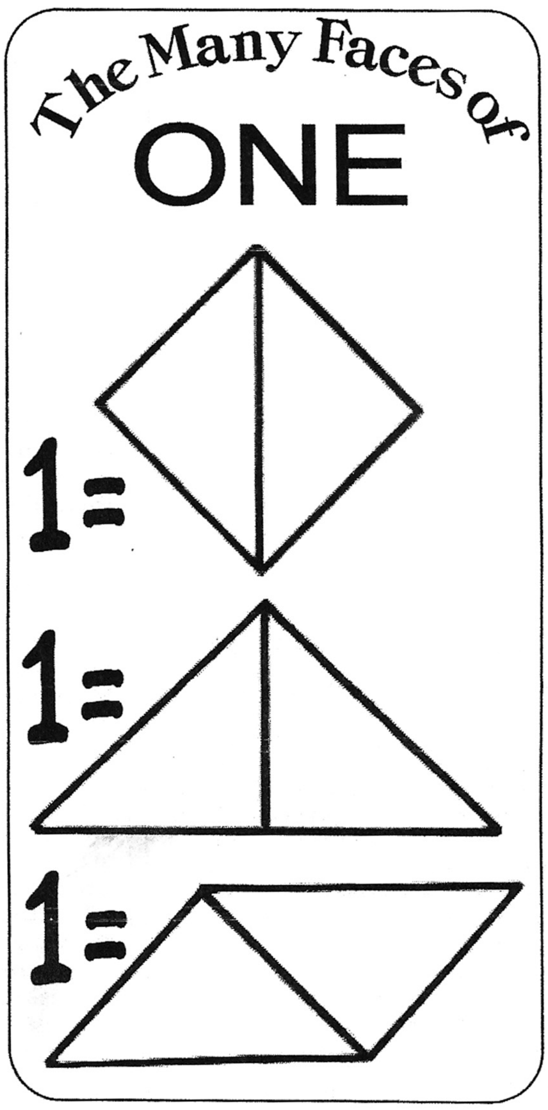
focus on Fractions 4

Fraction Fest! (A celebration of all things rational!)

We are pleased to bring you the final installment of this year's "Focus on Fractions" series, featuring in-depth lessons developed by fraction aficionado, Rachel McAnallen.

Topics Involved: Fraction concepts, algebraic concepts, mixed and rational numbers, Venn diagrams—everything but the kitchen sink!
Materials: Each group of partners will need a set of Fraction Shapes (available through Institute for Math Mania) an addition/subtraction hexahedra; a fraction octahedra; a math notebook and pencil, geo-dot paper.
Type of Activity: Includes both large and small group

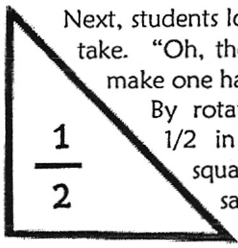


Who Is One *Today*?

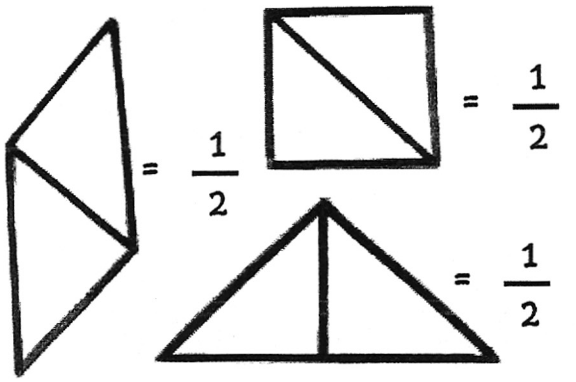
In this first activity, Rachel builds upon the framework of her Action Fractions lesson featured in our last issue. Moving beyond pattern blocks, students will apply established concepts to a different set of manipulatives and fractions.

The first step is to familiarize learners with the new shapes. In other words, students must know, "Who Is One Today?"

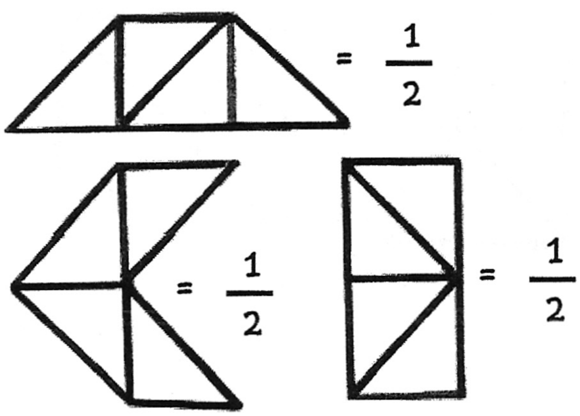
Placing a large, whole square on the overhead projector, Rachel does not mince words. "The square is One today," she announces, adding, "Today, One can take the form of other shapes—it doesn't always have to look like a square." She picks up two large triangle halves and builds a square that is standing on point, with it's vertices pointing North, South, East, and West. By rotating one triangle clockwise, and the other triangle counter clockwise, she creates a larger triangle. "This triangle is also One today," Rachel explains. Next she rotates the right-hand triangle clockwise and forms a parallelogram. "This, too, is One today," she says. "The square, the triangle, and the parallelogram are all equal in area. They aren't congruent, but they are equal—they aren't the same, but they are equal."

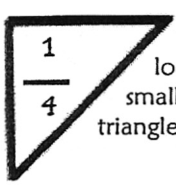


Next, students look at the shapes one-half can take. "Oh, there are lots different ways to make one half," Rachel remarks. By rotating 1/4 triangles, she builds 1/2 in the form of a triangle, a square, a parallelogram in the same way that she used the 1/2 pieces to build 1.

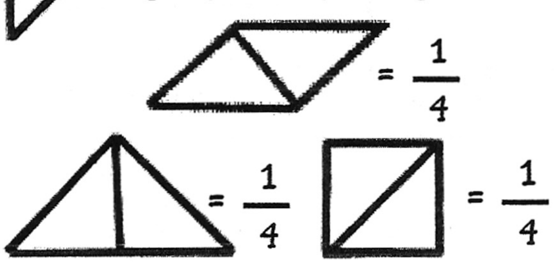


Using four 1/8 pieces, she discovers she can build 1/2 in the form of a trapezoid, and other interesting shapes. For example:

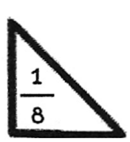




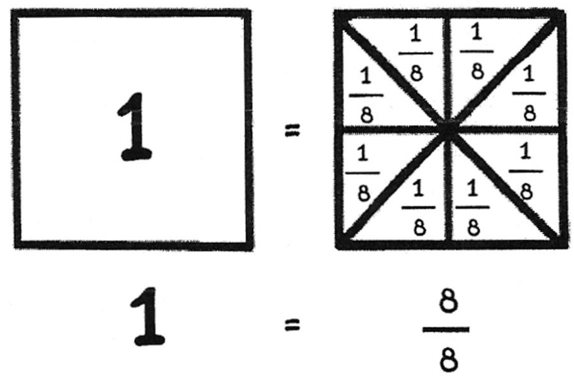
"Now, how does one-fourth look?" Using two 1/8 pieces, she builds smaller versions of the now-familiar, triangle, square, and parallelogram.



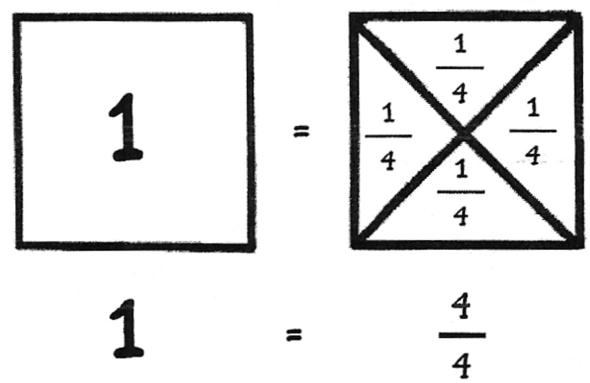
"And then we have one-eighth, which is the just a little triangle," she concludes.



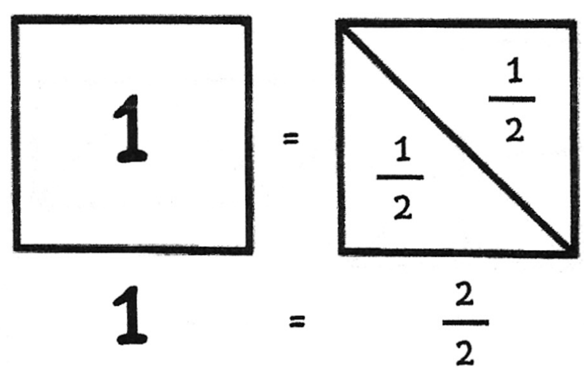
She picks up eight of the 1/8 triangles and places them on top of the square. "One is equal to eight-eighths."



Next she arranges four of the 1/4 triangles on top of the square, "One is equal to four-fourths."



Finally, Rachel places two 1/2 pieces on top of the square, "One is equal to two halves."



Once students have drawn out the equivalencies for One, they are ready to create fractional drawings from seven-eighths through one-eighth in their notebooks. They are encouraged to experiment— they begin imagining pictures, and mixing pieces together, Rachel is emphatic about setting aside classroom time for this activity. "There are loads of possibilities with these little shapes. It's important that they have the discovery process," she explains. "Not every student will find every shape, but one learner will find something no one else has found."

Even the act of tracing is a skill. "Some students will be sloppy with their tracing, and others will want it absolutely perfect," notes Rachel. As students explore ways to draw each fraction, Rachel walks around the room, offering strategies. "Try standing up while you trace," she suggests. "Ask your partner to hold the pieces while you draw around them."

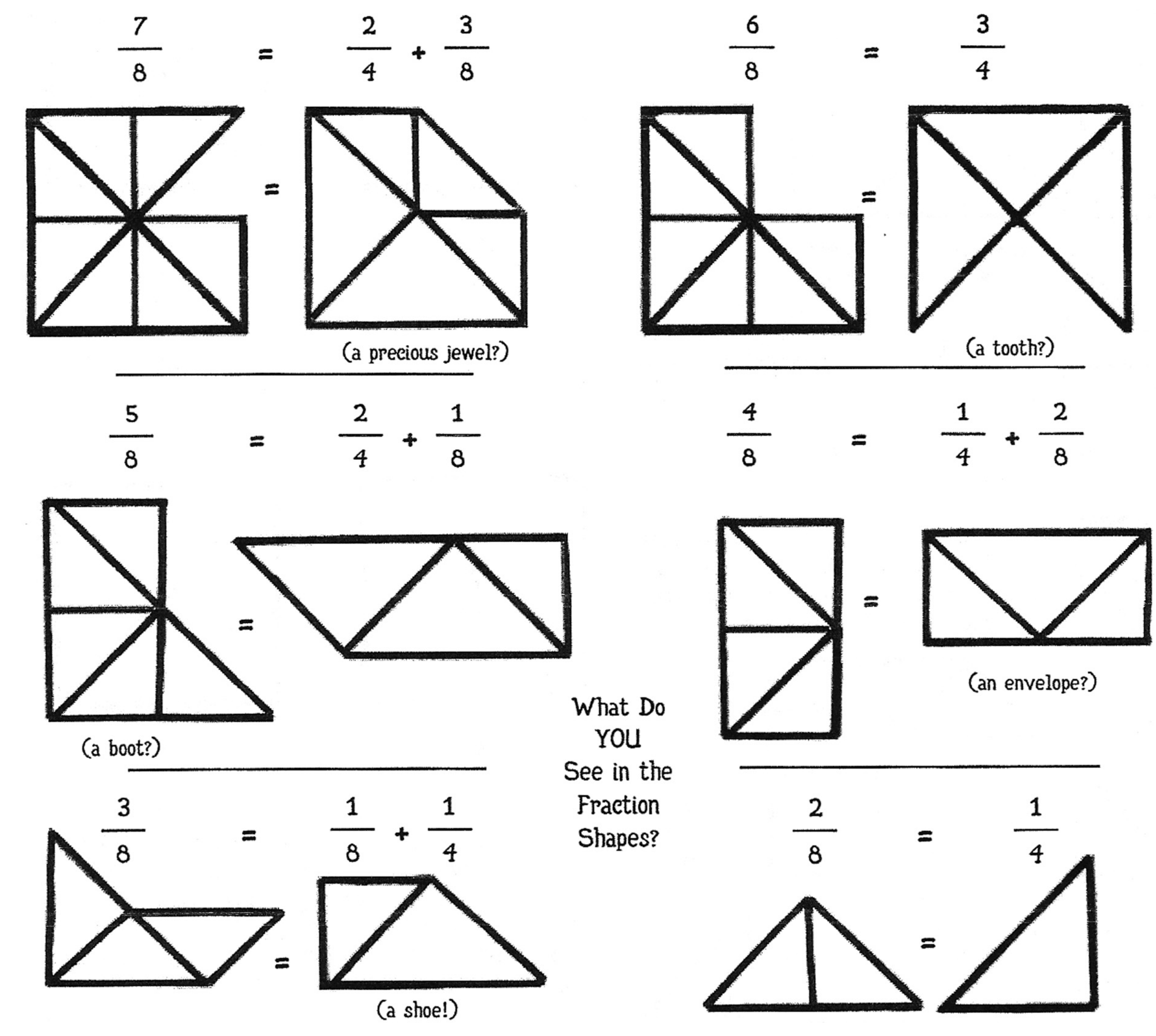
Occasionally, she asks a student to transfer their work on the overhead projector. "Why don't you go up and draw that one," she says, pointing to a tracing in one learner's notebook.

They spend up to a half an hour working on the drawings. "I do not rush the process," she stresses. "They must have time with it. We don't rush children in the process of reading, and we should not rush them through the process of mathematics."

When students have had sufficient time to draw, Rachel goes over the possibilities, "Are these all the pictures for seven-eighths?," she asks. "What are all the pictures for sixth-eighths?"

They go all the way through to one-eighth, making note of equivalencies. "We need to know that sixth-eighths is equal to three fourths, and that four-eighths is equal to one-half."

Just Some of the Ways to Make...



Rolling Fractions!

After students have drawn out all the combinations, they are ready to play a large group activity.

The structure of the game is the same as the "Pieces of Gold" game they played in Action Fractions--only the game pieces have changed. Each group of two is given a "bank" of fraction shapes and a tray with a octahedral fraction die and an operation die.

Everyone begins by adding two squares to a piece of paper they have labeled as their wallet. "How do you want to take out your two squares?" asks Rachel. "Do you want to take it out as two, or as four-halves, or as eight-fourths? Check with the person beside you—tell your partner how you took your two."

On a transparency, Rachel writes out the results of each roll, but she does not display the contents of her wallet during this activity. "If I demonstrate how I do it, the students who feel insecure tend to copy mine," she explains.

This is a cooperative game. Going around the room, one student "rolls it and reads it," the entire class repeats what their classmate has read, and then everyone adds to or subtracts from the contents of their own wallet.

"Subtract three-eighths."
"Add one-half."
"Subtract three-fourths."
"Subtract One."

Spotting that the next roll could put the class in the red, Rachel steps in and takes a turn. No matter what she rolls, Rachel will tell the class to add seven-eighths. She does not want the total to go below zero yet.

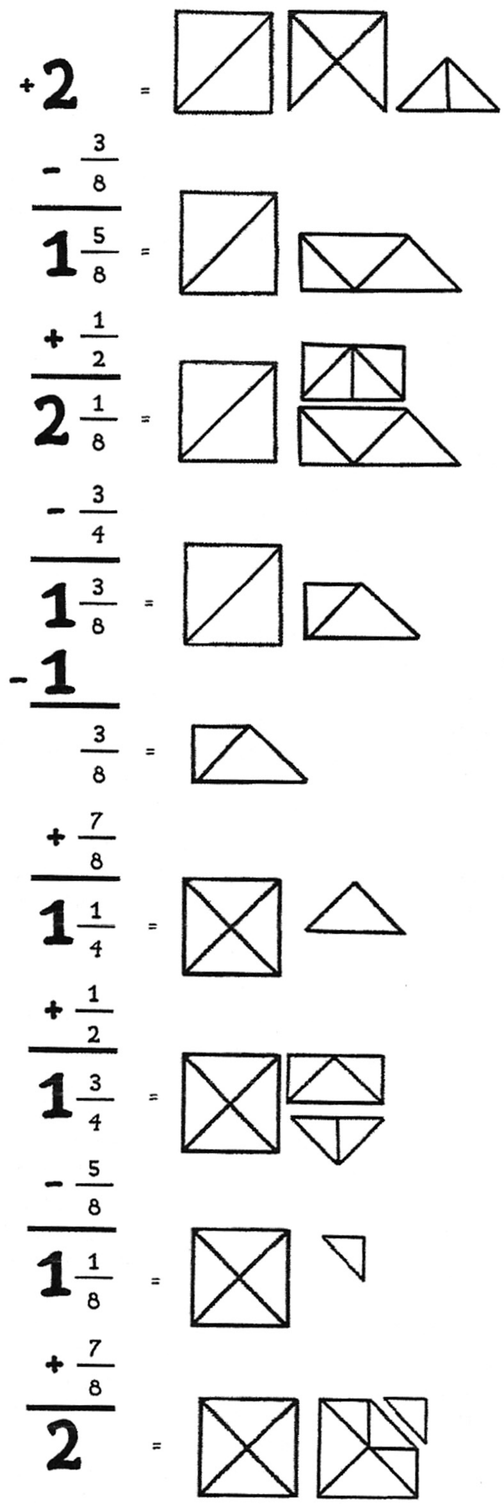
"Add seven-eighths."
"Add one half."
"Subtract five-eighths."
"Add seven-eighths."

Stopping the game, Rachel asks students how much they have in their wallet. The answers vary. "One and a half," one student tells her. "Great answer," she says, writing this down on a transparency.

"I have two," volunteers another player. "Oh, cool," says Rachel, adding it to the list. Still another student has two and one-eighth in her wallet. Each new answer is recorded. Unfazed by the assortment of numbers the class has generated, Rachel steps back to admire the list. "Look at all of these cool answers!" she exclaims.

Instructing students not to touch their wallets, Rachel uses her own fraction shapes and reviews each roll in the game. "Remember, we began by adding two to our wallet," she reminds them. "Then we subtracted three-eighths. If I subtract three-eighths, what should we have left?" "One and five-eighths."

Turn by turn, Rachel takes the class through the process. "Don't touch your own blocks—just watch me," she reminds them.



After careful review, the class agrees that the final amount left in the wallet is should be 2.

Students clear their wallets and begin a new cooperative game with a partner of similar ability. "The first person rolls it and reads it," Rachel reminds them, "the second person repeats it, then both people complete it." She instructs each team to compare how they complete the addition or subtraction with their partner. "How much are you starting with in your wallet?" she asks one pair of students. "Two and one-eighth," they tell her. "How did you take your two and one-eighth?" "I took out one, two halves, and one-eighth," answers the first student. "And how did you take out your two and one-eighth?" Rachel asks the second player. "I took out one, four-fourths, and an eighth," he tells her.

She continues to observe as the first student rolls and reads, "Add seven-eighths."

"Add seven-eighths," repeats his partner. They both add fraction shapes to their wallet.

"Now, tell each other what you have in your wallet," Rachel prompts them.

"I have three," says the first player. "I've got one, two halves, and four-fourths.

"I've also got three," agrees the second player. "I have one, three-halves, one fourth, and two-eighths."

"When students see that they can add and subtract in different ways from their partner, they begin to really understand equivalencies," notes Rachel. "They are working with the idea that one can be two-halves, it can be four-fourths, it can be eight-eighths, and it can also be one-half and two-fourths."

As she watches over the games, she differentiates for each group. When she spots two students who have picked up the concept very quickly, she tells them, "When you know the amount you have in your wallet, I want you to find a new way to say it—so if you have two and three-eighths, you might want to say, 'I have three subtract five-eighths'."

No one is writing anything down—all the computation is mental. "Teachers often want to know when students should start writing these down," says Rachel. "I have them begin writing it down once they have it mentally. When do we have them write down eucalyptus? When they have it mentally," she reasons. "Before a student can spell, they must know the pieces that make up the word—they must be able to visualize the letters. Students must be able to visualize fractions in the same way."

When a roll comes up that requires students to subtract more than they have in their wallet, Rachel allows them to borrow from the bank. The students keep track of their own debt, writing it down on an IOU. They are only allowed to borrow whole squares.

Once students are proficient with the cooperative game, they are permitted to move on to the competitive game. "There may be two shy kids who aren't comfortable with the competition yet," says Rachel. "They might want to continue to play the cooperative game. That's okay—they'll get there," she says assuredly. "Eventually they will all play the competitive game, because it's fun, but I don't have them compete until they are comfortable with the activity."

The rules for the competitive game are simple: One player rolls it, reads it, and completes it while their partner watches. When the first player has finished their transaction, the second player takes their turn. As they play, they must talk to one another. For example, if two players begin with 2 $^{3}/_{8}$ in their wallet:

P1: Rolls: – 7/8 "Subtract seven-eighths. I have one and one half. I have less than you."

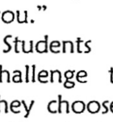
P2: Rolls: + 3/8. "Add three-eighths. I have two and three-fourths. I have it in five halves and one-fourth. I have a LOT more than you."

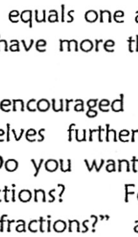
Players are visualizing the fraction shapes in their head, enabling them to mentally add and subtract fractions without finding a common denominator. Eventually, students will want to play the challenge game. The challenge game requires a player to roll two or more of the fraction octahedra. The player then adds the results of the roll together. They must roll the operation cube to discover whether they will add or subtract the total from their wallet. In a continuation of game from above, the partners decide to roll two octahedra:

P1: Rolls: 1/8, 1/4 "Three-eighths." Rolls: – "One and one half subtract three-eighths. I have one and one-eighth. I have less than you.

P2: Rolls: 7/8, 1/2. "One and three-eighths." Rolls: – "Two and three-fourths subtract one and three eighths equals one and three-eighths. I have more than you."

Students are encouraged to challenge themselves further if they choose. "Do you want to roll three fractions? Four fractions? Five fractions?" asks Rachel. "They love rolling more and more dice," she laughs. "The teachers can let the students take the game as far as they want."





The Wonderful Number One

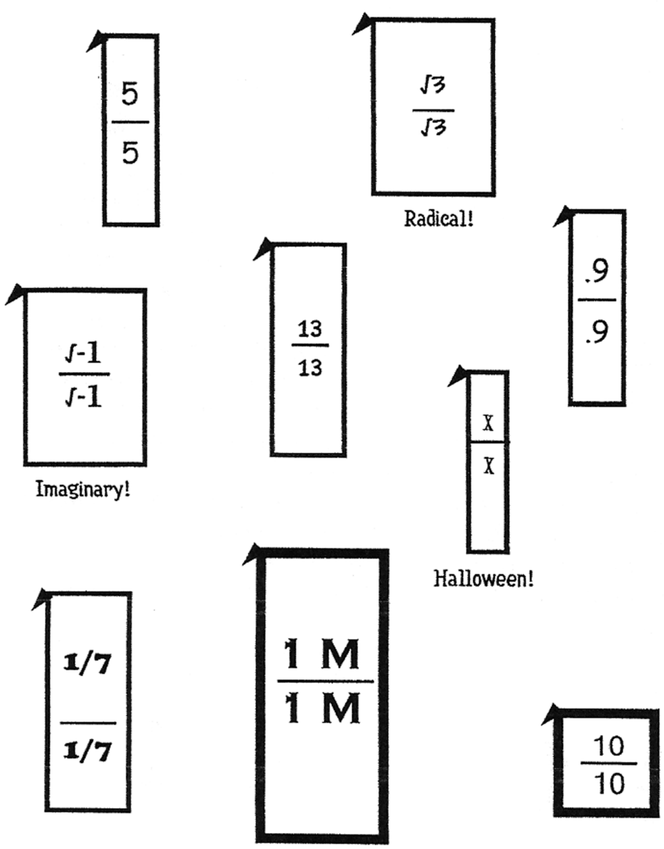
After a few more minutes of rolling fractions, students' are instructed to end their games. "You need a lesson on the Wonderful Number One," Rachel tells them. Eyeing the group of learners, she invites a student to come stand beside her at the front of the room. He is noticeably taller than she is. "Here is a very tall One," says Rachel, gesturing up towards her partner. Looking down at herself, she comments, "Notice that I am a short, squat little One." This observation is met with some giggles. "What is your favorite number?" Rachel asks her counterpart. "Seven," he answers. "Well," replies Rachel, "You can't be seven, because you are One—BUT," she adds dramatically, "you can be seven over seven!" Flipping on the overhead projector, she reveals the outlines of two 1's—one tall and skinny, and one short and squat.

Inside the tall 1, Rachel writes 7/7. "My favorite number is ten," she reveals. Inside the short 1 she writes 10/10.

"We're both still ones," she says, "We're both very unique. Uni—meaning one."

Rachel reveals another set of 1 outlines on the overhead projector. "One runs around looking a lot of different ways," she explains. "It might wake up and feel like being five over five." Rachel writes 5/5 within one of the outlines. "Now, let's suppose it's Halloween," she says, giving the teachers in the classroom a mirthful glance, "and One doesn't want anyone to know who it is—it dresses up as x over x!" Rachel shares several creative ways to make One.

"Draw a bunch of big ones in your notebook," she instructs the class. "Who do you want One to be today?"



Mixed Numbers & Rational Numbers

Returning to the fraction shapes, Rachel directs everyone to take out one square. "How many halves are in one?" she asks.

"Two halves," they tell her.

"How many eighths are in one?"

"Eight-eighths."

"Now," says Rachel, "Take out another half. How many halves are in one and one half?"

"Three halves."

"Oh, we can write that down," she exclaims. "One and a half is equal to three halves." On the overhead projector she writes:

$$1\frac{1}{2} = \frac{3}{2}$$

Students copy this down in their math notebooks. Working from the notes provided on the transparency, they are expected to write down the same information in their own notebooks.

"What's another way to write that?" Rachel asks. "How many fourths in one and one half?"

"Six-fourths."

$$1\frac{1}{2} = \frac{6}{4}$$

They write:

"How many eighths are in one and one half?"

"Twelve-eighths."

$$1\frac{1}{2} = \frac{12}{8}$$

"Notice how we are writing mixed numbers as rational numbers," Rachel says to the classroom teachers. Using the proper terminology is very important to this veteran algebra teacher—and she shows visible annoyance with one expression in particular.

"The term improper fractions drives me berserk," she declares. "It is not a term we use in the higher maths, and it is no longer used on standardized tests. Consider how children think about words. Does improper mean that five-fourths has bad manners and three-fourths has good manners? Does seven-fourths sass back to it's mother? Of course not!" she says vehemently. "We are preparing our students for algebra. Instead of telling our students that seven-fourths an improper fraction, we should be teaching them that seven-fourths is a rational number greater than one. There is nothing improper about it."

Determined that students in her classroom will not be subjected to improper math language, Rachel pauses their work with the fraction shapes for a brief lesson on rational numbers. "A rational number is a number that can be expressed as 'a over b,' but b cannot equal zero," she explains.

She writes:

$$\text{Rational Number } \frac{a}{b}$$

$$b \neq 0$$

On the overhead projector she writes a/b in large print. On another transparency she writes 4/5. She lays this second sheet over the first transparency, so that four overlaps a, and five overlaps b. "Who is 'a'?" Rachel asks. "Four," the class replies. "Who is 'b'?" "Five," they tell her.

She does this with a number of fractions, then throws down a transparency that simply reads:

$$7$$

"Who is 'a'?" Rachel asks.

"Seven," respond a few students.

"Who is 'b'?" There is a pause. Some students guess seven.

"Seven over seven would be one," reasons Rachel. "And we know that seven is not equal to one."

"Zero?" suggests another student.

"It's not zero," replies Rachel. "Remember, I said earlier that 'b' can't be zero." She tells them, "The answer is one." On the transparency she writes out:

$$\frac{a}{b} = \frac{7}{1}$$

"Mathematicians like to be lazy," Rachel reminds the class. "Whenever you see any whole number, there is a hidden denominator underneath it, and that denominator is always one."

She gives the class a special homework assignment to help them remember this idea. "Tonight I am going to allow you to write in your textbook," she announces. "If you turn to Chapter Five, you'll see that it begins on page 59. Take your pencil and at the bottom of the page, beneath 59, I want you to draw a line and write a one":

$$\frac{59}{1}$$

"Turn to page sixty, and do the same thing. Go through and do the same thing to all the page numbers in the chapter," Rachel urges. "From now one, you will always remember that when you don't see a denominator, it is a hidden one."

She writes:

$$\frac{5}{4} > 1$$

"Five-fourths is a rational number greater than one,'" reads Rachel. The class repeats this—it is important that students verbalize the sentences they are writing in their notes.

"If we have three-fourths, we say it is a rational number *less* than one." She writes:

$$\frac{3}{4} < 1$$

"Let's suppose we have ten-fifths," proposes Rachel. "We would write it as a rational number *equal* to two."

$$\frac{10}{5} = 2$$

"If we had twelve-fifths," she continues, "We would have a rational number greater than one, and it is also greater than *two*. We could even say that two is less than twelve-fifths, which is less than three":

$$2 < \frac{12}{5} < 3$$

In algebra, fractions are always left in rational form. "The reason we don't have students change fractions into mixed numbers in algebra," Rachel explains, "is because if they needed to use two and two-fifths later on they would eventually have to change it back into twelve-fifths. It makes sense to leave it in it's simplest form."

"Someone give me a rational number," she directs.

"Nine-tenths."

"What can we say about nine-tenths?"

"Nine-tenths is less than one."

On the overhead, Rachel writes:

$$\frac{9}{10} < 1$$

"Who else has a rational number?" she asks.
"Thirteen-eighths."
"Okay, what do you know about thirteen-eighths?"
"Thirteen-eighths is greater than one."
"Is it greater than two?"
"No."
Rachel writes:

$$1 < \frac{13}{8} < 2$$

Students are put together with a partner to continue the activity in their notebooks, taking turns choosing a rational number. They both write down the number and together they decide upon a sentence to write. For instance:

$$0 < \frac{4}{5} < 1$$

Rachel tours the classroom, surveying the students' work. "Who thinks they have one that no one else has?" she asks. Almost all the students' hands go up. She sends some of them to write their sentence on the overhead.

"They all think they have one," laughs Rachel. "And there is always a learner in the class who will be thinking outside the octahedron. Teachers have to be prepared for a student to choose a rational number like negative three-fourths"
(One possible sentence for – ¾ :

$$1 < \frac{-3}{4} < 0$$

Going back to the fraction shapes, Rachel tells students,

"Take out two and one-fourth." Everyone places two squares and a triangle fourth in front of them.

"Let's look at all the ways we can write two and one-fourth," she says. "How many fourths are in two and one-fourth?"
"Nine-fourths."
They write:

$$2\frac{1}{4} = \frac{9}{4}$$

"How many eighths are in two and one-fourth?"
"Eighteen-eighths."

$$2\frac{1}{4} = \frac{18}{8}$$

"How many *halfs* are in two and one-fourth?" Rachel asks. This gives the students pause.
"Four...," says one student, cautiously, "...and...half of one half?"
"That's correct!" says Rachel. "Look—take four half squares and lay them out over your two squares." She arranges her own pieces on the overhead projector. "Now take your little fourth and put that on top of another half piece. One-fourth is what fractional part of a half?"
"Half."
"So we have four and a half halves."
She writes:

$$2\frac{1}{4} = \frac{4\frac{1}{2}}{2}$$

"Remember our funny One?" she asks the class. "We can use One to write two and one-fourth in all kinds of ways."

$$2\frac{1}{4} = \boxed{\frac{2}{2}} + \boxed{\frac{3}{3}} + \frac{1}{4}$$

$$2\frac{1}{4} = \boxed{\frac{4}{4}} + \boxed{\frac{4}{4}} + \frac{1}{4}$$

"How many ways you can think of to write two and one-fourth?" challenges Rachel.

$$2\frac{1}{4} = \boxed{\frac{2}{2}} + \boxed{\frac{8}{8}} + \frac{1}{4}$$

$$2\frac{1}{4} = \frac{3}{1} - \frac{3}{4}$$

$$2\frac{1}{4} = 100 - 97\frac{3}{4}$$

I have discovered that 3rd, 4th, and 5th graders are so creative with fractions," she smiles. "They come up with ideas that make their teachers faint."

Teaching Creatively with the Textbook!

Traditionally, a standard math textbook offers few opportunities for creativity. "A chapter on fractions in a typical textbook starts out the same way," says Rachel. "There is a big circle cut into four parts--it's a pizza," she speculates, "or maybe it's a cookie. Three of the pieces are shaded in. We're told that three-fourths of the pizza has been eaten--they don't bother to mention the one-fourth of the pizza leftover." Using this example, the book goes on to define three as the numerator, and four as the denominator. "The books say the denominator is divided into equal parts," contends Rachel. "That is true in theory, but in reality, there is no way you can divide those pieces into equal parts." Nor does the book offer any explanation for working with group fractions, such as people. "A questioning student is going to point out that not everyone in a group of people are equal," Rachel says knowingly. "That child is going to want to know how the denominator can still be in equal parts. There is always a learner who will force a you to go back and explain what the book has left out."

If textbooks provide a dry and sometimes incomplete picture of fractions, teachers can still find imaginative ways for students to solve the problems supplied in the book. Even the task of finding equivalent fractions can be fun. As always, Rachel is careful about language—she does not use the word reduce when working with equivalent fractions. "The word reduce means to make smaller in quantity," she explains. "Children in the elementary grades are very literal when it comes to language—if I tell them we are going to reduce 9/12, they think we are making 9/12 smaller. But what we really want to do is find fractions that are equivalent to 9/12."

She asks the students, "What is the one number you can multiply another number by and not change it's value?"

"One," they tell her.

"Let's make sure," says Rachel. "What is four times one?"

"Four."

"What's fifty times one?"

"Fifty."

"What is one million times one?"

"One million."

"That's right. One is the only number that you can multiply any number by and not change it's value," she agrees. " It's called the multiplicative identity."

They play with this word, emphasizing the consonant sounds, then stretching out the vowels. "Say it," Rachel urges. "Stick your tongue out when you say it!"

On the overhead she writes:

"If we multiply five times one, we know our answer is five." she say. Continuing to write, Rachel

$$5 \times \boxed{1} = 5$$

reviews, "We already learned that another way to write one as two over two," she explains. "And we know that we can write five as the rational number five over one":

"What is another way of writing ten over two?" she asks.

$$\frac{5}{1} \times \boxed{\frac{2}{2}} = \frac{10}{2}$$

"Five."

"Let's do another one," says Rachel. "What is your favorite number today?"

$$\frac{10}{2} = 5$$

"Eight.

"How do you want to write one today?"

"Three-thirds."

$$\frac{8}{1} \times \boxed{\frac{3}{3}} = \frac{24}{3}$$

$$\frac{24}{3} = 8$$

"Pick another number."

"Ten."

"Let's divide this time":

$$\frac{10}{1} \div \boxed{\frac{10}{10}} = \frac{100}{10}$$

$$\frac{100}{10} = 10$$

On the overhead, Rachel writes:

$$\frac{9}{12}$$

"We're going to simplify nine-twelfths. We don't want to make it smaller—it has to be equivalent," she explains. "What is the one thing I can do to nine-twelfths that won't change it's value? I can multiply or divide by one."

The students decide to divide by one in the form of 3/3:

$$\frac{9}{12} \div \boxed{\frac{3}{3}} = \frac{3}{4}$$

"Think of a fraction. It can be a rational number greater than one, or less than one. We are going multiply or divide by one to change it into an equivalent fraction.

"One half," offers a student.

"Do you want to multiply or divide?"

"Multiply."

"What is the only thing you can multiply by and not change it's value?"

"One."

"What is your one going to be in the form of?"

"Four over four."

$$\frac{1}{2} \times \boxed{\frac{4}{4}} = \frac{4}{8}$$

Another student wants to multiply one half by one in the form of 9/9:

$$\frac{1}{2} \times \boxed{\frac{9}{9}} = \frac{9}{18}$$

"Give me another fraction," she says.

"Four-sixteenths,"

"What do you want to do?

"Multiply by two over two."

$$\frac{4}{16} \times \boxed{\frac{2}{2}} = \frac{8}{32}$$

Another student wants to multiply:

$$\frac{4}{16} \div \boxed{\frac{2}{2}} = \frac{2}{4}$$

After working on a few more examples as a class, Rachel announces, "Okay, I want everyone to pick a fraction—just yell one out to me—and I'm going to make a list." Students are calling out fractions as fast as Rachel can write them. "Oh, you can't use one-half," she tells a student, "Someone's already gotten that one. Five-tenths is okay—it's equivalent, but it's not the same." The room is buzzing as Rachel furiously jots down each fraction. "Oh, okay, two-thirds is gone—no one else can use two-thirds," she calls out.

Once everyone has their fraction, Rachel tells them, "You have five minutes to take your fraction and write out all the equivalent fractions you can for it. You must decide what form your one will take, and choose whether you will multiply or divide. I want you to show your work—that means left-hand side equals right hand side."

When the five minutes are up, the students exchange notebooks with a partner. "Check over your partner's work," instructs Rachel. "Do they have their equals sign? Is their one in other forms? If you think they have made a mistake, tap them gently on the shoulder and tell them you think they may have erred."

Rachel advises students to be bring their notebooks to class the following day. "Make sure this is all in your notes—I am giving an open notebook quiz tomorrow and I expect everyone to have one hundred percent."

For the quiz, Rachel selects a single fraction that is not in simplest terms:

$$\frac{24}{48}$$

"I want you to give me six examples of equivalent fractions—three in which you have multiplied by one, and three in which you have divided by one. Show your work—left hand side equals right hand side."

If a student makes an error, Rachel gives them a chance to correct it. "My job as a teacher is to make sure everyone has the concept," she explains. "The job of the reading teacher is to make sure that everybody learns how to read. My job as a math teacher is to make sure that everybody learns how to do math."

Prime Factorization

Another way to simplify fractions is through prime factorization. To teach this method, Rachel begins with the basics. "If you don't know what a prime number is, and you don't know what prime factors are, you can't do this activity." reasons Rachel. "A prime number is a number that has only two distinct factors—one and itself." The word distinct is important to the definition, because it means that the factors must be different. In other words, 1 is not a prime number.

"The word factor is a noun," says Rachel, "but it is also a verb--if I tell you to factor twelve, that is a verb. If I ask you to give me the factors of twelve, that is a noun," she explains.

"When you hear the word factor," Rachel continues, "I want you to think--multiply!" Quickly, she calls out, "Factor!" "Multiply!" responds the class. "Factor!" she repeats enthusiastically. "Multiply!" the students shout back. Rachel points to them urgently. "Factor," they say to her. "Multiply!" she says with glee.

She returns to the overhead projector and writes:

Factor × Factor = Product

"If four is one factor, and two is the other factor, what is your product?"

"Eight."

"Seven is a factor!" calls Rachel. "One is a factor! What is the product?"

"Seven!"

They generate several equations in this manor:

Factor	×	Factor	=	Product
4	×	2	=	8
7	×	1	=	7
6	×	5	=	30
3	×	8	=	24
10	×	5	=	50

"Notice I've been giving you the two factors and you have been giving me the product," Rachel says to the class. On the overhead she writes the number 12.

"Someone give me two factors that are equal to twelve," she asks.

"Three times four," volunteers one student.

"What's another pair?"

"Two times six."

"What's another pair?"

"One times twelve."

$$\left.\begin{array}{l} 1 \times 12 \\ 2 \times 6 \\ 3 \times 4 \end{array}\right\} = 12$$

"That's great," Rachel tells them. For now, she is only asking for whole number positive integers. "Later on, we could list negative twelve and negative one, and so on, " she admits, "but in the beginning lesson we are only using positive whole number factors."

She picks another product, "What are some factors for fifty?"

Students call out pairs of factors at random, but Rachel writes them down in the following order:

$$\left.\begin{array}{l} 1 \times 50 \\ 2 \times 25 \\ 5 \times 10 \end{array}\right\} = 50$$

"I think I see a pattern," she says deliberately, looking at the factors she has just written. "It looks like there is an even number of factors for all products." She pauses for a minute to let students consider this idea, then suggests, "Let's try factoring thirty-six."

$$\left.\begin{array}{l} 1 \times 36 \\ 2 \times 18 \\ 3 \times 12 \\ 4 \times 9 \\ 6 \times 6 \end{array}\right\} = 36$$

"How many factors does thirty-six have?" she asks.

"Ten," offers one student.

Rachel considers this answer. "Well" she says, "We can't count six twice."

"Nine," the student corrects herself. It seems that the pattern Rachel thought she saw does not work after all. They try some other numbers:

Eleven has only two factors. $\left.1 \times 11\right\} = 11$

Twenty-five has just three factors. $\left.\begin{array}{l} 1 \times 25 \\ 5 \times 5 \end{array}\right\} = 25$

"Ask students if they can find which products will have an even number of factors and which ones will have an odd number of factors," suggests Rachel. "They will discover that the square numbers that will have an odd number of factors."