focus on Fractions

Action Fractions!

"Action Fractions" is another comprehensive lesson developed by Rachel McAnallen, a self-proclaimed math maniac and an enthusiastic teacher with 45 years of classroom experience. Building upon the lessons previously featured in our "Focus on Fractions" series, this issue's activities incorporate the same delightful sense of play that has become their creator's trademark.

Topics Involved: fraction concepts, algebraic concepts, and fun! *Grades:* 1st and up

Materials: Pattern Blocks—not to be confused with Attribute Blocks. The pattern block shapes are as follows: a yellow hexagon, a red trapezoid, a blue rhombus, a green triangle, a white rhombus, and an orange square; addition/subtraction hexahedra; fraction hexahedra; paper, pencil, and three 5" x 7" note cards.

Type of Activity: Includes both large and small group activities

Design Time

To begin, students are paired with a partner and each group of two is given their own substantial pile of pattern blocks. No matter what their grade level, Rachel allows the students time to make designs with the blocks before she begins the formal lesson. "It is very important that the children play with the manipulatives," she

explains. "If you don't, you will have students making patterns while you try to teach the lesson."

After about ten minutes, Rachel announces, "Okay, now you are going to have to mess up your design." After allowing the class to give one collective moan of disappointment, Rachel

assures them that she will allow them time to design at the beginning of each lesson. "I can decrease the design time

each day, but they still need to play with them," she notes. "If math class is after recess or after lunch, it's helpful to have the blocks already out on the desks. When the students come back into the classroom, they can settle down and start to design

right away."





red trapezoid





blue rhombus

green triangle

LHS

Left Hand Side

When design time is over, students are asked to sort the blocks. Rachel collects the skinny white rhombuses and the orange squares. These blocks will not be used because they are not significant to the lesson. "Even the oldest kids will want to use them in their equations," explains Rachel, "so it is just easier to remove them."

Pattern Block Equations

Next, Rachel provides each student with three 5"x 7" note cards. They are instructed to arrange the items on their desks as follows: pattern blocks at the top of the desk, index cards in the middle, and their math journal directly on front of them.

Moving to the overhead projector, Rachel places a yellow hexagon, a red trapezoid, a blue rhombus, and a green triangle in a line along the top of the screen. Students replicate the arrangement at the top of their notepaper. "We are going to trace around each shape," instructs Rachel, drawing around the yellow hexagon. "Do NOT write anything inside your tracing!" she stresses. "Underneath, but not inside, I want you to write yellow hexagon."



Equals

Following the example on the overhead, learners trace the remaining pattern blocks and label them:

Red Trapezoid, Blue Rhombus, Green Triangle.

"We are going to make math sentences with these pattern blocks," Rachel explains. "For example, one yellow hexagon is equal to two red trapezoids." With her marker poised to write, she pauses and asks skeptically, "Do you want to have to write out yellow hexagon?"

"No," several students respond.
"That's right," agrees Rachel,
"because it takes a long time to
write out yellow hexagon.
Mathematicians are lazy," she
confides. "Usually we choose just
one letter to represent
something."

She gives an example, "When a mathematician wants you to pick a number, they don't bother to write out 'pick a number.' Instead, they use a letter to stand for the word 'number.' Now, can anyone guess what letter they use?"

"N," one learner suggests.
"Why did you pick n?" Rachel asks her.

The girl gives Rachel a puzzled look, as if the answer was obvious, "Because it's the first

RHS

Right Hand Side

letter in the word."

"Of course!" Rachel says. "Now, what letter do you think we could use for the hexagon?"

"H," another student suggests.

"Yes," Rachel tells him, "but don't write that down yet." She's leading up to something. "What would we use for trapezoid?" The room is filled with a chorus of T's.

"Yes," Rachel says slowly, "but then what are we going to use for triangle?"

"Tr?" one learner volunteers.

She thinks this over, then points out, "But trapezoid is also spelled t-r."

"We could use colors," offers one girl.

"Oh," exclaims Rachel, "What a good idea! What letter would you like to use for the yellow hexagon?"

"Y."

Inside the hexagon tracing, students draw a great big Y. They label the red trapezoid as R, the blue rhombus as B, and the green triangle as G.

Rachel now calls the students' attention to the three 5" x 7" cards she handed out earlier. "On the card on the left hand side, I want you to write LHS," she instructs, writing out her own

example on the overhead.

"What do you think LHS stands for?" she asks. The response depends on the age of the learners—older students will often correctly guess, "Left Hand Side."

Along the bottom of the card, they write out, "Left Hand Side." Moving on to the center note card, the class draws a large equals sign in the middle. On the bottom they write out, "Equals."

Finally, the right hand card is labeled RHS, with the smaller explanation, "Right Hand Side" on the bottom edge.

"Now we are going to make equal sentences. So now what we are going to do is make equal sentences," Rachel declares. "Everyone take a Y and lay it on top of the LHS on your left hand side," she says, placing her own Y block on the overhead. "I want you to take other blocks and build them on top of the yellow. Once you have build blocks that match, slide them over to the right hand side card."

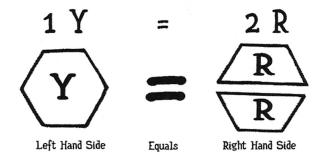
After a minute or so, Rachel asks, "Someone tell me what they have."

"Two reds."

"Read me the whole sentence," she urges.

"One yellow equals two reds."

"How many of you have one yellow on the left hand side and two reds on the right hand side?" Many students raise their hands. "Here is what we're going to write": 1 Y = 2 R or 1 Y = 1 R + 1 R



On the overhead, Rachel builds her own equation using two blues and two greens. She writes:

1Y = 2B + 2G

"Who has something else?" she asks.

"One yellow equals three blues."

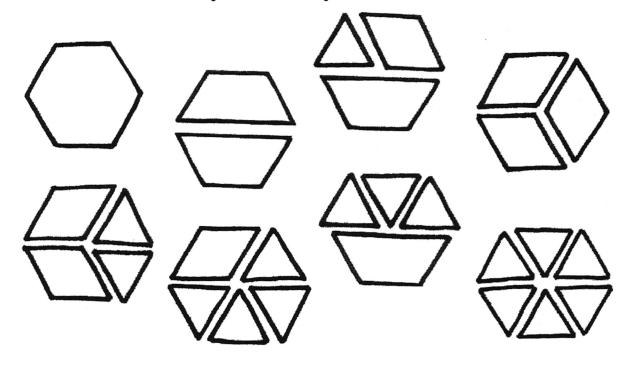
1Y = 3B

"Who has another one?"

1Y = 1R + 1B + 1G

"Teachers will sometimes tell me that their fifth and sixth grade students still do not have the concept of equals," notes Rachel. "To be very truthful, I often have more difficulty teaching this concept in a workshop full of grown-ups than I do in a classroom of second graders."

How Many Ways to Make 1?



Some adults have a tendency to build inequalities rather than For example, they equations. might place one yellow hexagon on the left hand card, and on the right hand card they will stack six green triangles on top of three blue rhombuses, which are stacked on top of two red trapezoids. "That is not equal," Rachel explains. "What they are saying is that one yellow is equal to two red, plus three blue, plus six green." In other words: 1Y = 2R + 3B + 6G—which is not true.

The creator of an offending non-equation will often argue with Rachel, "The yellow is equal to each one of those!"

"It's true--six is equal to three plus three, and four plus two, and five plus one," she agrees, "but not all at once. What I've found is that many adults may not have the concept of equals!"

No matter what the age of the learner, they are only allowed to build one equation at a time. Rachel likens it to balancing a set of scales. "If we were to put just the one hexagon on the left side of the scales and a stack of trapezoids, triangles and rhombuses on the right hand side, they would not balance—the right hand side would be heavier."

Working together, the class writes out all the possible ways to balance one yellow hexagon.

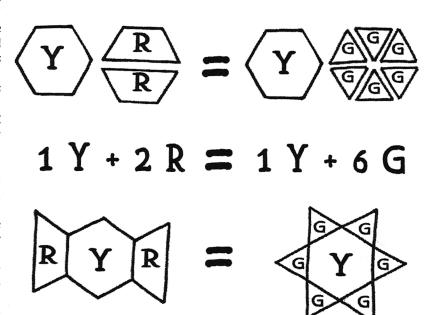
Once they have completed their list of equations, Rachel gives a new direction. "Pick up the yellow from the left hand side and put it back in your bank of blocks. You must substitute it with something that is equal."

A student who had a sentence that read 1Y = 2R might trade in their Y for one blue and four greens. The new sentence would then read: 1B + 4G = 2R

When they have written out their new equation, the students exchange notebooks with their partner. "Check and see if what

LHS = RHS

Left Hand Side Equals Right Hand Side



your partner has written works out," Rachel tells them. After verifying their partner's equation, students clean off their LHS and RHS cards and build a new sentence. "Students are constantly exchanging notebooks," says Rachel. "They build an equation, write it down, and then someone reads their work, and they read their partners."

Throughout the lesson Rachel continues to build sentences on the overhead. On the LHS card she places a hexagon, then carefully positions two trapezoids to create an arrangement that resembles a yellow face with two enormous red ears. On the RHS card she arranges three blues and six greens in a flower formation. "Equal does not always mean the same," she explains. "These may not look the same, but they are still equal."

Another way to illustrate this idea is by using coins. "If I have a quarter and a nickel in one hand, and three dimes in another, then I have an equal amount in each hand. They are not the same coins, but they have the same value."

The students do several large equations with Rachel before she announces, "You can now go make your own sentences-but," she adds, "you cannot use more than twenty pieces total." This limitation not only conserves pattern blocks, it incorporates an opportunity for students to work on problem solving. "If a learner builds with sixteen blocks on the left hand side, they only have four blocks to work with on the right hand side," Rachel smiles. "They might have to go back and redo. It's a puzzle they have to work out."

Some students create elaborate sentences, while others build more simple equations. After about ten minutes, Rachel lifts the limitation on blocks and replaces it with an interesting new restriction. "I'm going to allow you to use as many blocks as you want," she tells them, "but this time one person must build the left hand side, and then your partner must build the right hand side." The new twist requires teamwork. When the equation is balanced, both partners write down the entire sentence they have created.

An extension of this activity is to assign monetary values to a block according to students' developmental level. For instance, if a green triangle is worth \$.05, students must then work out the values of the other blocks. Students can also build sentences and then fill in the values for each side to see if it works out. For instance, if a green triangle is worth \$.05, then the sentence 2R = 3B works out to \$.15 + .15 = \$.10 + \$.10 + \$.10 "You can make this activity as easy or difficult as you want," declares Rachel.

Pieces of Gold

When the class has become experienced in building pattern block equations, Rachel introduces them to the first Action Fractions game. "We are going to play a fun game called Pieces of Gold," she announces. "Imagine that the pile of pattern blocks is a gold mine. I want everyone go into the gold mine and take out the block you think is a piece of gold." The majority of the class selects a yellow hexagon.

"Yes," Rachel confirms. "The yellow hexagon is one piece of gold." On the overhead projector, she traces around a yellow hexagon. Each student has turned to a clean sheet of paper in their notebooks-before they play the first game the class creates a fractional code sheet. Following Rachel's lead, they place a yellow hexagon in the upper left hand corner of the page and trace around the "piece of gold." Next, they draw a large 1 inside the tracing, leaving plenty of space on the right. Underneath the hexagon, they write: "Piece Of Gold."

 $-\Delta = \frac{\frac{6}{6}}{\frac{6}{6}} = \frac{1}{6}$

Now they take two red trapezoids out of the gold mine. On the overhead, Rachel places them over the tracing of the hexagon. "What fraction is this?" she asks.

"Two out of two." They write: 1 = 2/2

Rachel slides one of the trapezoids to the left of the hexagon outline. "What fraction is this?"

"One half." They trace around the trapezoid, and write ½ inside.

Now three blue rhombuses are placed over the POG outline. "What is the fraction?" Rachel asks. "Three out of three." They write this inside the hexagon. Next they make a tracing of a blue rhombus and label it 1/3.

"What do you think the green triangle is worth?" asks Rachel. This time she does not build over the hexagon tracing. Although most students answer one-sixth, some learners may think they see a pattern based on the previous fractions one-half and one-third. "When I first taught this lesson," she recalls, "I did not teach the left hand side equals right hand side activity. Then I would ask what fraction the green triangle represented and the students would all say one-fourth. I wondered what was wrong with these kids! Well, there was nothing wrong with them—I had left something out of the lesson. If they have enough practice building the sentences, they usually come up with one-sixth."

Rachel reminds students, "If you have one out of six, then you must have two out of six, three out of six, four out of six and five out of six, and six out of six." She instructs them to put six one-sixths together and write 6/6 inside the hexagon tracing. "Now we need five out of six. What are we going to do?"

"Subtract one-sixth," suggests one learner.

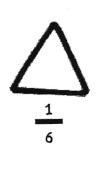
Placing a finger on one of the green triangles in sixth-sixths, Rachel slides back and forth, in and out of position. "Five-sixths, one-sixth, five-sixths, one-sixth. Partner fractions! If we subtract one-sixth from six-sixths, what do we have left?"

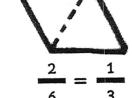
"Five-sixths."

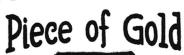
Drawing around the five-sixths can be tricky, so students are given a choice, "Do you want to trace around the hard way or the easy way?" asks Rachel. The class chooses the easy method. "Trace around

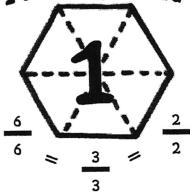
the yellow hexagon," Rachel tells them, "but leave one edge open." She makes her own tracing on the overhead, explaining, "Now put your green triangle in the opening and draw around that." The result reminds Rachel of an old video game character. "It looks like Pac Man," a student agrees.

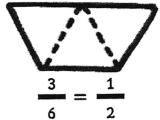
Fractional Gold Game

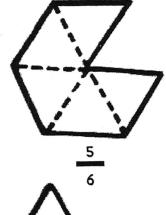


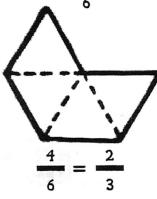














Following Rachel's example on the overhead, students position and reposition their green triangle to create the dotted outline of five triangles within the tracing.

Using their pattern blocks, they subtract another one-sixth from the group of five green triangles. "How much have we subtracted in all?" asks Rachel.

"Two-sixths."

"What do we have left?"

"Four-sixths."

Students explore different methods for drawing this fraction—some trace around a yellow hexagon and leave two edges open, others trace around two blue rhombuses.

"It looks like a baby buggy," one student observes. Using a dotted line, they fill in the images of the four triangles that make up the shape and label it 4/6. "Wait a second," says Rachel. "Four-sixths is also equal to what?"

"Two-thirds." To the left of 4/6, they write = 2/3.

"We have a pattern going on," Rachel notes. "We've had sixth-sixths, five-sixths, and four-sixths. What is next?"

"Three-sixths." Looking at the three green triangles, students decide that they do not need to draw around them because they have already have traced around the red trapezoid. Instead, they dot the outline of three triangles inside the existing tracing and write = 3/6 to the left of the 1/2 label.

The class subtracts another one-sixth from their pattern block image. "Do we already have the drawing for two-sixths?" asks Rachel.

"Yes—one-third," they tell her. They dot in two triangles and add to the label: = 2/6.

Once the class has completed their own code sheet, Rachel gives each student a hand-out with the code sheet at the top and an outline of a bag of gold below. This sheet will be used to play several games. The information in the hand-out is identical to what the students have created in their notes. "There is a reason I don't give them my code sheet in the first place" explains Rachel. "When the students draw out the fractions themselves they have a much better understanding of the material."

Roll It Read It "Add one-half." Repeat It

Large Group Gold Rush

The first game is a large-group activity. Each set of partners share a tray containing a two hexahedra—an action die and a fraction die. The addition/subtraction cube will determine the action or operation that must be applied to the fraction rolled with the second die.

"The object of this game is to collect gold," explains Rachel. "You will add to your bag of gold, or subtract from it depending on what you roll with the addition/subtraction cube." The fraction die features the following six numbers: 1, 5/6, 2/3, ½, 1/3, and 1/6. "Rolling the other hexahedron tells you how much you must add or subtract," she continues. "Every fraction is on here, but some are hidden--let's see, is six-sixths on here?"

"Yes," students tell her.

"How is it written?" she asks.

"One."

"That's right—how about four-sixths?"

"Two-thirds."

"Every fraction or it's equivalent is here," Rachel tells them. "Remember, you can look at your code sheet if you need help finding an equivalent."

To begin the game, players are instructed to put two pieces of gold in their bag of gold. Most of them select two yellows. Smiling, Rachel waits a moment. "Oh," she adds, "Your two pieces of gold can't be the same." There is a flurry of activity as students make exchanges.

"From now on, I want you to use the fractional name when you talk about your pieces of gold," Rachel explains. "Instead of having three blues, you would say you have three-thirds."

Going around the room, students use fractional language to describe their pieces of gold.

For the sake of classroom management, the rules for rolling the hexahedra are strict. The dice remain in the tray until it is the student's turn to roll. Players must shake and then roll the dice in the tray—if a die lands on the floor or if the player takes too long shaking, they lose their turn.

To emphasize the importance of speaking the language of fractions, Rachel asks the class to observe a simple formula: Roll it. Read it. Repeat it. Complete it. In other words, the player rolls the hexahedra, reads what they have rolled, and the class repeats it. Finally, the entire class completes the transaction that their classmate has rolled.



The game begins. The first player rolls and reads, "Add one-half." The class repeats this. "Everyone go and add one half," Rachel tells them. Each student decides for themselves how they will add one half to their bag of gold. Some players might simply add one trapezoid to their pile, others may decide to add three green triangles, or three-sixths, and still others might prefer to make more complex trades.

"How much do we have?"

"Two and one-half."

On the next roll, players must subtract one-third. Some students do not have one-third to subtract from their bag of gold. In order to subtract, they must trade a larger piece of gold in for smaller blocks.

"What is our new total?"

"Two and one-sixth."

Rachel plays as well. Throughout the game she invites students up to the overhead projector to show how they choose to make a transaction.

When the class has subtracted for a number of rolls in a row, Rachel intervenes. "Oh, it's my turn!" she exclaims. It's a bit of a shady operation—no matter what she rolls, the students are told to add 5/6 or 1. "I'm not ready for them to deal with negative numbers yet," she explains. "When I see that possibility, I take charge of the numbers!"

The game continues until each student has had a turn. "Sometimes teachers will comment that this takes an awfully long time," acknowledges Rachel. "It does. But middle school teachers will never complain that your students come in not knowing fractions."

The game is a wonderful way to pass recess on a rainy day.

Cooperative Game

At the beginning of the next lesson, Rachel offers students the option of playing a quick large group game, or a small cooperative game with a partner. She groups students carefully, depending on the situation. Students who appear to have a good grasp of the activity are paired together. Other factors are also taken into consideration. "Every class has one or two students who are motor mouths, and every class has a couple of students who are shy," observes Rachel. "I never put a shy student with a motor mouth, because the motor mouth will take over and the shy kid will never talk. I always put the motor mouths together so they can drive each other crazy," she reasons with a laugh, "and I put the shy students together so that they have a chance to talk."

To begin the cooperative game, students are told to start out with 2 1/6 in their bag of gold. As the students play, Rachel announces, "If you get in a situation where don't have enough gold in your bag to subtract your roll, wave to me." When this problem arises, she steps in with well-timed gift. The owner of

the gold mine has decided to give the partners two pieces of gold each. "Now don't expect this to happen all the time," Rachel warns. "The banker isn't always going to be this generous."

Competitive Game: Greater Than/Less Than

Once students have gotten comfortable playing the cooperative game, Rachel asks, "Who is ready to play the competitive game with your partner?" Students are allowed the choice of continuing to play the cooperative game if they decide they aren't ready to compete yet.

Each student begins with two and a half pieces of gold. The first player rolls the dice, reads the result and makes the transaction. Their partner watches carefully to ensure that they don't make a mistake.

Next, the first player must compare their own bag of gold with their partner's and declare whether they have more gold or less gold. For instance, if the first person to roll subtracts one half, they must say, "I have less gold than you."

Borrowing from the Bank

As the competitive games continue, Rachel reminds the class, "If you have done a lot of subtraction and you don't have enough gold, wave to me." It is not long before a hand is waving furiously in the air. The students are allowed to finish their last transaction, then, with a flip of the switch on the overhead projector, Rachel calls attention to the front of the room.

Ms. Math's Bank of Gold IOU Name Owes

"Step right up," she calls. When the needy student has joined her at the front of the room, Rachel introduces herself. "Hello, it's nice to meet you. I'm the bank president," she says with a professional air. "Tell me your situation."

"I only have one-sixth and I rolled subtract one half." explains the unfortunate player.

"Well, welcome to Real Life!" exclaims Rachel. Amidst laughter, she offers the student a loan and explains the terms. "I will only lend you whole pieces of gold," she says. "You may borrow up to five pieces of gold." This last statement is met with enthusiasm from the class. Holding up her hand, Rachel adds, "BUT, you have to pay me back!" She points to a large IOU transparency. on the "How much do you want to borrow?" she asks the player.

Though his classmates urge him to take five pieces of gold, the borrower is cautious. "I'm going to borrow two."

Rachel has the player write his name and the amount he has borrowed under the IOU. When he returns to his seat, he completes his subtraction, leaving one and two-thirds in his bag of gold. Rachel listens carefully. "I have more than you," the borrower tells his partner.

"Oh no!" Banker Rachel corrects him, "You don't really have one and two-thirds, because you still owe me two!"

Unhappily, the player corrects his mistake. "I owe Ms. Math, so I have less than you."

Students may or may not be able to figure out how deep they are in debt. Rachel walks around the room as the game goes on. When she sees that a borrower has enough in their bag of gold to repay the loan, she will collect what is due.

Challenge Action Fractions!

When the class has had plenty of practice rolling the addition/subtraction die, the game can be moved to the next level. The students gather around a table for a demonstration. "This is going to get a little more complicated," Rachel tells them. "Now you are going to learn to add fractions in your head." She selects a willing

partner from the group to play with her. Each of them begins with two and a half pieces of gold. Rachel picks up two fraction cubes and rolls a half and three-fourths. "I'm going to add these in my head," she explains, taking a moment to think. "One and one-fourth!"

To add to the suspense, Rachel rolls the operation die last—will she add or subtract? She rolls an addition sign.

Now it is the student's turn. "Remember, roll the two fraction dice and add them together," Rachel reminds her.

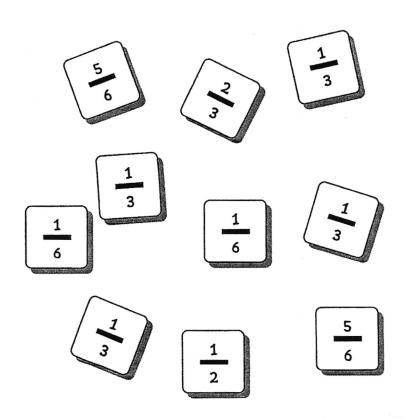
The student rolls two-thirds and one-half. "One and one-sixth." Tension is high as she rolls the operation cube. When it lands on subtract, Rachel gives a gleeful cheer.

On her next turn, Rachel rolls four fraction dice. "In this game," she explains, "you can choose how many fraction dice you want to roll." Her total for the roll is 2 ¾. When she rolls a subtraction sign, it is the students turn to cheer—they are rooting for their classmate to beat Ms. Math.

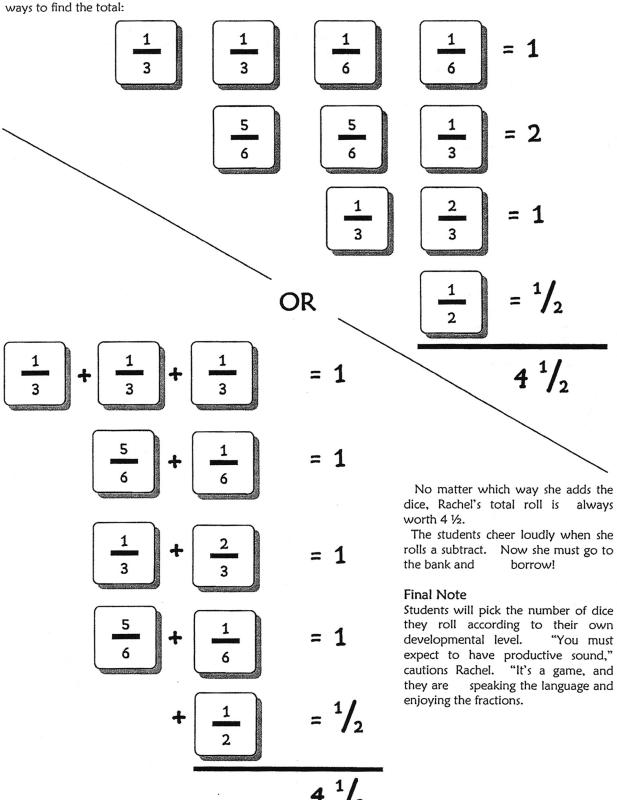
The more dice each player chooses to roll, the more exciting the game. On a sheet of paper, they keep track of IOU's to the bank. There is no longer a limit on what a player can borrow. Depending on the grade level, Rachel will eventually begin to charge interest on what players borrow. Although they may borrow any amount, for every piece of gold they are loaned, they must pay back one and one-sixth, or 16.6% interest. "That makes them stop and think!" laughs Rachel. "The teachers love it!"

"I'm going to roll TEN fraction dice," announces Rachel boldly. There is a ripple of excited chatter amongst the students.

Rachel rolls the following fractions:



Unphased by the large list of fractions she must add, Rachel reminds the class that they can pick out fractions that add up to one. "Remember knocking off ones?" she asks. She shows them two of many different ways to find the total:



Epiphanies & Ernie

by Rachel R. McAnallen

A number of years ago, when I was first developing my lesson "Action Fractions," I visited a school in southern Vermont where the principal was also the first grade teacher. It was still autumn, and while I was there, he asked me to demonstrate how I work fractions with beginning first graders.

After spending some time with his class using "jungue fractions," I decided they were ready to move on to the pattern block fractions. First they created pattern block equations using the left hand side equals right hand side cards. I was amazed at how quickly these young learners picked up on the algebraic activities. We played a big group game with our "pieces of gold," and then I paired them off to play Action Fractions with a partner.

As the partner game continued, a little blond-haired boy raised his hand. His name was Ernie, and he was a bright-eyed and bushy-tailed six-year-old. "This is getting boring," he informed me.

At that point, I had not developed the game using more than one fraction hexahedron, but I picked up two fraction dice and rolled a one-half and a two-thirds. Without missing a beat, little Ernie piped up, "One and a sixth."

I knew he could not have found a common denominator. "How did you get your answer?" I asked him. I still remember his exact words. He told me, "Well, I picture a half is a blue and a green, which is a third and a sixth. Then I take the one third and put it with the two-thirds and I get one. And so I have one and one sixth."

Then a little red-headed girl raised her hand. "I did it differently. My favorite color is red, so I thought of two-thirds as a red and a green. That's a half and a sixth. Then one half is another red, so I put the two halves together and I get one and one sixth."

For a moment I just stood there, reeling from the fact that I had just listened to these six-year-olds adding fractions in their heads. The principal and I looked at each other in amazement. Regaining my composure, I announced, "Here is what we are going to do—everybody will roll two fraction dice and you'll add them. Then you will roll the action die to see if you add or subtract it from your bag of gold."

I experienced a moment of epiphany that day. This fresh crop of first-graders were able to add fractions in their head because they had color coordinated them, and "chunked" them. Traditionally when we teach students to add one-half and two-thirds, we make them take the half and change it into sixths, which would be three green triangles. Then we tell them to take their two-thirds and change that into four-sixths, or four green triangles. Next we have them put all their triangles together to get sevensixths. Finally, they put six of the triangles together to make one, with one sixth left over. Using the Ernie method, I can now add loads of fractions in my head. Whenever a math teacher challenges me about the efficiency of taking the time to teach process, I give them the three examples above and I ask them, "Which one is the most inefficient?"

From my experience in that first grade classroom I was able to develop the concept of "knocking off ones." It's been at least sixteen years since my epiphany with Ernie, and I often wonder how he did in school as he went along. He was such a wonderful little mathematical thinker! Did his later teachers force him to find a common denominator, when he was able to add fractions so much more simply?

Many students remember their teachers and the lessons they were taught in school. After forty-five years, I look back and remember my students and all the important lessons I have learned from them.