

\section*{Folding Fractions \\ | 0 | $1 / 2$ | $2 / 2$ |
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Requiring nothing more than a few strips of colored paper and a glue stick and her common sense approach to mathematics, Rachel MicAniallen has developed a comprehensive lesson on fractions and their relationship with the number one.
"In order to have a fraction, you must first have a One," says Rachel matter-of-factly. "Fractions are qualitative terms, not quanitiative terms. You can't have a fraction unless you know who the one for the unit is-to really understand fractions," she explains, "students must see how the fractions relate to One, and how they all relate to one another.

## Who is One today?

"Who is One today?" begins Rachel, playing with emphasis of each word in the phrase. ""WHO is one today?" Who is one today? Who is ONE today? Who is One TODA ${ }^{\prime \prime}$
"If we look at the clock, a quarter of an hour is fifteen minutes, because an hour is One today," she explains. "If we talk about a quarter of a dollar, that is twenty-five cents, because a dollar is One today. If you talk about a quarter of a mile, the mile is One today and a quarter would be one thousand three hundred twenty feet."


"So, before we can have a fraction, we have to have a Orie," Rachel continues, handing each student a blue strip of paper. " "Let's have this strip of paper be One today," she suggests. "Think of it as your favorite long candy bar." Now students label their blue strip with a large 1. On the far left edge they write $0 / 1$, and on the far right edge they write $1 / 1$.

Each student is now given three strips of pink paper. "Take one of your pink strips and fold it in half," Rachel instructs. She demonstrates specific directions, for labeling the strip. "We have two parts-two halves-so we know that we will have three fractions," she reminds them. "On the extreme left hand side, we write zero out of two," she says, writing $0 / 2$ on the far left edge of her strip. "Then; where our fold line is, that is one out of two, or one half." Rachel draws a line and writes $1 / 2$. "On the extreme right hand side we have two out of two," she says, writing $2 / 2$.


Using a glue stick, students paste their fraction strips down onto a large sheet of paper as they work. The "fold, label and glue" sequence helps students keep their strips together.
"There are three really difficult folds in this lesson, and this next one is the easiest of those three." Rachel cautions the class as she passes out three green strips of paper to each student. "Take one of your green strips and wiggle it until you have thirds," she directs.
"Notice that when you folded your strip in half, the two ends of the strip were together, and now when we fold it into thirds, the ends are opposite." Rachel observes. "I want you to begin to look for a pattern with the ends of paper."

Although she requires each student to struggle with the wiggle method, Rachel also provides a template strip of thirds, fifths, and sevenths for students to work from should they need it.


Taking another pink strip of paper, students fold it in half and then in half again.

Rachel encourages students to speak the fractions as they write them out on their strip. "We have zero out of four, then we have one out of four, then two out of four, three out of four, and finally on the other end we have four out of four."

"This is another tough one," Rachel tells the class as she hands them each two yellow strips. "Try and wiggle your yellow strip into fifths. If you are have trouble, bring your strip up to the front and lay it underneath mine and mark it."

Students are now beginning to see patterns to folding the strips. Holding up another green strip. Rachel asks, "How can you make. a sixth from thirds?" "Fold in into thirds and fold it in half," one student tells her quickly.
Working in groups of four. students often help each other problem solve. "I don't have to wiggle thirds again to make my sixths strip." realizes one learner, "I can just lay the it- under my thirds strip and make marks."
"Ooooh kids," exclaims Rachel in an ominous tone," "this next fold is a killer!" Passing out one strip of orange paper to each student, she challenges, "See if you can wiggle your strip into sevenths:"

Several students choose to work without the template. A problem arises when some of them notice that their one seventh fold is greater than their fold for one sixth.
"That can't be right," puzzles one student.
"Yes," agrees Rachel, "what you have is an inaccuracy in the folding."

Rachel recommends teachers provide enough strips to allow for mistakes. "Some students are very accurate folders," she observes." "Other learners do not yet possess the fine motor skills for precise folding, so I cut enough strips to give them a chance to develop those skills.
The next pink strip is folded in half three times. "Make nice "crispy creases," Rachel reminds the class. "Can you see that that the two ends are still together?" Opening the strip, they label the fractions. "Now we have zero
out of eight, one out of eight, two out of eight, three out of eight, four out of eight, five out of eight, six out of eight, seven out of eight and eight out of eight:"
"One ninth,", volunteers a girl.
"Okay," says Rachel, writing: 1/9. "Three fourths can either be less than, equal to, or greater than one ninth. When you were little kindergartners and didn't know anything, how many of you thought that one ninth would be more than three fourths?" she teases.

Many hands go up in the air amidst giggles.
"You're all laughing because now you know better!" Rachel says.
Using their last green strip, students fold their paper into ninths and label it. Finally, they fold and label the remaining yellow strip into tenths.

One student discovers a pattern for the ends of the strips, "When you fold a strip into even numbers, the edges are together, and when you fold it into odd numbers, they are opposite."
"With this activity, the students self-differentiate," notes Rachel. "Some learners go very slowly because they want to be absolutely accurate, and other students rush through it. Some students' strips look great, and 'others' are messy as the dickens. At this point, I do not care so much about accuracy. I want them to see the patterns.".
One of the most beautiful patterns in the lesson is found when looking at the completed page of fraction strips. The decreasing effect on the left side from one half to one tenth and on the right side from one half to nine tenths is what many students call the "pyramid."
"When you have your whole strip sheet made, raise your hand," Rachel announces. As soon as the first two students raise their hands, Rachel brings the class' attention to overhead projector.
"Any two numbers have one of three relationships with each other," she explains. "The first number can only be less than, equal to, or greater than the second number.". On the overhead she writes two whole numbers:

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68
$$

"For example, is six less than, equal to, or greater than eight?" Rachel asks.
"Six is less than eight," one learner tells her.
On the overhead, Rachel writes:

$$
6<8
$$

"The same is true for fractions," Rachel continues. "Using my fraction strips, I can see that five sixths is greater than one fourth." She write:
$5 / 6>1 / 4$
"Someone give me a fraction," she asks.
"Three fourths.".
Rachel writes this down: 3/4 "Someone give me another fraction."
"Someone give me a Fractioneze sentence about an inequality they see," she asks.
"Two fourths is greater than one third," one learner tells her.
Rachel writes:

$$
2 / 4>1 / 3
$$

"Two fifths is less than two thirds." observes another student.

$$
2 / 5<2 / 3
$$

As students complete their fraction strips, Rachel pairs them off to write Fractioneze sentences containing equalities and inequalities. "One of you says a fraction and you both write it down," she instructs. "Then the other person says another fraction and you both write that down. Together you must decide whether the first fraction is less. than, greater than or equal to the second fraction."

## Fractional Equalities \& Inequalities


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Rachel suggests making a large version of the fractions strips for the classroom. "It's a beautiful bulletin board," she says, "and the students can see them all the time. It's a great little sponge activity to have students look at the fractions chart and make up Fractioneze sentences on equalities and inequalities.

## Ruler Math

After all the students have had an opportunity to create sentences in pairs, they are given another blank strip of paper. Using their halves, fourths, and eighths strips as templates, students will create a new fraction strip. - By placing their halves strip above their fourths strip, students can see that the marks for $1 / 2$ and $2 / 4$ line up. "One half is equivalent to two fourth," says Rachel. In their notes, students write: $2 / 4=1 / 2$.
Next, students line up their eighths strip beneath the halves and fourths strips and find more equivalencies. "Two out of eight is equal to one fourth." $2 / 8=1 / 4$
"Four out of eight is equal to two fourths." $4 / 8=2 / 4$
"Four out of eight is equal to one half." $4 / 8=1 / 2$
"Six out of eight is equal to three fourths." $6 / 8=3 / 4$
On their strip they mark out the following from left to right: $0,1 / 8,1 / 4,3 / 8,1 / 2,5 / 8,3 / 4,7 / 8$, and 1 . ""The hardest fractions to find on a ruler are $3 / 8,5 / 8$, and $7 / 8$," Rachel tells the class. "But I'm going to teach you a secret! If you know halves and ones, you know just about anything. If you want to find five eighths, find one half and go up one eighth--five
eighths is one half plus one eighth."
She writes: $\quad 5 / 8=1 / 2+1 / 8$
Now, if you want to find three eighths, you jump up to one half and subtract an eighth. Three eighths is one half minus an eighth. $3 / 8=1 / 2-1 / 8$
Now if you want to find seven eighths, what do you think you do? You jump up to one and subtract an eighth!" $1-1 / 8=7 / 8$.
"It's just sort of a jumping around thing," she says simply.

Using their notes and their ruler strip as a guide, students work on writing down addition and subtraction fraction facts.
"They do not have to find a common denominator," stresses Rachel.
For example:

$$
\begin{aligned}
& 1 / 4+1 / 8=3 / 8 \\
& 3 / 8+4 / 8=7 / 8 \\
& 1 / 4+3 / 8=5 / 8
\end{aligned}
$$

One student wants to add five eighths and one half. "Let's write this down," Rachel tells him.
$1 / 2+5 / 8$
"What we can do is break five eighths down into one half plus one eighth," she explains. She writes this new math sentence on the overhead: $1 / 2+1 / 2+$ 1/8
"We know that one half added to one half equals what?" "One."
"So we have and answer of one and one eighth," says Rachel. $1 / 2+1 / 2+1 / 8=11 / 8$



## Knocking Off Ones Using Partner Fractions

"If you know these three important facts, you can add up a long list of fractions." Rachel tells the class. "We are going to find combinations of one using partner fractions." On the overhead projector, she writes:

$$
\begin{aligned}
& 1 / 2+1 / 8=5 / 8 \\
& 1 / 2-1 / 8=3 / 8 \\
& 1-1 / 8=7 / 8
\end{aligned}
$$

"I want everyone to give me a fraction-it must be less than one, with a denominator of halves, fourths, or eighths," she instructs. Going around the room, they generate the following list:

$$
\begin{aligned}
& 1 / 4 \\
& 7 / 8 \\
& 1 / 2 \\
& 1 / 8 \\
& 3 / 4 \\
& 2 / 4 \\
& 5 / 8 \\
& 7 / 8 \\
& 3 / 4
\end{aligned}
$$

"Now, we are going to knock off some ones," says Rachel. "Loo k-one fourth added to three fourths equals one." She crosses off one fourth and three fourths from the list and writes out: $1 / 4+3 / 4=1$
"Seven eighths and one eighth equals one," she continues, crossing these off the list. She writes:
$7 / 8+1 / 8=1$
"One half and two fourths is one," volunteers a student. Rachel crosses them off and writes:
$1 / 2+2 / 4=1$
"Look, we have taken care of the first six fractions!"
she tells them. "Now, let's see what's left. We know that five eighths is equal to one half plus one eighth," she says, writing $5 / 8=1 / 2+1 / 8$ "And look, now we can add that one eighth to seven eighths and that is another one!" Rachel crosses off one eighth and seven eighths and writes: $1 / 8+7 / 8=1$. "Now, we can break down three fourths into one half plus one fourth," she says, writing: $3 / 4=1 / 2+1 / 4$ "Then we know that one half plus one half equals one," Rachel writes: $1 / 2+1 / 2=1$
"And now all we are left with five and one fourth,!" she says delightedly.
"No text book out there has students adding up nine fractions," contends Rachel, "and if they did, students would be forced to go and find the common denominator, which is eight-what a pain!" "Students love adding up a really long list," Rachel says. "If there are twenty kids in the class, then we write down twenty fractions to add up!"

Rachel uses halves, fourths and eighths for this activity because they are the most commonly used fractions on a ruler in the United States. "I want to get students comfortable with these particular fractions because these are the ones that we use with the ruler," she reasons, joking, "Never at any time in real life have I had to add 11/17 and 14/19 together." students could practice knocking off ones using thirds. sixths and ninths on a different day. "The metric system is divided into tenths," adds Rachel, "so the next time they might want to mess around with tenths:"

Rachel considers this lesson a lead up to the fractions work students will do in the textbook. "Common denominators are in the book-these are the activities we should do before we get to the pizza in the first chapter of fractions in the textbook."


