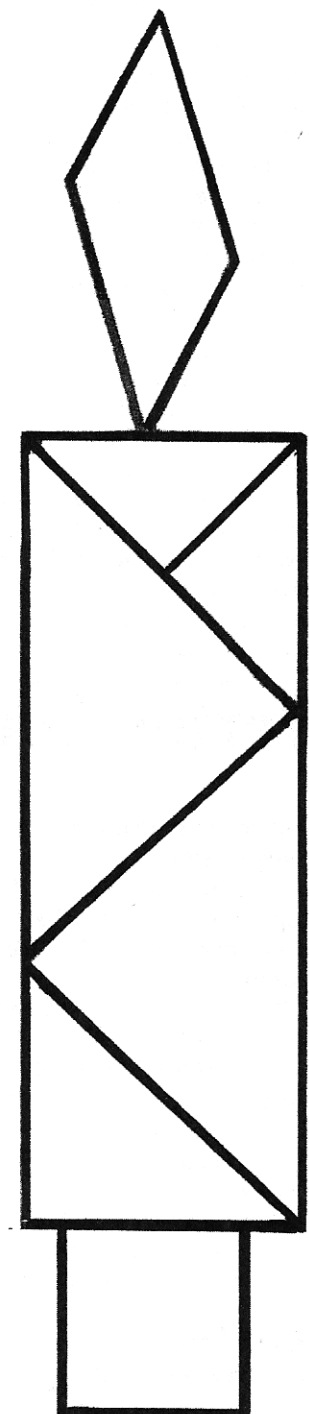
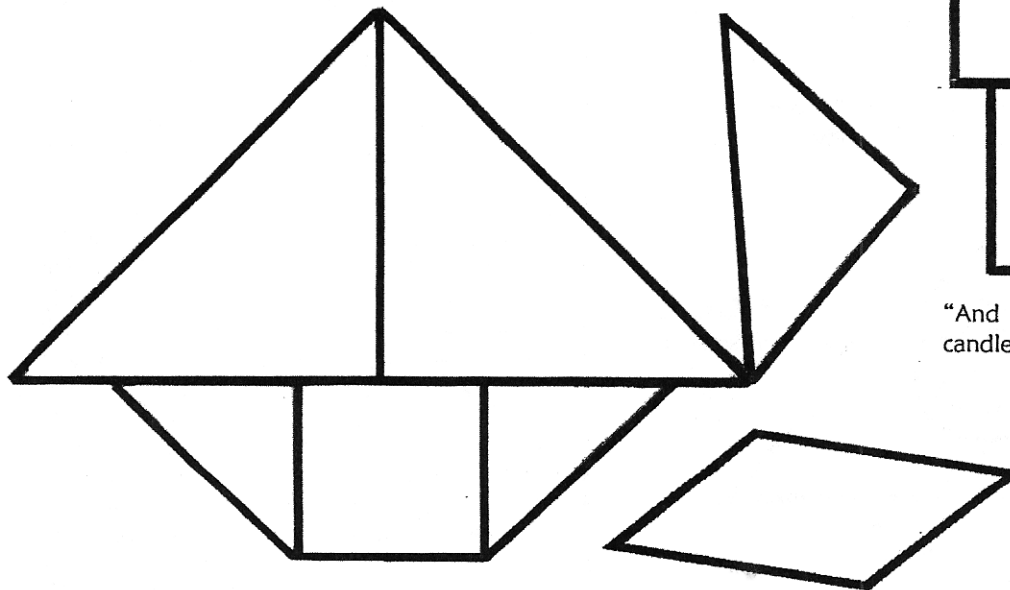
tangrams might be large and made out poster board, though ordinarily she works with a set of standard-sized plastic tangrams. But whether she is surrounded by a group of wiggling kindergartners, or standing in front of a conference hall full of teachers, Rachel does little to alter the lesson. She begins by holding up her own bag of tangrams. "In this bag I have seven pieces," she announces, tipping them out onto an overhead projector. "I have two big triangles, a medium triangle, two small triangles, a square, and a parallelogram." As she speaks, Rachel carefully arranges the five triangles. "I've made a tree," she says, adding the last two shapes to her picture. "The square makes the trunk, and I can hang the parallelogram on the side as a decoration."

Leaving the tangram tree up on the screen, Rachel passes out a set of tangrams to each student. "Open your bag," she instructs the class. "I want all of you to make the tree. Check to see that you have all seven pieces. Make sure that all your pieces are the same color. If you are missing a piece, wave to me furiously and tell me what shape you need. When you have made the tree, put your hands in your lap, look at me and smile."

There is a brief buzz of activity as a forest of tangrams trees sprout up around the room. When every student is smiling at her, Rachel calls their attention back to the overhead screen. "There are lots of things that you can do with these shapes," she says, quickly rearranging the pieces. "I can make a teapot":



"I can make a sailboat and a fish" She moves the fish along to make it swim under the boat":



"And I can make a candle."

Rachel builds these three particular pictures because each requires a different arrangement of the two largest triangles. To make the teapot, the big triangles fit together to make a square. In the sailboat, the same big triangles are combined create an even larger triangle. To build the candle, the two large triangles must form a parallelogram.

After the demonstration, Rachel mixes up the pieces on the overhead and announces, "I'm going to give you some time to make any shapes you want to make. If you would like to see one again—the teapot, the candle, or the boat—wave to me and I will show it to you."

Some students prefer to design their own pictures, but others want to learn the puzzles from the overhead. "Could I see the teapot again?" asks one learner.

Rachel builds the teapot on the overhead and leaves it on the screen for a while. Sometimes a student attempting to copy the teapot will have trouble placing the parallelogram. In some cases, they simply touch the vertex of the shape against the body of the teapot to make the spout. "I don't worry about it," says Rachel. "They simply haven't realized that they need to flip their piece to get it to fit where they want it."

Many students choose to make their own version of the candle,

rather than copying Rachel's design. As she walks around the room, Rachel will occasionally ask a learner to put their design on the overhead for the class to see. "It's funny," she smiles. "A kid will go right up to the front of the room and then once they get there, they realize they can't remember how they did it!" she says laughing. "So then they'll be running back and forth!"

Rachel enjoys the amount of creativity children express with the tangrams. "Kids don't necessarily want to use all seven pieces, and they are more likely to build pictures with spaces in them," she observes. "Adults don't want to leave any empty space at all."

## Vertex, Edge & Face

After the students have had time to make several designs with their shapes, they are asked to stop what they are doing. "Slide all your pieces to the top of your work area," Rachel directs. "Bring down one big triangle. We are going to learn some vocabulary."

"Pick up your triangle and feel this point right here," she tells them, demonstrating with her own shape. "What do you call that?"

"A corner," says one learner.

"Corner is one word we can use," agrees Rachel. "Mathematicians also call it a vertex. Say that—vertex."

"Vertex," the class repeats.

"Say vertex like you're happy!" says Rachel.

"Vertex!" the students respond brightly. "Say vertex

like you're mad!" "Vertex," the class grumbles amidst their own giggles.

"When we are talking about more than one vertex, we say 'vertices'."

Drawing a finger along the side of her triangle, Rachel directs the students, "Run your finger from one vertex to another—that is an edge, or a side."

Now everyone lays their triangles down on their desks. "How many vertices does your triangle have?" asks Rachel.

"Three."

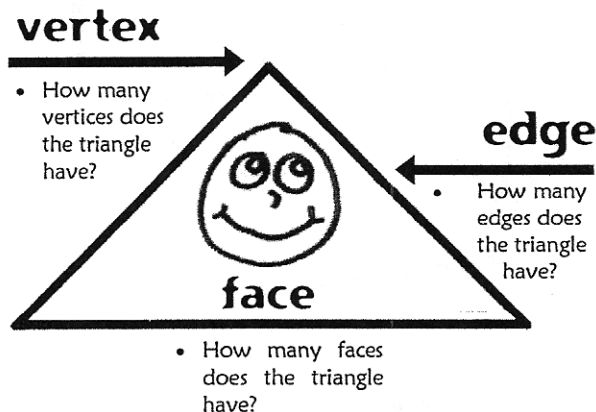
"How many edges does a triangle have?"

"Three."

"Run your hand over the triangle," she says, making a back-and-forth motion across the flat plane of the her own triangle. "This is called a face." With a marker she draws a smiling face on the triangle and spells out f-a-c-e.

"This is a surface," explains Rachel, patting the triangle. "If you spell the word surface, the word face is in there."

"How many faces does a triangle have?" can prove to be a tricky question for students. "You must be careful," Rachel cautions teachers, "If students pick up their triangle off their desk, they will usually answer that there are two faces." Some students will notice that their tangram piece actually has five faces. "A true triangle has only one face," explains Rachel. "You cannot pick it up because it can only exist on a flat surface—basically, you can only draw a triangle. A triangle is two dimensional, but our tangrams are three-dimensional because we need to be able to pick the shapes up in order to work with them."





# Flip! Slide! Turn! (Geometry Gymnastics)

"Today we will be doing transformational geometry," Rachel declares. "We are going to transform shapes into other shapes. There are three verbs we will use when we are working with the tangrams," she announces. "Flip. Slide. And turn." As she calls out each verb, she models the corresponding action with her triangle. "In plane geometry—in surface geometry—you would not be able to pick up a shape and flip it over," Rachel reminds students. "You would only be able to turn and slide it."

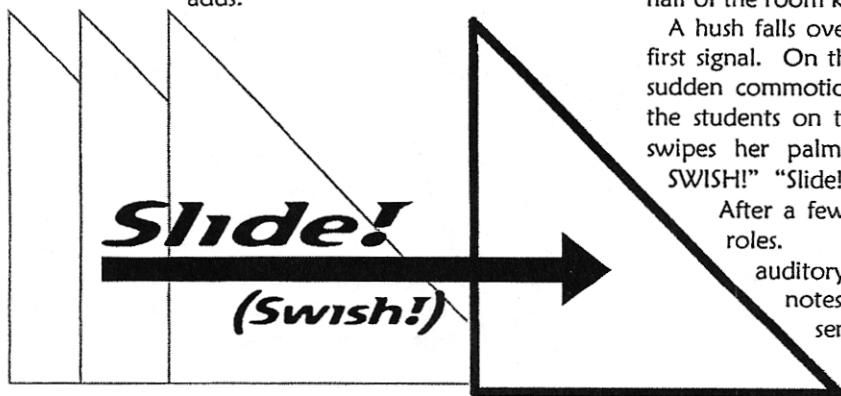
Picking up her triangle, Rachel quickly turns it over and gently slaps it down on the overhead, producing a distinct sound—thwap!



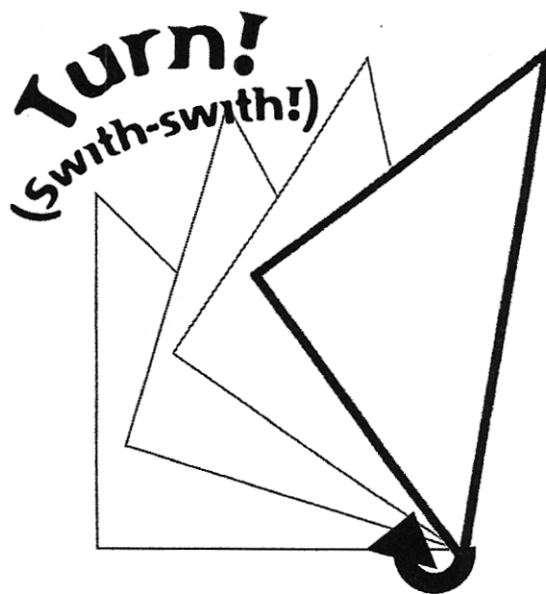
"Flip your triangle," she urges students. A cacophony of flipping tangrams fills the air. "Another word for flip is reflect," Rachel exclaims over the din. "Listen to the sound of reflection! Close your eyes—notice how the flip feels."

The next movement is the slide. "Slide your triangle to the right," instructs Rachel. Swishhh! "Slide it to the left!" Swishh! The sound is softer than the flip. "Feel the slide," she advises.

"Slide your triangle to the center and reflect it!" Swishh! Thwap! "Sometimes a slide is called a translation," Rachel adds.



Finally, rotating her triangle in steady increments, Rachel demonstrates the turn. "Turn your triangle clockwise." Swish-swish! "A turn is also known as a rotation," she says. "Listen to the turn—turn your triangle counter clockwise." Swish-swish!



"Now close your eyes," calls Rachel.

"Flip!" Thwap! "Slide it to the left." Swish!

"Turn it clockwise!" Swish-swish-swish-swish! "Flip it!" Thwap! "Slide it to the center!" Swish!

She divides the classroom in half. "Put your hands in your lap and close your eyes," Rachel tells the students on the left side of the room. "You're going to listen." Addressing the students on the right, she says, "I'm going to give you signals. If I do this," she holds out her hands palm-up, then quickly turns them palm-down, "that means flip. If I do this," Rachel continues, making a small turning motion with her hand, "turn it. If I do this," she says, moving her outstretched hand from left to right in front of her, "slide it. We'll see if the other half of the room knows what we've done."

A hush falls over the room and Rachel gives the first signal. On the right side of the room there is sudden commotion. THWAP! THWAP! "Flip!" the students on the left cry out. Silently, Rachel swipes her palm from side to side. "SWISH! SWISH!" "Slide!"

After a few more signals, the groups switch roles. "This activity incorporates the auditory, the visual and the kinesthetic," notes Rachel. "I call it geometry sense."

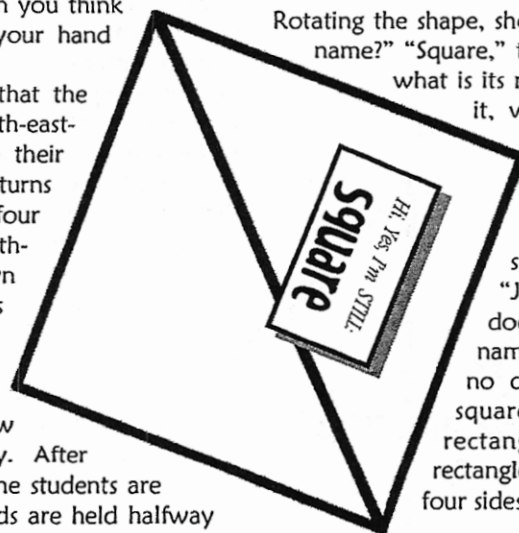
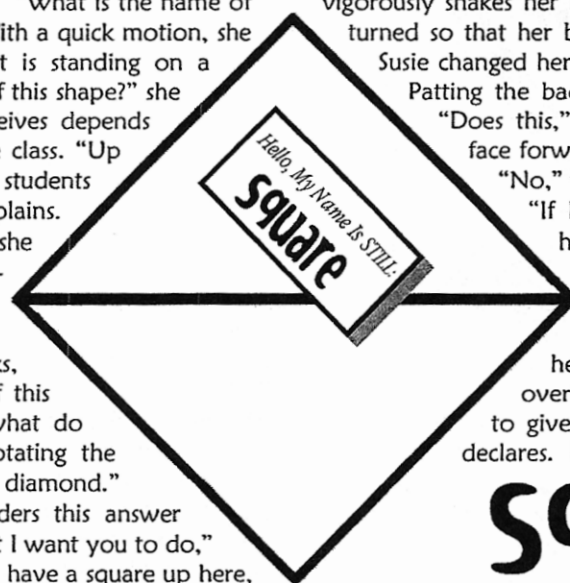
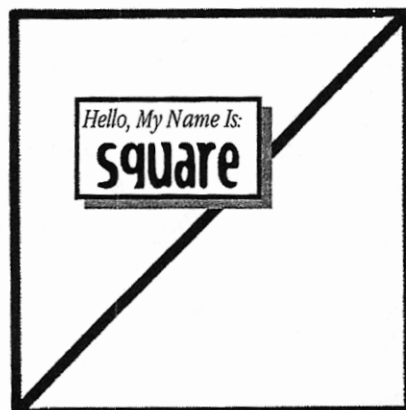


# It's a Square! It's a Square! It's a Square!

Moving on to the next activity, Rachel tells students, "Get out your two big triangles and put them together to make a square."

She does not need to look to know that all the squares have been positioned the same way on the desks. "Almost one hundred percent of the population will build the square so that it is sitting flat on its bottom," she notes. Flipping on the overhead projector, Rachel calls students attention to the square on the screen. "Look up here," she says, patting her handiwork. "What is the name of this shape?" "A square." With a quick motion, she rotates the shape so that it is standing on a vertex. "What's the name of this shape?" she asks. The answer Rachel receives depends upon the grade level of the class. "Up until about the sixth grade, students say it's a diamond," she explains. If a class gives this response, she makes time for a quick side-lesson. Turning the square back to its more familiar position, she asks, "Now what is the name of this shape?" "Square." "And what do you call this?" she asks, rotating the shape back on its vertex. "A diamond." "Mm-hmm," Rachel considers this answer for a moment. "Here is what I want you to do," she says. "When you think I have a square up here, put your hand in the air. When you think it is no longer a square, put your hand down."

She positions the square so that the four edges are facing north-south-east-west. All the students have their hands raised in the air. Rachel turns the shape quickly. Now the four vertices are pointing north-south-east-west. The hands go down just as quickly. She alternates between these two positions a few more times, watching as the hands are raised and lowered accordingly. Now Rachel rotates the square slowly. After a certain amount of rotation, the students are unsure what to do. Some hands are held halfway up.



Rachel asks for a volunteer to come and face the front of the room. Pointing to the girl's nametag, Rachel asks the class, "What is her name?" "Susie." With a hand on the student's shoulder, Rachel guides Susie so that her back is to the class. "What is her name now?" "Susie." Quickly, she turns a giggling Susie to face her classmates. "What's her name now?" "Susie." "Should I flip Susie?" teases Rachel. Susie vigorously shakes her head, "No!" Again, Susie is turned so that her back is facing the class. "Has Susie changed her name?" asks Rachel. "No." Patting the back of Susie's head, she asks, "Does this," quickly she turns Susie to face forward, "look the same as this?" "No," the students reply.

"If I flip her, slide her, or turn her, does she change her name?" Rachel asks.

"No!"

While the volunteer takes her seat, Rachel returns to the overhead projector. "I'm going to give this shape a nametag," she declares. In big letters, she writes:

## square

Rotating the shape, she asks, "If I turn it, what is its name?" "Square," the class tells her. "If I flip it, what is its name?" "Square." "If I slide it, what's its name?" "Square."

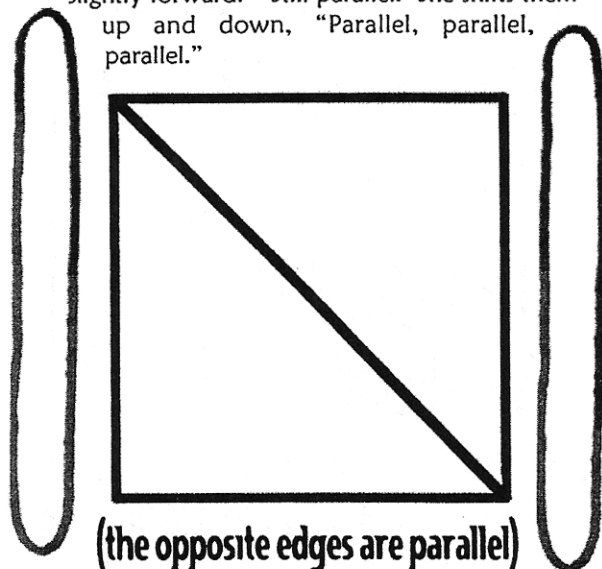
Together, all the students rotate their squares, chanting at each turn. "It's a square, it's a square, it's a square, it's a square."

"Just because you turn a shape doesn't mean it's changed its name," reasons Rachel. "There is no diamond in mathematics. A square is a specific type of rectangle—it is an equilateral rectangle. It is a rectangle with all four sides equal in length."

# Parallel & Perpendicular (Popsicle-stick Vocabulary)

Using the characteristics of the square, Rachel decides to introduce some more vocabulary to the class. She provides each student with two wooden Popsicle sticks. Demonstrating on the overhead projector, she places a Popsicle stick on the left and right sides of the square. "Our Popsicle sticks are on opposite sides of the square," Rachel remarks.

Next, they carefully slide the square out from between the sticks. "What relationship do the Popsicle sticks have to one another?" asks Rachel. The students are unsure. "They are parallel," she explains. Now they slide their squares back into place. "The sticks are still parallel to one another," Rachel comments. On the overhead, she slides one stick slightly forward. "Still parallel. She shifts them up and down, "Parallel, parallel, parallel."

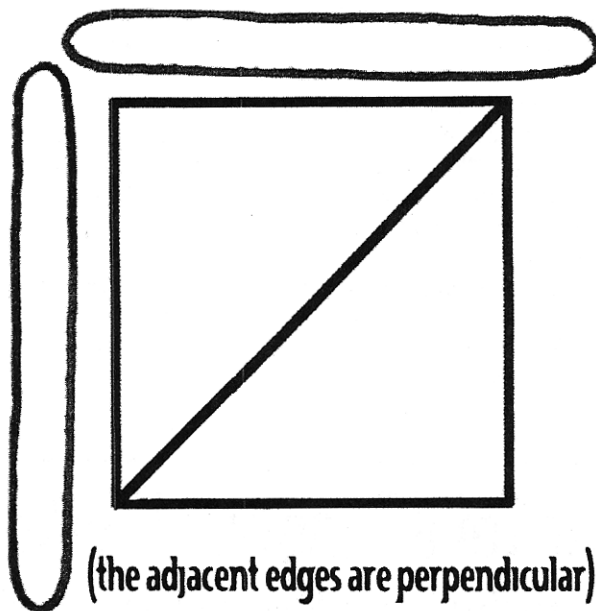


The students hold the Popsicle sticks out parallel in front of them. "I want you to find parallels in the room," says Rachel. "When I say 'Go', I want you to get up and walk around until you find two parallel line segments."

"After you find a set of parallels," she continues, "put your Popsicle sticks over them and hold them there. Find your parallels before I yell, 'Freeze!'"

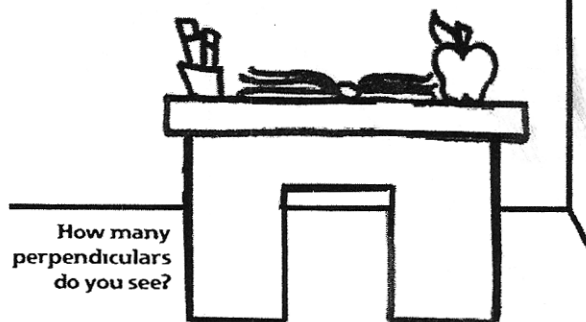
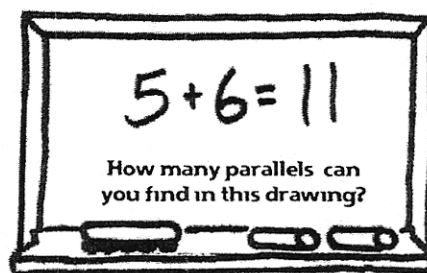
On the word, "Go," the students rush to find a set of parallels. "Freeze!" calls Rachel after several seconds. There are Popsicle sticks along opposite edges of a desk, a notebook, the light switch plate, the tiles on the floor and more. They do this parallel search three or four times before students return to their seats. "Do you see how much geometry is in your classroom?" says Rachel gleefully. "You are surrounded by mathematics!"

Next, they look at the adjacent edges of the square:



Sliding the square out from the Popsicle sticks, Rachel asks the class, "What relationship do the sticks have to one another? 'Adjacent?' replies one student. "That is what they form," she agrees, "but I asked for their relationship--the Popsicle sticks are perpendicular to one another." Sliding the square back into place on the overhead, Rachel demonstrates, "In a square, the adjacent edges are perpendicular, the opposite edges are parallel."

Now the students search their classroom for perpendiculars. "Tonight when you go home, see how many parallel and perpendicular lines you can find in your kitchen," suggests Rachel.

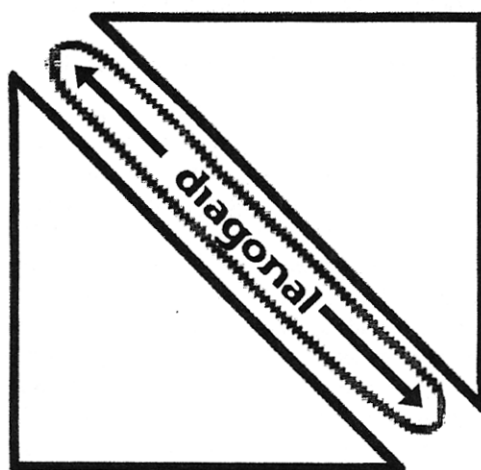




On the projection screen, Rachel repositions her square so it sits on an edge.

"Notice this line going from vertex to vertex when we put the triangles together to make the square," she says. "Pull your triangles apart and lay your Popsicle stick along that line. This is a diagonal—a line segment that connects two vertices is called a diagonal."

Rachel moves the stick and lays it across the square the other way, "There is another diagonal here," she explains, "it's just not split apart. A square has two diagonals."



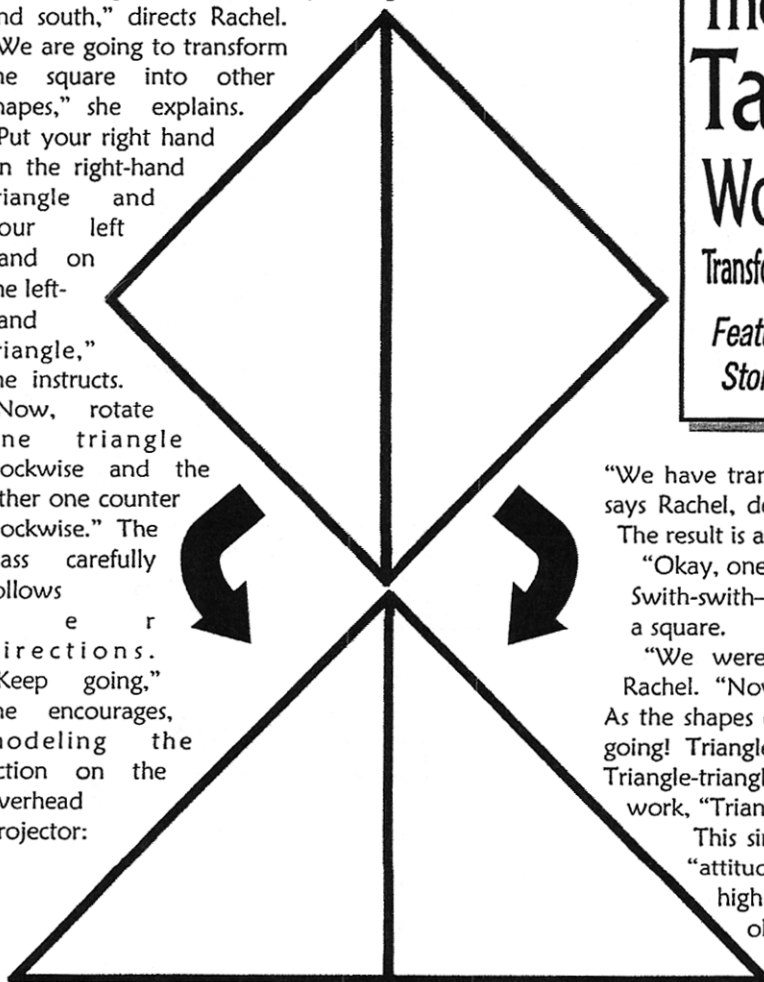
## The Total Tangram Workout (Transforming Shapes in Seconds)

I would like you to position your square so that the diagonal is vertical—pointing north and south," directs Rachel.

"We are going to transform the square into other shapes," she explains.

"Put your right hand on the right-hand triangle and your left hand on the left-hand triangle," she instructs.

"Now, rotate one triangle clockwise and the other one counter clockwise." The class carefully follows her directions. "Keep going," she encourages, modeling the action on the overhead projector:



## SHAPES

Magazine

### The Total Tangram Workout!

Transforming Shapes In Seconds!

Feature **The Quadrilaterals**  
Story: **How This Family Stays in Shape**



"We have transformed our square into a triangle," says Rachel, delighted. "Let's do another rotation." The result is another triangle.

"Okay, one more time!"

Swish-swish—the triangle is transformed back into a square.

"We were rotating towards ourselves," says Rachel. "Now let's rotate away from ourselves."

As the shapes emerge, she narrates, "Triangle! Keep going! Triangle! Square! Listen to that pattern—Triangle-triangle-square!" The students chant as they work, "Triangle-triangle-square."

This simple activity is occasionally met with "attitude" by some students. "Especially in high school geometry classes," Rachel observes. "I notice a lot of eye-rolling and big sighs."

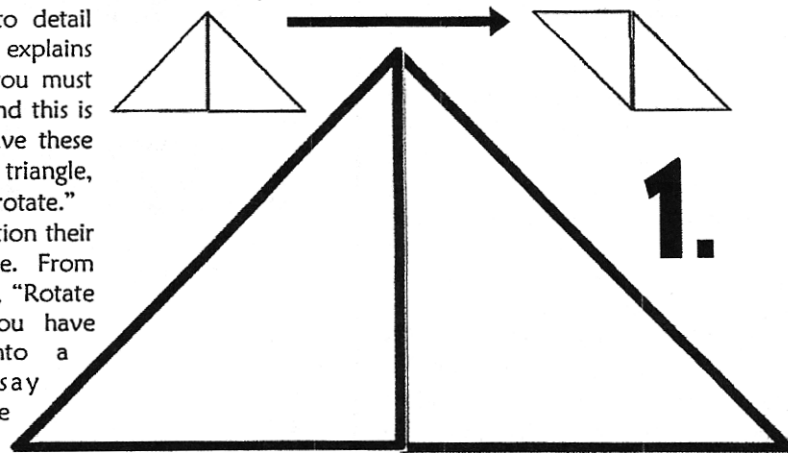


With an impish grin, she cautions restless pupils, "I know you think this is elementary, but I will catch you on it later!"

All the repetition and attention to detail serves purpose. "This is a warm-up," explains Rachel. "When you play a sport you must practice. I am your tangram coach and this is tangram practice. Any time you have these pieces and you need to make a triangle, remember to go to your square and rotate."

For the next exercise, students position their two big triangles into a larger triangle. From the overhead screen, Rachel instructs, "Rotate just one of the triangles until you have transformed the big triangle into a parallelogram. Everyone say parallelogram." "Parallelogram," the students repeat.

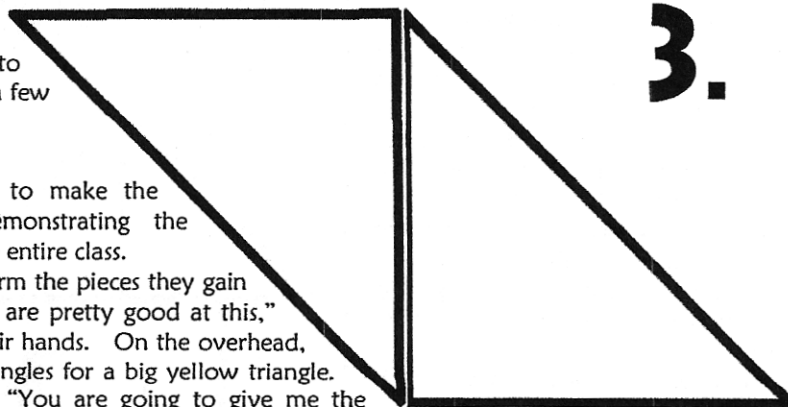
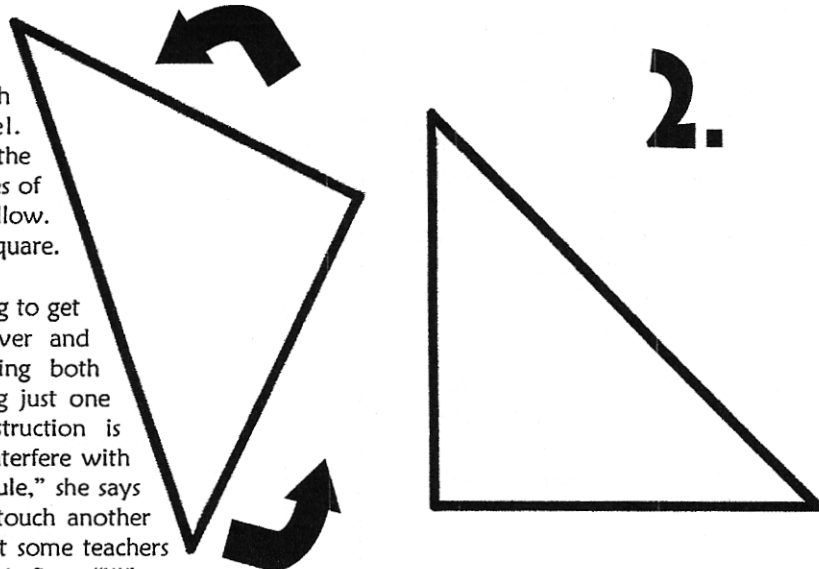
# Triangle to Parallelogram



They practice moving back and forth from parallelogram to triangle.

"Let's start over, beginning with our square," suggests Rachel. Working with her own pieces on the overhead projector, she gives a series of commands for students to follow. "Make a triangle. Go back to a square. Triangle. Parallelogram!"

When she spots a student struggling to get the parallelogram, Rachel goes over and gives directions. "You're moving both pieces," she explains, "try rotating just one triangle." This one-on-one instruction is completely verbal—she does not interfere with the student's tangrams. "I have a rule," she says seriously. "No one is allowed to touch another person's pieces." It is an edict that some teachers find difficult to follow, but Rachel is firm. "When you teach a child to ride a bicycle, you can ride the bike all you want, but the child won't succeed until she's allowed to ride it herself—eventually you have to let her fall down and skin her knees a few times."



"You can also slide the triangles to make the parallelogram," Rachel offers, demonstrating the move on the overhead screen for the entire class.

As the students continue to transform the pieces they gain more confidence. "Who thinks they are pretty good at this," Rachel asks. A few students raise their hands. On the overhead, Rachel trades one of her big red triangles for a big yellow triangle. Selecting a volunteer, she explains, "You are going to give me the orders. Tell me which of the three shapes you want me to make—square, triangle, or parallelogram—and tell me which color piece you want me to move—red or yellow."

Using the square as her starting point, Rachel asks the hesitant student, "Which color do you want me to move?" "Yellow." She places her hand on the yellow piece and waits patiently for further instruction. "Make a triangle," he orders. With a quick rotation, Rachel moves the yellow shape to form a triangle. "Red, parallelogram. Yellow, square." She performs each direction the student gives her without faltering.

"Now, here's how good I am," says Rachel. "I'm going to let you tell me which hand I can use." The student pauses. "Are you right or left-handed?" he asks thoughtfully. "I'm right-handed," she tells him. The volunteer has Rachel put her right hand behind her back. "Left hand, yellow, parallelogram," he instructs. Deftly sliding the yellow triangle into position, Rachel invites the rest of the class to give her further directions. After a few more one-handed transformations, she asks, "Who thinks this is easy?" Almost every hand is raised in the air. "Okay," she smiles, surveying the response, "Now it's your turn. Exchange one of your triangles with someone else so that you have two different colored triangles. Once the flurry of trading is complete, Rachel puts the learners through their paces. "Ready?" she asks. "Put one hand behind your back. Put one hand on a color. Make a triangle." There is a collective pause from the class—perhaps this isn't as simple as it looks. "It's not so easy!" exclaims Rachel, looking over the room. "You can only touch one color," she reminds them.

She notes that some students are not able to work with just one hand. "If the child needs to use two hands, I give them that option." Continuing to call out directions, Rachel allows the class time to practice. "Who is getting better?" she asks. Several learners raise their hands. "Was it hard in the beginning?" "Yes," they admit reluctantly.

# Puzzle Time!

For the next activity, students clear the two large triangles from their work area, making sure to return the odd colored shape to its rightful owner—and replace the pieces with the two smallest triangles and the square. "We're going to do a puzzle using these three pieces," Rachel announces. "I

am going to tell you which shape to build."

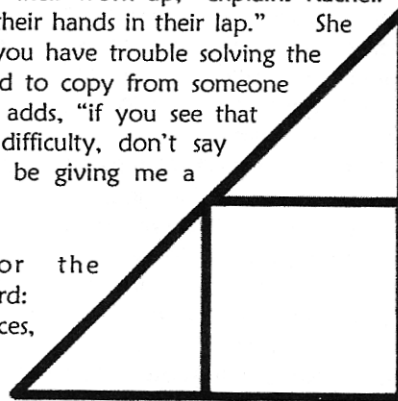
Before they begin, she establishes a few guidelines. "When you solve the puzzle, don't jump up and yell," she advises, "because that gives other people in the room anxiety. Instead, I want you to put your hands in your lap and give me a *knowing* smile." Her eyes fixed in a significant gaze, Rachel beams at the students and gives a series of slow, deliberate nods. "Show me your knowing smiles," she says, scrutinizing their technique. "Shake your head up and down—very good. That's the *knowing* smile."

This silent posture serves a double purpose—it does not distract the students still working on their puzzle, and it allows them to refer to a neighbor's completed shape. "I don't want students to cover their work up," explains Rachel.

"That is why they put their hands in their lap." She informs the class, "If you have trouble solving the puzzle, you are allowed to copy from someone else. Remember," she adds, "if you see that somebody is having difficulty, don't say anything! You should be giving me a *knowing* smile."

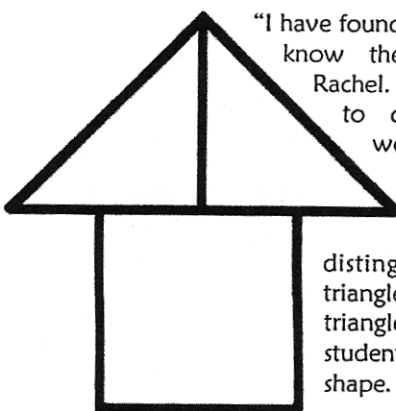
The directions for the puzzle are straightforward:

"Using all three pieces, make a triangle."



Some students—particularly in the primary grade classroom—will build a triangle, but ignore the square. "The rule is you must use all three pieces," Rachel reminds them.

Other students will build with all three pieces, but the result is not a triangle:

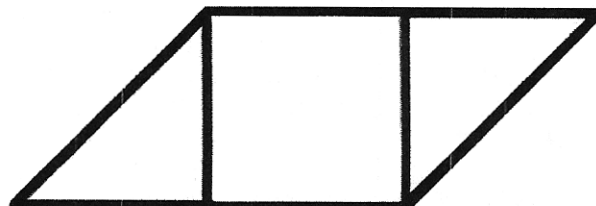
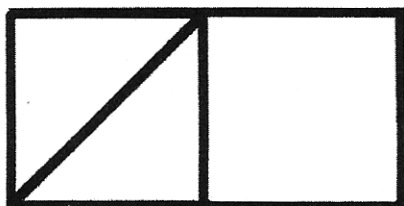


"I have found a lot of learners still don't know their shapes," comments Rachel. "The student may be able to circle a triangle in the workbook, but when they build something incorrectly, it tells me they don't know how to distinguish what is *not* a triangle." "That is not a triangle," she explains to a student who has built an arrow shape. "Count the edges."

From the triangle, students now transform their triangle into a rectangle.

"What do we know about a rectangle?" asks Rachel. On the overhead, she places the Popsicle sticks along the both pairs of opposite edges of the rectangle. "The opposite sides of a rectangle are parallel and congruent," she explains. "A rectangle is an equiangular parallelogram," she adds. "It is a parallelogram in which all angles are congruent."

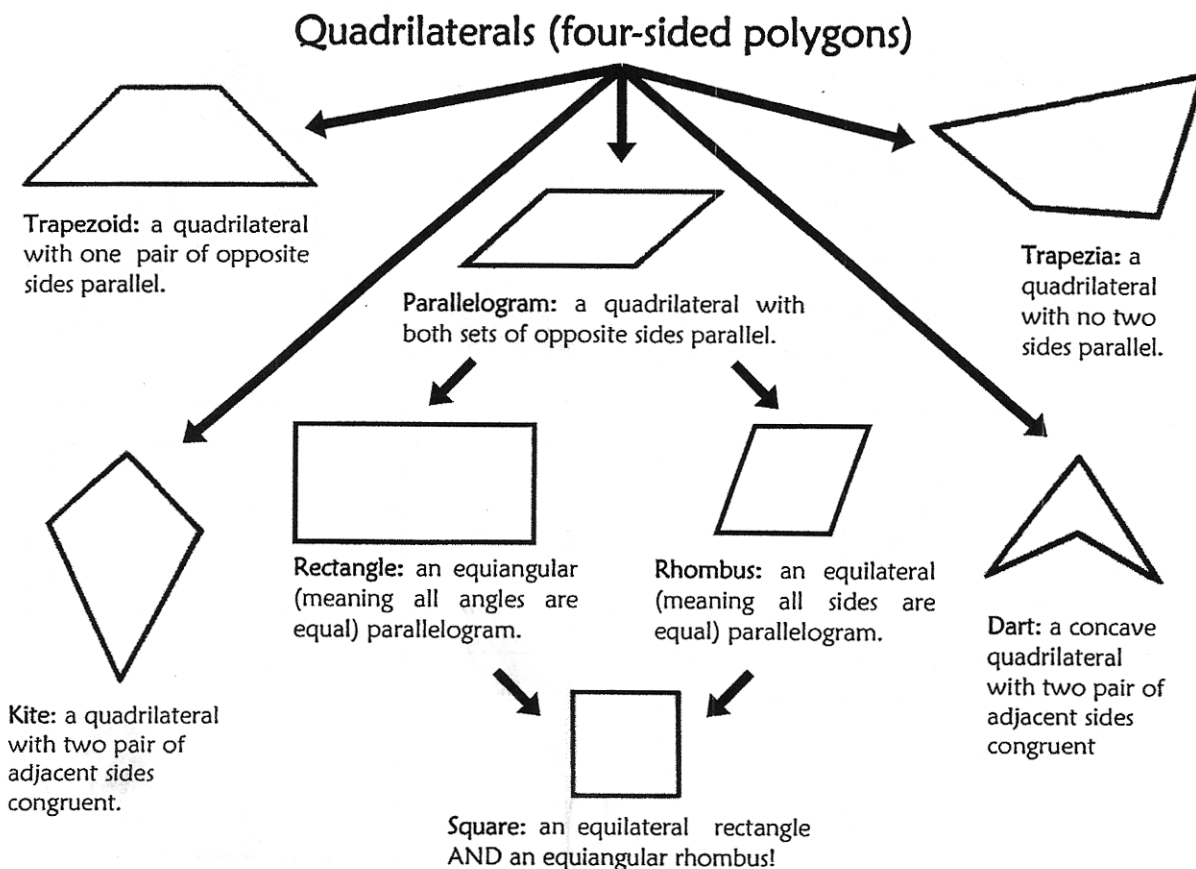
They transform the rectangle into a non-rectangular parallelogram. "The opposite edges are parallel," notes Rachel, illustrating with the Popsicle sticks. "The reason it's called a parallelogram is because both pairs of opposite sides are parallel and congruent. The adjacent angles are not necessarily congruent," she explains. "In a rectangle, the angles must be equal. We will look more at angles later on."



## The Quadrilateral Family

Following the same logic, Rachel informs the students, "A square is a rectangle." Occasionally a teacher will disagree with her on this point. "Some teachers have led their students to believe that a rectangle must have two long sides and two short sides," she explains. "That is not the mathematical definition—a rectangle can have all sides congruent. A square is an equilateral rectangle."

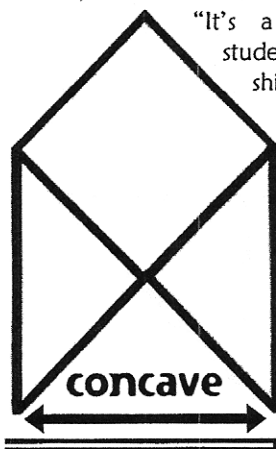
She draws a tree to show how the different quadrilaterals relate to one another





# Counting Edges

Next, Rachel builds a new shape on the overhead:



"It's a rocket ship," says one student. "Yes, it is a great rocket ship," agrees Rachel.

Together, they count the sides of the shape. "What we have is an irregular hexagon," she observes. "It has six sides, but they are not equal. It is also concave." Standing a Popsicle stick on edge, Rachel uses a portion of it to connect the vertices. When she reaches the two vertices that connect outside the shape, she

explains, "If a line segment connecting two vertices passes outside the shape, we call the shape concave. When all the diagonals are inside, the shape is convex," she adds."

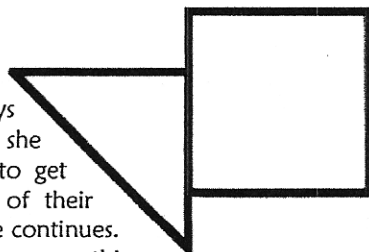
Using the square and two triangles, Rachel allows the class to experiment building other polygons. She gives them a quick vocabulary lesson.

"I'm going to teach you two words that carpenters use in geometry. You are allowed to laugh for one second, because they are funny words. The first word is 'butt'." Rachel conducts the class' laughter, "Butt-butt-butt! Ha-ha-ha!" On the overhead, she uses the small triangle and the square for a demonstration. "When you butt something up against another object, it means you place them so they are touching."

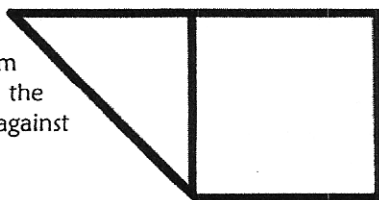
**(butt the triangle up against the square)**

"The second word carpenters use is 'flush,'" says Rachel. Again, she gives them time to get the laughter out of their systems before she continues. "When you make something flush means

that you are making it even." Rachel explains. "I am going to butt the triangle flush against the square."



**(butt the triangle flush with the square)**



# What is (and what isn't) a Polygon?

**Polygon:** A flat or plane figure made up of at least three line segments.

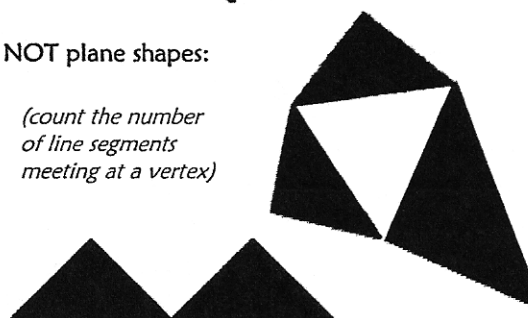
**Plane:** a flat (2-dimensional) closed surface having two and *only two*, line segments meeting at a vertex.

**Polygons/Plane Shapes:**



**NOT plane shapes:**

*(count the number of line segments meeting at a vertex)*



# Sides	Type of Polygon
• 3	• Triangle
• 4	• Quadrilateral
• 5	• Pentagon
• 6	• Hexagon
• 7	• Heptagon
• 8	• Octagon
• 9	• Nonagon
• 10	• Decagon
• 11	• Undecagon
• 12	• Dodecagon

# Building on Previous Knowledge

"Your next puzzle is to take out the parallelogram and two small triangles," announces Rachel. When the students have the shapes in front of them they are directed to make a triangle with all three pieces. "Remember, once you get your puzzle made, put your hands in your lap and smile," she reminds them.

"You have to allow students a lot of time with this," notes Rachel. "especially if they have never messed around with tangrams."

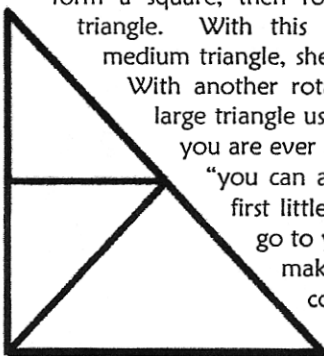
The students transform the same three pieces into a rectangle, and then a trapezoid.

Next, they set aside the parallelogram and retrieve the medium triangle. "Using the two small triangles and your medium triangle, make a triangle," instructs Rachel. She gives them some time to solve it themselves, then demonstrates on the overhead projector. She puts the small triangles together to form a square, then rotates them to form a triangle.

With this new triangle and the medium triangle, she builds another square.

With another rotation, she has made a large triangle using all three pieces. "If you are ever unsure," she comments, "you can always fall back on this first little skill you learned—just go to your square. If you can make a square with a couple of pieces, you can eventually get to the triangle."

Using the same three pieces, they also build a square, a parallelogram, a rectangle, and a trapezoid.



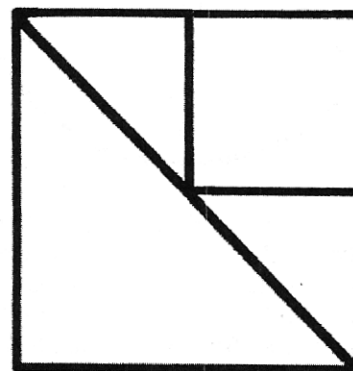
"Take out a big triangle, a square, and two small triangles," Rachel directs. She waits a few seconds for the students to get these four shapes in front of them, then calls out, "Make a square." Walking around the room, Rachel notices many students are having trouble building the shape. "You've done this one before," she encourages, "I want you to think about previous knowledge—that is what a good scholar does." Moving back to the overhead projector, she calls the class's attention to the screen. She builds a square with the two large triangles. "Have I ever done this with you before?" asks Rachel. "Yes," the class agrees.

Next, she uses the square and the two small triangles together to form a big triangle. "Have any of you ever built this before?" she asks. "Yes," the students answer.

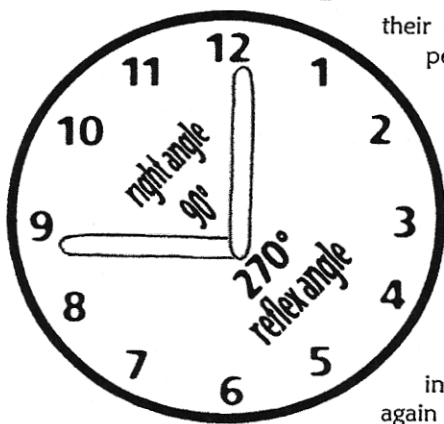
"Now, let's put these together to make our square."

On the screen, they watch as she smoothly uses the three-piece triangle to replace one of the large triangles in the square. "Many of the puzzles are made up of previous knowledge," Rachel explains.

With the same four pieces, students build a triangle, a parallelogram, and a trapezoid.



## Exploring Angles



On the overhead, Rachel draws a clock face. "We are going to learn about angles," she informs the class. In their notebooks, the students also draw and label their own clock. "Take your two Popsicle sticks and make them perpendicular," Rachel instructs. "Perpendicular lines can look like a T, or look like an L," she describes.

In this case, the students make the L shape, placing the Popsicle sticks at 9 o'clock. "This space inside the Popsicle sticks is an angle," explains Rachel. "But the space outside is also an angle. Together, the small angle and big angle complete the circle," she continues. "A circle has 360 degrees." Rotating the Popsicle sticks, Rachel demonstrates. "Turning 360 degrees is known as one revolution. A half turn is 180 degrees. A fourth, or a quarter turn would be a 90 degree angle."

Rachel has the class stand up. "Face the front of the room," she instructs. "Do a 360 degree turn." The students turn until they are once again facing Rachel. "Do a 180," she orders. The students comply. "Good," comments Rachel to the backs of twenty heads. "Now, do another 180."



The class turns to face front once more. "Raise your right hand," she tells them. "Do a 90 degree turn to your right." The students show Rachel their left profiles. "Okay, now do a 360." Some students stop where they began, others turn only until they are facing Rachel. They play a few more turns before returning to their seats.

"When the hands of the clock are 9 o'clock, or  $90^\circ$ ," Rachel continues, "we call that a right angle. A right angle doesn't mean it is always right," she clarifies. "It doesn't mean right or left, it is just the way mathematicians define an angle at  $90^\circ$ ." Placing the Popsicle sticks at 3 o'clock, she says, "Notice, this is also a right angle."

Next, they place both hands of their clock at 12. "We have a  $0^\circ$  angle," Rachel says. "Open it up just a teeny bit— isn't it cute?" she asks. "It's so cute, we call it an acute angle."

Slowly, they open up the angle, chanting, "Acute, acute, acute, acute."

When they reach three o'clock, or ninety degrees, Rachel calls out, "Right!"

"Any angle that is greater than  $0^\circ$  and less than  $90^\circ$  is called an acute angle," she reviews. "When it is exactly  $90^\circ$ , it is a right angle."

"Suppose our angle is greater than  $90^\circ$ ," poses Rachel. "What is it?" There are two possibilities, but one is often left untaught. "An obtuse angle is greater than  $90^\circ$ , but less than  $180^\circ$ ," stresses Rachel. She is concerned that many teachers define an obtuse angle as simply greater than  $90^\circ$ . Moving

one hand of their clock, students chant, "Obtuse, obtuse, obtuse."

When they reach 6 o'clock, they call out, "180!" "It is not a straight

line," notes Rachel, pointing to the center of the clock. "It is a straight angle, because we know where our vertex is."

"If we continue to go around past  $180^\circ$ ," she explains, "we don't have an obtuse angle, we have a reflex angle. A reflex angle is an angle greater than  $180^\circ$ , but less than  $360^\circ$ ."

"Reflex, reflex, reflex," chants the class. "We're looking at this angle," Rachel says, sweeping her hand around clockwise from the 12 to the 10, "not this little one over here." She explains, "Every acute angle has a reflex angle. The acute angle and the reflex angle equal a whole revolution—if my acute angle is  $30^\circ$  then my reflex angle is  $330^\circ$ ." Rachel puts the Popsicle sticks in the 9 o'clock

position. "What if my angle is  $90^\circ$ ?" she asks. What would the reflex angle be?" " $270^\circ$ ."

"When you have two angles whose sum is  $90^\circ$ , they are called complimentary angles,"

Rachel adds. "When you have two angles whose sum of their degrees is  $180^\circ$ , we call them supplementary angles."

Using the two large tangram triangles, the students build a bigger triangle. "

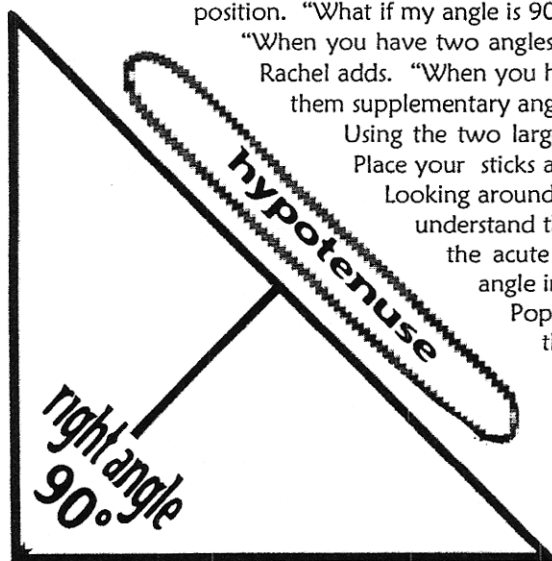
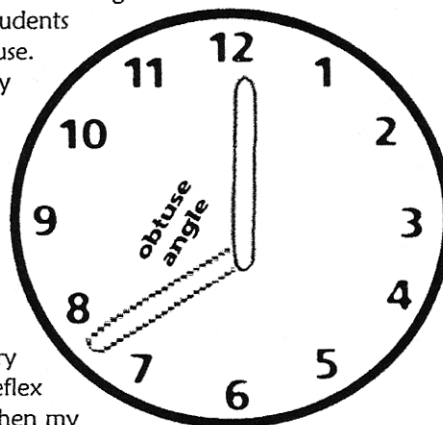
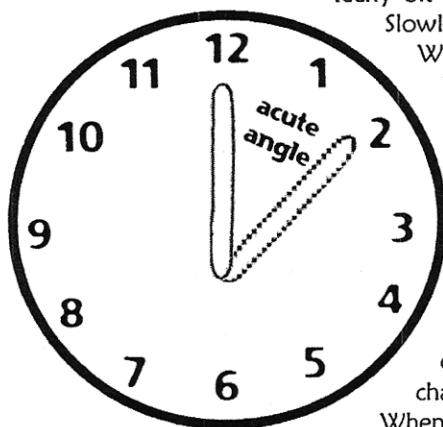
Place your sticks against the edges that form the right angle," instructs Rachel.

Looking around the room, she can immediately assess whether the students understand the concept of right angle. They move their Popsicle sticks to the acute angles of the triangle. "When a triangle has one right

angle in it, it is called a right triangle," Rachel explains. Placing the

Popsicle sticks on the right angle again, she adds, "Notice that these two legs of the triangle are congruent—we call this an isosceles triangle."

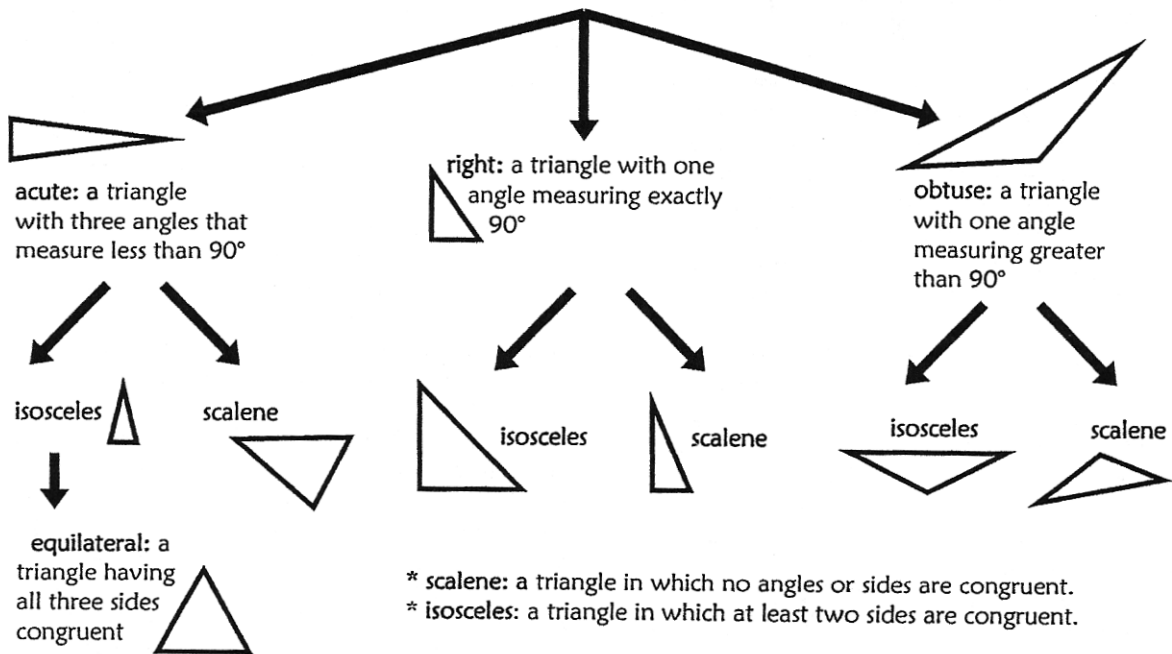
Rachel lays just one Popsicle stick along triangle's longest side. "This edge of the triangle directly across from the right angle is called the hypotenuse. Only a right triangle has a hypotenuse—it is the side or leg that falls opposite the right angle."



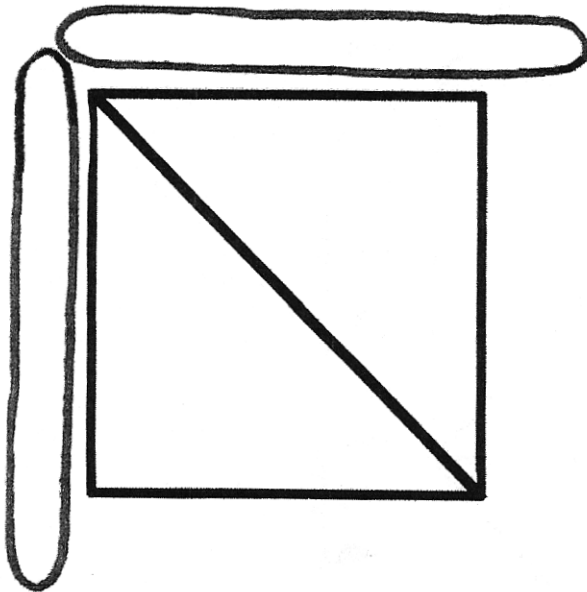


# The Triangle Family

Triangles (three-sided polygons)



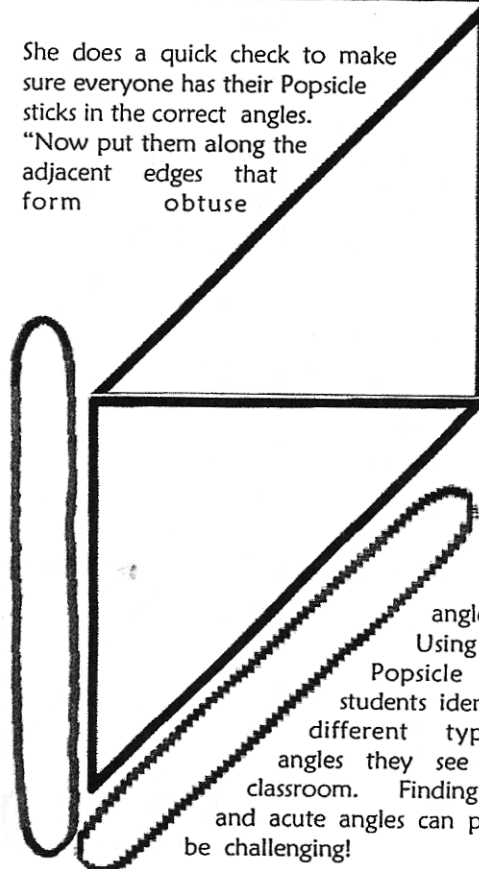
Next, the class transforms the triangle into a square.



"Notice all the perpendiculars," Rachel observes. "That means we have right angles."

Finally, they build a parallelogram. "Put your Popsicle sticks against two adjacent edges that form an acute angle," instructs Rachel.

She does a quick check to make sure everyone has their Popsicle sticks in the correct angles. "Now put them along the adjacent edges that form obtuse



angles." Using their Popsicle sticks, students identify the different types of angles they see in the classroom. Finding obtuse and acute angles can prove to be challenging!

# Bringing It All Together

The final activities allow students to practice their new geometry vocabulary while building with the tangram pieces. Gathering the class around a table, Rachel selects a volunteer. "I'm going to make shape," she tells the student. "I will give you directions for building the same shape—I'll be able to watch you, but you must listen," stresses Rachel, "because you won't be able to see what I am doing."

Facing opposite one another, both Rachel and the student have a set of tangrams. On their work surface, each of them has a sheet of note paper labeled with directional words—North, South, East, and West. Rachel props a book up between them to serve as a divider and stands so she can see the student's tangrams.

She gives very specific directions:

- Take your two big triangles and put them together to make a square.
- Rotate that big square so that the edges are facing North-South-East-West.
- Take your medium triangle and place the hypotenuse on the North side of the big square. Butt it flush up against there.
- Take your two small triangles and put them together to make a triangle.
- Take the hypotenuse of that right triangle and butt it up against the South side of the square.

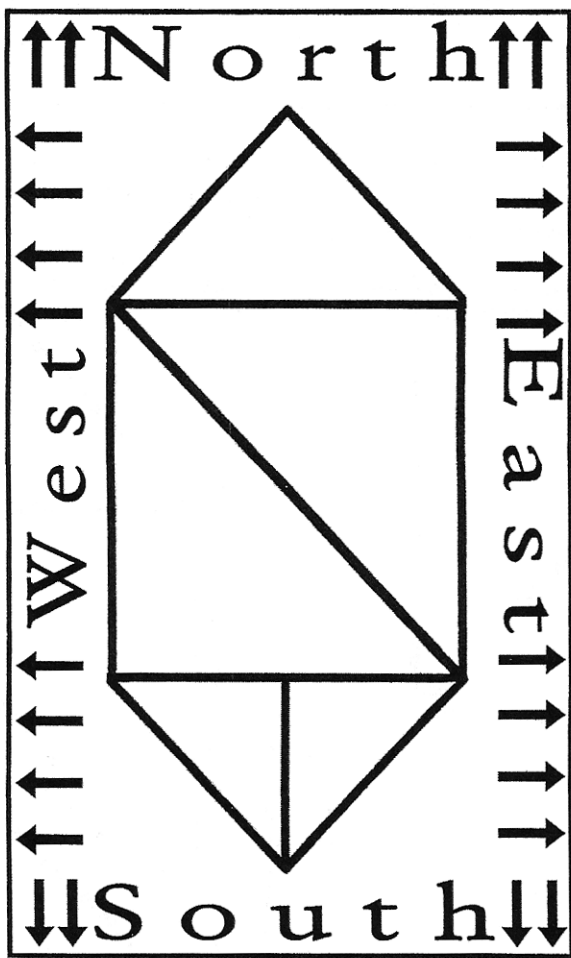
"Tah-dah!" Rachel lifts the divider, revealing that both she and her volunteer have the same completed shape in front of them. Next, the student is allowed to give Rachel directions for building a new shape.

After the large-group demonstration, Rachel puts the students in pairs so they can take turns giving and receiving directions with a partner. The direction-giver is allowed to watch the listener. "If they aren't allowed to look during this first activity," explains Rachel, "the students get discouraged." "Sit opposite one another, not adjacent," she advises. "Prop open a book between you so the listener can't see what your doing!" It is important that students have enough time to practice using the vocabulary—Rachel allows the teams five or six turns with the game.

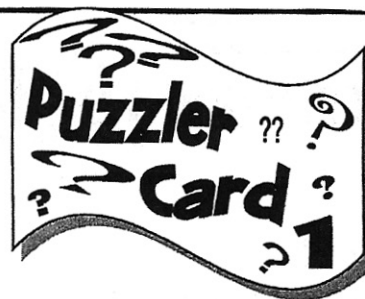
The second activity is more challenging. Gathering the class around once more, Rachel selects a new volunteer. "This time the person giving directions is not allowed to watch what the other person is doing!" she announces. "The listener is allowed to ask questions." Onlookers are forbidden to give hints. Without looking at her volunteer's pieces, Rachel successfully guides the student through building the correct shape. "I've had a lot of practice at giving directions," she explains. When they switch roles, the student has more difficulty. When in doubt, Rachel asks clarifying questions.

"Do you want me to butt this little triangle flush against the square?"

Following the second demonstration, the partners team back up for the double-blind activity. To succeed, students must rely on their vocabulary and listening skills alone. It requires practice—in most cases, the first time the divider is lifted, students are surprised to discover that two different pictures are revealed! "Giving directions, listening to directions, and then following those directions are skills," Rachel notes. "The learners discover they must be very specific with their language, be careful listeners, and they must use their vocabulary to ask good questions."



# Perplexing TANGRAM Pictures

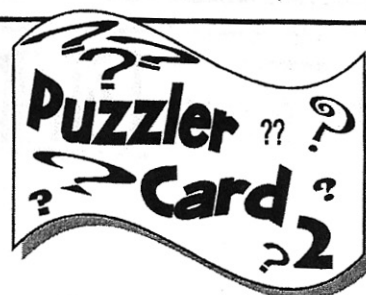


Sitting Cat

Use all 7 tangram shapes to build the picture.

*If you are having trouble with a shape, remember—flip, slide and turn!*

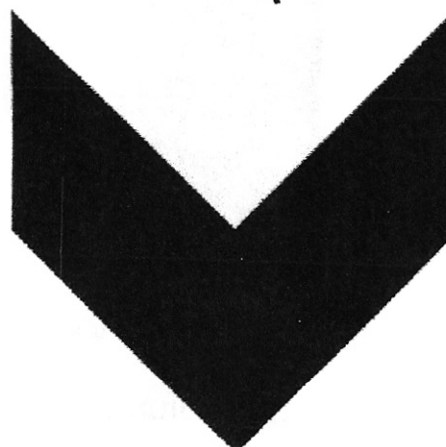
# Perplexing TANGRAM Pictures



Use all 7 tangram shapes to build the picture.

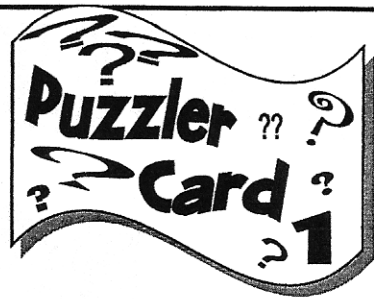
*If you are having trouble with a shape, remember—flip, slide and turn!*

V - Shape

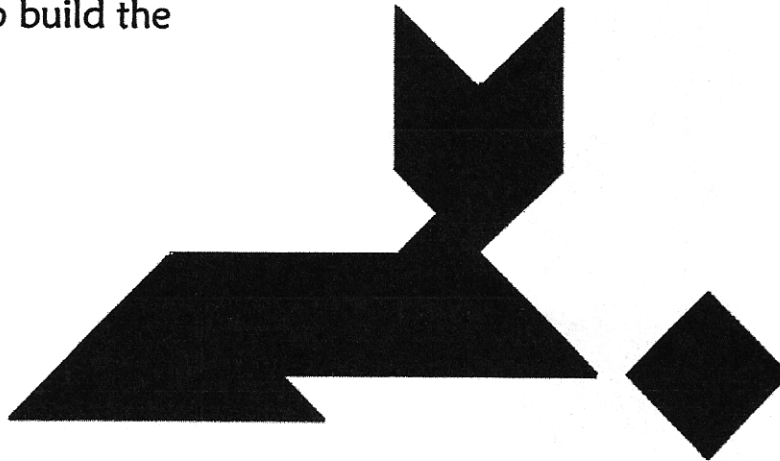




## Perplexing TANGRAM Pictures

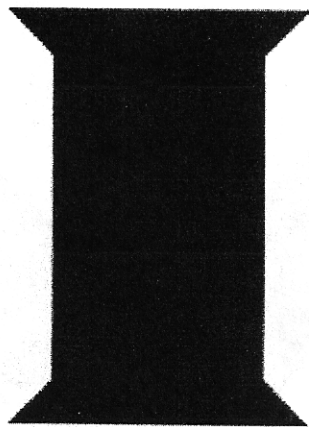
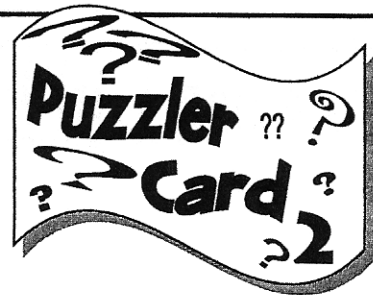


Use all 7 tangram shapes to build the picture.



Cat with a Ball of Yarn

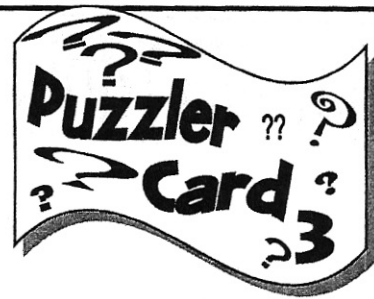
## Perplexing TANGRAM Pictures



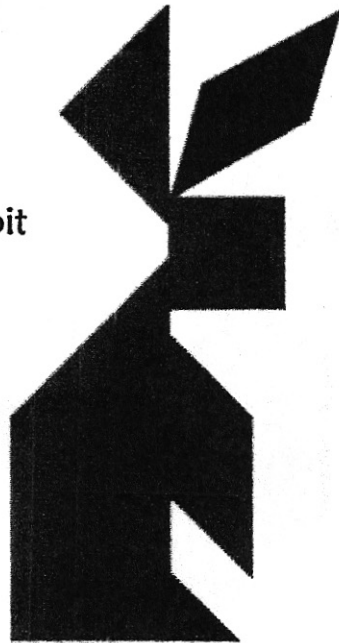
Use all 7 tangram shapes to build the picture.

Wide Column

## Perplexing TANGRAM Pictures



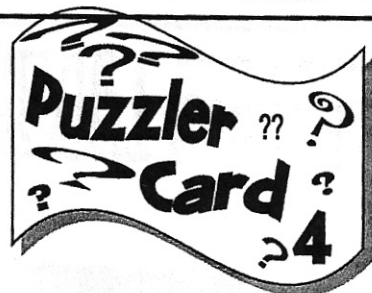
Bunny Rabbit



Use all 7 tangram shapes to  
build the picture.

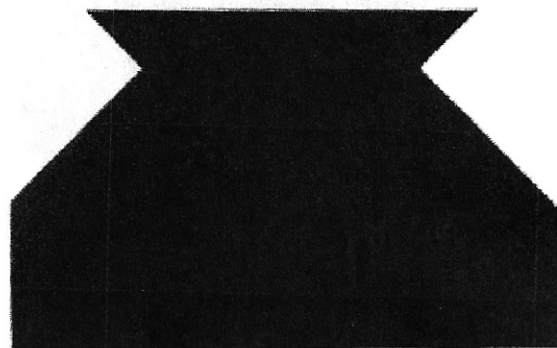
*If you are having trouble  
with a shape, remember—  
flip, slide and turn!*

## Perplexing TANGRAM Pictures



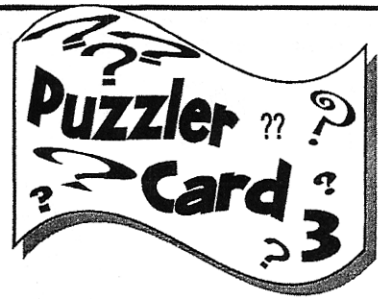
Use all 7 tangram shapes to  
build the picture.

*If you are having trouble  
with a shape, remember—  
flip, slide and turn!*



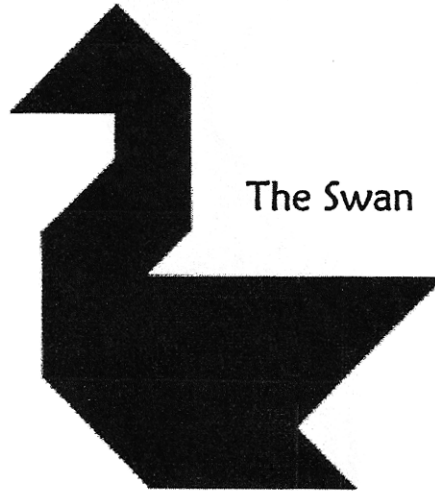
Black Cauldron

# Perplexing TANGRAM Pictures



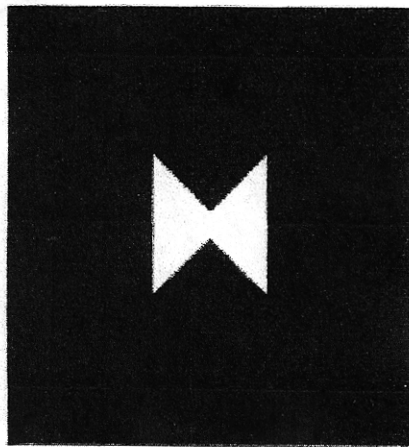
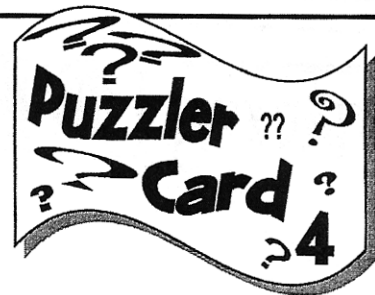
Use all 7 tangram  
shapes to build the  
picture.

*Try building your own  
birds and animals with the  
tangram pieces!*



The Swan

# Perplexing TANGRAM Pictures



The Bow-tie

Use all 7 tangram  
shapes to build the  
picture.

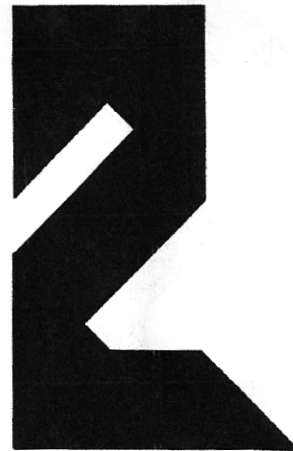
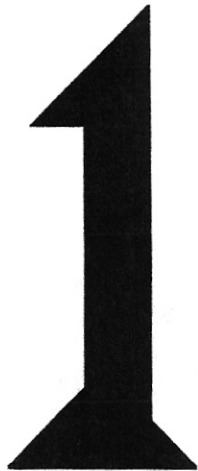




# Puzzler Card

## 5

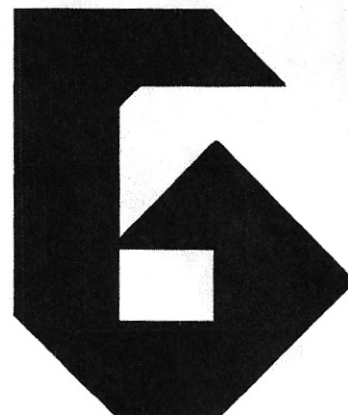
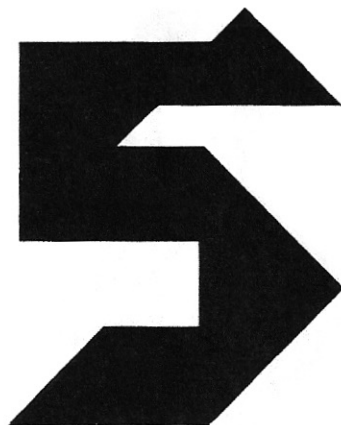
Use your seven tangram pieces to build each digit pictured below.



# Puzzler Card

## 6

Use your seven tangram pieces to build each digit pictured below.

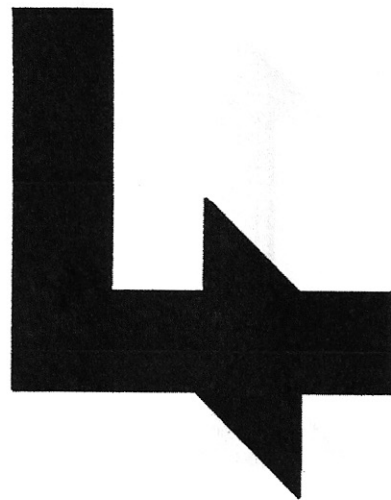
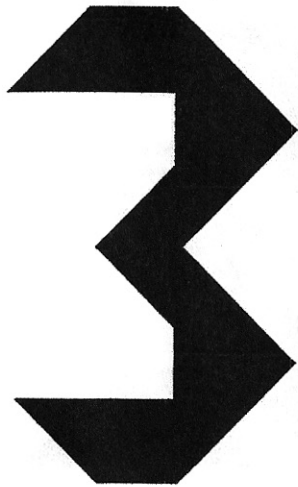




# Puzzler Card

## 5

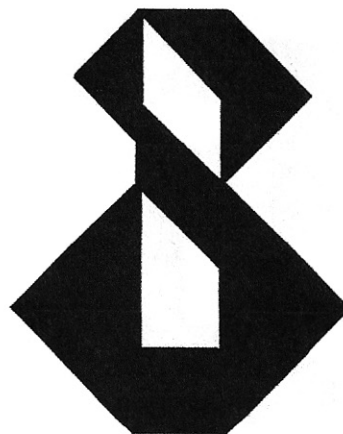
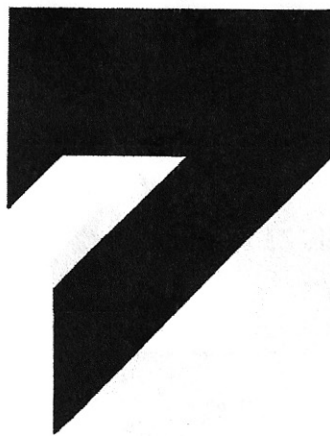
Use your seven tangram pieces to build each digit pictured below.



# Puzzler Card

## 6

Use your seven tangram pieces to build each digit pictured below.

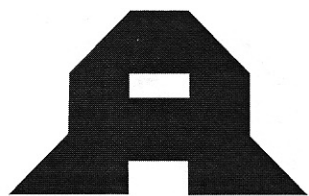




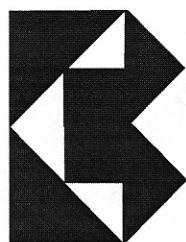
# Puzzler Card

## 7

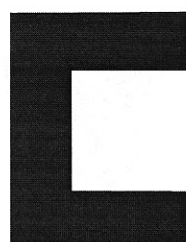
*Can you use the seven tangram pieces to create each of these letters?*



7a



7b



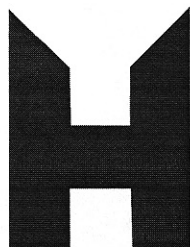
7c



# Puzzler Card

## 8

*Can you use the seven tangram pieces to create each of these characters?*



8a



8b





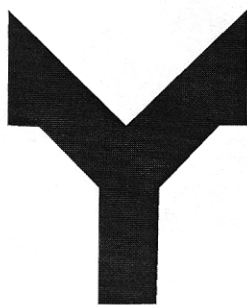
## Puzzler Card

# 7

*Can you use the seven tangram pieces to create each of these letters?*



7d



7e



7f



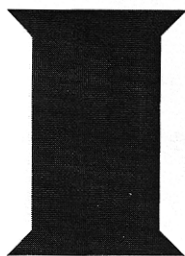
## Puzzler Card

# 8

*Can you use the seven tangram pieces to create each of these letters?*



8c



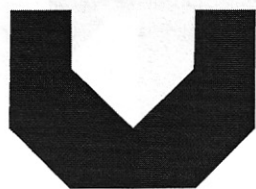
8d



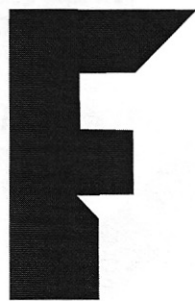
## Puzzler Card

# 9

*Can you use the seven tangram pieces to create each of these letters?*



9a



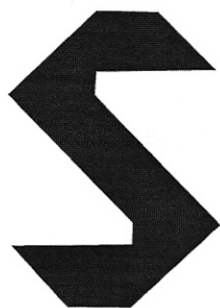
9b



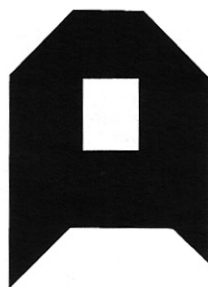
## Puzzler Card

# 10

*Can you use the seven tangram pieces to create each of these letters?*



10a



10b



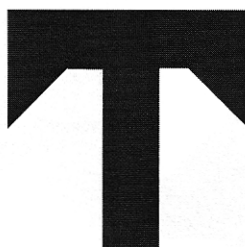
## Puzzler Card

# 9

*Can you use the seven tangram pieces to create each of these letters?*



9c



9d



## Puzzler Card

# 10

### Tangram Word Scramble!

*Unscramble the ten characters you built in Puzzler Cards 8, 9 and 10 to reveal a secret message!*

\_\_\_\_\_

*What other words can you build using...*

2 letters

3 letters

4 letters

5 letters

6 letters

7 letters or more...

*...from the sentence above? What if you include the letters from Puzzler Card 7?*