

# focus on Fractions

# 1

## Speaking Fractioneze



(Understanding  
the Language of  
Fractions!)

Rachel McAnallen presents a lesson which sprang from her realization that her students did not have a full understanding of fractions.

"I was only teaching a fractional part of fractions," quips the woman whom most students know simply as "Ms. Math." "For instance, most text books don't explain that you can't have one third without its partner fraction, two thirds. And you can't have one out of three without having had three out of three to start out with—that is vital." Drawing a pie chart cut into thirds, she continues, "Over the years I have changed the way I introduce fractions. Usually we start this way—the pizza thing." She fills in one section of the pie chart. "Now, we've eaten one slice of the pizza. We teach students that this is the numerator":

1 n

"And this is the denominator":

3 d

Typically students are taught that the fractional parts in the denominator must be equal parts. "We make that the Golden Rule," observes Rachel, "and then when we teach parts of a group our students get confused, because they can see that not all things in the group are equal."

Now when she teaches a lesson on fractions, Rachel has learners begin by working with parts of a group, rather than parts of a

whole. Students learn the different ways to speak and write a fraction—and they learn how the English language sometimes muddies the mathematics. As a result, students gain a better understanding of the language of fractions—a language Rachel likes to call "Fractioneze."

### What Is a Fraction?

On this particular day, Rachel is working with a group of fifteen second graders and eight teachers. "Today we're going to work with fractions," she explains. "Who loves fractions? Raise your hands." Several hands go in the air. "That's great," she says with a smile. "If you can tell me what a fraction is, raise your hand." Only two or three hands are up.

One learner offers, "A fraction is when you have five pieces in a puzzle and you take two away, and that's a fraction."

"Okay, you took two out of five," acknowledges Rachel. "That's correct."

"A fraction is a picture that has different parts that it says to take away with you and you leave part in," another student volunteers.

"Then you have to color in and see how much [is left]."

"The magic word that you said there about fractions is *part*," hints Rachel. "Who else thinks they know what a fraction is?"

"I think a fraction is when you have eight people and two of them have blue shirts, and five have red shirts," says another boy.

Rachel thinks a minute. "Let's see, there's eight people and two of them have what color shirt?" she asks, holding up eight fingers for him to see.

"Blue," he replies.

Rachel puts two fingers down. "And five of them have what?"

"Red!"

She puts five more fingers down, leaving one remaining. "There's another person left," she says. "What are they wearing?"

Realizing his mistake, the boy quickly corrects his example. "There are six wearing red and two have the blue shirts."

"And what does that have to do with fractions?" asks Rachel.

The student pauses. "I don't know."

"Well, you're pretty close," she urges.

The first student to offer a definition raises his hand again. "I think that there are six people and five people go home. That is a fraction."

"Oh, that's a fraction? How many are left?" Rachel asks him. "One."

This last exchange inspires a girl to raise her hand. "Well, if there are six people and five go home, then five out of six would be gone," she reasons.

"Ooo!" exclaims Rachel, "Five out of six would be gone. We're getting there!" Rachel turns to the teachers seated in the corner of the room. "A fraction is basically a comparison of two numbers by division. But you can also compare two numbers by subtraction. If I have six people and one of them goes home, how many are left? Five out of six are left. Or, one out of six went home."

Moving to the overhead projector, Rachel informs the class, "I'm going to teach you people how to speak Fractioneze." She writes *Fractioneze* on the overhead, declaring, "This is a new word I made up."

"When you speak

Fractioneze," Rachel explains, "I want to always hear your mouth saying two numbers." She writes: *Say 2 numbers*

## Junk (Junque) Fractions

"I have some cool Fractioneze stuff here," announces Rachel. With a sly smile, she slaps a large plastic fly on the overhead projector. The response from students is enthusiastic. This is what Rachel refers to as her "Fractioneze Junque." "It's very high class," she assures the teachers. "There are loads of thing around the classroom to use—paperclips, Popsicle sticks, scissors." For junk of a more creepy, crawly nature, Rachel recommends shopping around Halloween time, noting that students' collections from home are also a wonderful resource fractions.

Reaching into her bag of tricks, she selects two more items—a little troll doll and a pair of miniature of eyeglasses—for the overhead. Pointing to the

little troll on the screen, she explains, "I want to talk about this little guy, because he has hair. Now listen—*one*," she picks up the troll and sets it down, "out of *three* items," she emphasizes, picking up all three in turn, "has hair."

Students are instructed to repeat this sentence, using their fingers: "ONE," each student holds up one finger, "out of THREE items," they all hold up three fingers, "has hair."

Rachel thinks of another sentence "Let's see, TWO out of THREE items could be alive." Again, the class repeats the Fractioneze, always using their fingers to illustrate the two numbers they are speaking.

Now a large rubber spider is added to the overhead projector. "Who can say a Fractioneze sentence for me?" Rachel asks.

"One out of four items has wings," offers one learner.

"Everybody, let's repeat that," directs Rachel. "ONE," she touches the fly, "out of FOUR," she touches all four objects, "has wings."

"TWO out of FOUR have pink.

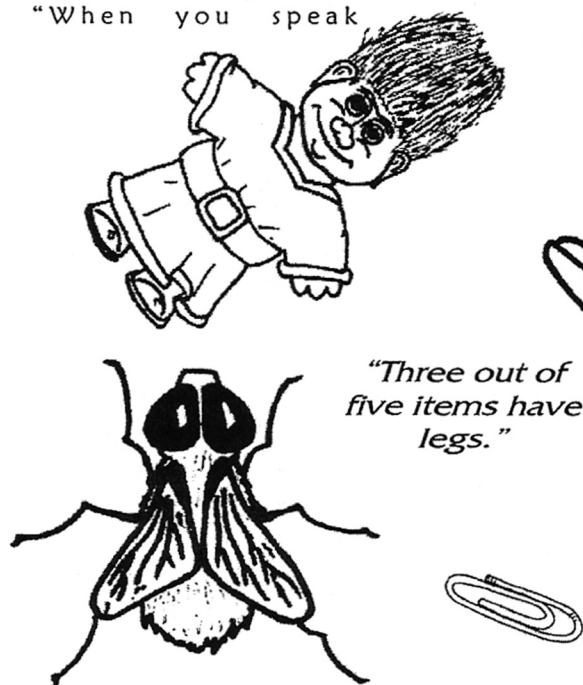
Going around the room, students invent and repeat more Fractioneze sentences as Rachel continues to change the items on the overhead.

Rachel switches items again. Everyone counts. Now there are six objects. "Two out of six are shiny.

"Six out of six are up on the screen," says Rachel. "Zero out of six are on Mars right now! They are all right here on Earth!"

Next, students are teamed with a partner. "You and your partner are going to take turns saying Fractioneze sentences about the fun stuff I put up here on the projector," Rachel instructs the class. "Be very careful, because I will change it!" she warns.

For the next several minutes





the classroom is filled with the murmur of students speaking Fractioneze.

Occasionally Rache removes an item from the group of things on the projector, adding one or two objects in it's place. There are never more than six items in the group at a time. She walks around listening to each team's sentences to ensure that they are using the correct language. "You're all getting really good!"

## People Fractions

Once the class has spent some time creating sentences, Rachel invites four students to come stand at the front of the room. "We've worked with my Junque Fractions," she says, "But did you know that I also have people fractions?" She gives some examples, "Four out of four students have name tags on. Two out of four are wearing shorts. Raise your hand if you see a Fractioneze sentence about the people up here." The students catch on quickly. "One out of four people have blonde hair." After a few sentences, Rachel adds another person to the group. "Three out of five people are boys."

## Partner Fractions

"Every fraction has a partner," Rachel informs the class. "If three out of five people are boys, then that means that two out of five are *not*," she reasons. "What is the fraction I'm thinking of? If two out of five people are wearing jeans, what is the partner fraction? Three out of five are *not* wearing jeans."

"I want you to find a Fractioneze sentence and then I want you to say the partner fraction," explains Rachel. The student she calls on hesitates. Rachel quickly directs the students

at the front of the room. "Don't move! You can breathe and blink."

She whispers to the timid student, "How many people have hands in their pockets?" "Two," replies the girl. "Good, now say that sentence," Rachel encourages.

"Two out of five have hands in their pockets."

Rachel walks up to the group and sorts out the students according to whether or not they have their hands in their pockets. "Now who is the partner fraction?"

"Three don't."

"Three out of how many?" coaxes Rachel.

"Three out of five don't," the student replies.

"Don't what?" asks Rachel. "Don't have elephants on their heads?"

Giggling, the student clarifies, "Three out of five don't have hands in their pockets."

Now Rachel invites a teacher to join the group at the front of the room. The class easily spots a new fraction. "One out of the five are grownups."

"What is the partner fraction?" asks Rachel.

"Five out of six are not grownups."

When the group of people fractions grows to seven, Rachel decides to change the rules a bit.

"This time I'm going to *give* you the fraction that I want your sentence to be about," she explains. "Someone give me a seven out of seven sentence."

"Seven out of seven have eyes."

"Seven out of seven have ears."

"Seven out of seven have arms."

"Seven out of seven are up there."

"Seven out of seven alive."

"Now I want a six out of seven, and those can be hard to find," says Rachel. "This is where partner fractions come in! If I look for a one out of seven, that helps me find a six out of seven."

"One out of seven has hands on his head," observes one student.

"If one out of seven has hands on their head, then what do you know?" asks Rachel.

"Six out of seven don't have hands on their heads."

"Okay, now I want five out of seven," continues Rachel. "I see one!" She pulls two girls in the group aside. "What's so special about those two girls I pulled over on that end?" she asks.

"Two out of seven have pink shirts," a student tells her.

"But I don't want two out of seven, I want five out of seven," Rachel reminds him.

"Five out of seven *don't* have pink shirts."

"Yes, you solved the puzzle!" she exclaims.

Students continue to find people fraction sentences, from four out of seven down through to one out of seven. Finally, Rachel announces, "I want a zero out of seven." The class thinks this over.

"Zero out of seven has two heads," pipes one witty student.

"Is that right?" asks Rachel amidst the laughter. She turns to the group, "If a person up there has two heads, raise your hand. No? Oh good! So how many have two heads," she asks the class.

"Zero!"

This exchange inspires a flood of sentences: "Zero out of seven are jumping up and down." "Zero out of seven are dead." "Zero out of seven are dancing." "Zero out of seven have their hands up."

"Seven out of seven please go sit down," says Rachel, dismissing the group from their post.

## Room Fractions

Next Rachel gives a thoughtful look around the classroom. "I'll bet we have some *room* fractions here," she muses. She counts silently to herself, then says, "Ten out of ten windows in this room are closed."

Students are now craning their heads looking around the room to find fractions of their own: "Twelve out of twelve lights are on." "One out of eight teachers is wearing a red shirt." "One out of two computers is on."

## Home Fractions

"So far we have junque fractions, people fractions, and room fractions," lists Rachel. "Did you know that we have home fractions?" She selects a teacher to be her partner. "You and I are going to talk about our home fractions," she explains. "Here is a home fraction—one out of my two animals at home is a cat named Freddie."

Now it's her partner's turn. "Five out of the seven people in my family are girls."

"One out of the two bedrooms in my house is very, *very* messy," reveals Rachel. "It's *mine*!" she admits in a delighted whisper.

Students are given a few minutes to share their own home fractions with their partners.

## Writing Fractions

For the next part of the lesson students are provided with a sheet of paper and a pencil. Rachel instructs them to write their first and last name at the top of the paper. "You are going to take some notes," she explains. "I want you to follow my directions. Today you are going to learn how to write fractions."

Moving back to the overhead projector, Rachel models taking notes for the students. "I want you to write this at the top of your paper, so we will know what your notes are about:"

### Writing Fractions

"When you have written down, look at me and smile."

While she waits, Rachel places three pattern blocks on the overhead—one hexagon and two

trapezoids. Once the entire room is grinning at her, she asks, "Would someone give me a Fractioneze sentence about what I have here.?"

"One out of three is yellow," suggests a student. The class repeats the sentence.

"Now, don't any of you write this down," says Rachel. But do you think this is the way we write one out of three?" On the overhead, she writes:  $\frac{13}{3}$

"NO!" the class tells her emphatically.

"Well, why not?" Rachel demands.

"Because people might think it was thirteen," explains one student.

"Yes, they would," agrees Rachel, crossing out her example. "So we don't write it that way. We write one out of three this way:"  $\frac{1}{3}$

Students write  $\frac{1}{3}$  in their notes.

"There are more Fractioneze sentences here," Rachel says, pointing to the pattern blocks.

"Two out of three are red."

"Don't write this, but do you think people write two out of three this way?" asks Rachel, writing:  $\frac{23}{3}$

"No!"

"Who thinks they know how to write two out of three? Come up here and write it for me," Rachel says, selecting a volunteer to come to the overhead projector.

The student carefully writes  $\frac{2}{3}$ .

"Excellent!" Rachel tells him.

"Everyone write two out of three!"

When everyone has finished writing, Rachel continues. "There are some more sneaky fractions around here. Can anyone find a new one?"

"Three out of three are on the paper."

"Everyone say it!" says Rachel.

"Now, don't write this, but do we write three out of three this way?" She writes:  $\frac{33}{3}$

"No!" the class replies.

"Everyone tell your partner why

we don't write three out of three

## Writing Fractions

one out of three

$$\frac{\cancel{13}}{3} = \frac{1}{3}$$

two out of three

$$\frac{\cancel{23}}{3} = \frac{2}{3}$$

three out of three

$$\frac{\cancel{33}}{3} = \frac{3}{3}$$

zero out of three

$$\frac{\cancel{03}}{3} = \frac{0}{3}$$

this way."

Another volunteer is chosen to come up and write three out of three:  $\frac{3}{3}$  The class now writes three out of three in their notes.

"Now, it is the teachers turn. Teachers, if I have a three out of three, and a two out of three, and a one out of three, there must be—what?" All over the room students have their hands stretched high. "Oh, teachers, too bad! You didn't get your hands up fast enough!" Rachel laughs, calling on one of the eager students instead.

"Zero out of three!"

"Oh! Give me a zero out of three sentence," Rachel tells her.

"Zero out of three have eight sides."

"Zero out of three have eight sides," Rachel repeats, counting. "Is she right?"

"Yes," the class confirms their classmate's answer.

"Am I going to write zero out of three this way?" asks Rachel.  $\frac{03}{3}$

"No!"

Another volunteer comes up and writes:  $\frac{0}{3}$  Students add this newest fraction to their notes.



Next, Rachel places a blue parallelogram on the screen. "Raise your hand if you think you know the different fractions we will have."

Soon, students have written the following fractions in their notes:

0/4  
1/4  
2/4  
3/4  
4/4

## Any Number + One

"I think I see a pattern!" Rachel announces.. "I have one, two, three, four things here," she counts. "Now, when I had three things, I had..." students watch as she quietly touches and counts the four fractions written on the overhead. "When I had four things, I had..." again, she silently counts the five fractions listed in her notes. She pauses a moment to allow students mull over this information, then asks, "I wonder how many fractions I will have if there are five things?"

"Six," pipes up one learner.

"How did you get six?" Rachel asks him.

"Well, every time the number goes up by one," he tells her, "There are four fractions for three, because—you might think there are only three—but [there is] zero out of three, and then we know there is one out of three, two out of three, and three out of three. So for the other ones there would be zero out of four and a zero out of five, so..."

"Let's see," says Rachel. "Let's check how good you are. If I had nine things, how many fractions do you think I could write down?" she asks him.

"Ten," he replies quickly.

"If I had one hundred?"

"One hundred one," he smiles.

"If I had a one million one?"

"One million two!"

"What if my number was just  $n$ ," she asks.

This stumps him. "I don't think you can do it?"

"Oh, yes!" Rachel tells him. "Mathematicians just love talking about  $n$ ! The  $n$  stands for any number. So our pattern would be  $n + 1$ . Any number plus one! Give me a number of things, and I'll tell you how many fractions we would have."

"Six."

Rachel counts on her fingers. "Seven!"

"Five hundred million five thousand nine," he challenges.

"Five hundred million five thousand nine *plus one more*!" They both laugh.

Turning back to the problem at hand, Rachel asks, "So I have five things, and you think we'll have how many fractions?"

"Six," he reminds her.

"Let's write them in order. Someone give me either the lowest fraction, or the highest fraction."

Zero out of five.

The class lists all the fractions for the five items in ascending order, writing each one in their notes. They must also create a Fractionese sentence for each fraction—students discover that finding a sentence for the pattern blocks can be tricky. "Be a fractions detective," Rachel urges them.

Three out of five have four corners."

When students struggle, Rachel moves the pattern blocks around.

"Here, four out of five are *not* green. Four out of five are *not* blue. Four out of five are *not* yellow!"

Five out of five sentences come more easily for the second graders. "Five out of five are shapes." "Five out of five are on the paper." "Five out of five are pretty."

## Other Ways to Say Fractions

"Turn your notepaper over to the blank side," instructs Rachel. Following her example on the overhead projector, students write *Other Ways to Say Fractions* at the top of their paper. On the left hand side of the page, they write 9/10.

"What is one way to say this fraction?" Rachel asks.

"Nine out of ten."

"Let's write that," she says, printing out the words for them to see: *Nine out of ten*.

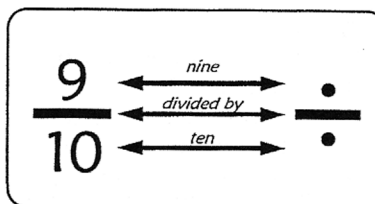
"Now, I'm going to teach you another way to say this," Rachel tells them, writing: *Nine divided by ten*. Directly underneath that she writes:  $9 \div 10$ . Rachel points to the first line, "Let's say this out loud."

"Nine divided by ten." Pointing to the numerical equivalent beneath, learners repeat, "Nine divided by ten."

"Now," says Rachel in a conspiratorial tone, "I want to tell you a little known math secret. Would you like to know it?"

Of course, they all want to know the secret.

"I don't tell many people this," she cautions. She opens the door and peeks out to check that there are no math spies in the hallway. "Good, no one is listening," Rachel assures the class. Walking back to the overhead projector, Rachel points to the division sign in her notes. "Do you see this little division sign?" she asks. "See the line? That is the line in the fraction sign!" she explains, pointing to the line in 9/10. "And the two dots are replaced by numbers."



## Other Ways to Say Fractions

$$\frac{9}{10}$$

nine out of ten

nine divided by ten

$$9 \div 10$$

nine is to ten

$$9 : 10$$

nine tenTHS

Bringing their attention back to the 9/10 written near the top of the page, she says excitedly, "Look how we can say this!" In unison they point to the top number in the fraction and read, "Nine..." then move their pencils to the small dividing line, "...divided by..." their pencils rest on the bottom number, "...ten."

"We don't usually teach this next way until fifth or sixth grade. Would you like to learn it in second grade?" Rachel asks the class. The students decide that they would.

"Let's write it this way": *Nine is to ten* Underneath, they write, 9 : 10

"Teachers," says Rachel, pointing to 9 : 10 on the overhead, "Where do you think the two dots in the ratio symbol comes from?"

A second-grade girl eagerly raises her hand. "Oh, you're too late, teachers!" says Rachel, "One of your students has her hand up first. Where?" she asks.

"From the two dots in the division sign," the student tells her.

"Teachers, just because it isn't in the second grade books doesn't mean we can't teach it," Rachel notes. "Keep in mind that most of the knowledge we test for are the *minimum* standards. Not the maximum."

## Thh-Thh-Thh!

Once again, Rachel directs the students' attention back to the 9/10 on the overhead. "Earlier you told me there was *another* way to say this," she says to one learner. "What was that?"

"Nine tenths," answers the girl.

"Nine tenths," repeats Rachel. "We are going to write it, and then I am going to show you some fun ways to say it." On the overhead projector she writes, *Nine TenTHS* "If you want to, you can put a little ths by the 10." *9/10ths*

"Thhh-thhh-thhh," stresses Rachel. "Do this with your tongue—thh-thh-thh. She goes around the room, calling on each group of students, "This table, right here!" "Thh-thh-thh," they answer. "Teachers!" "Thh-thh-thh," reply the teachers.

Silently Rachel plays with them, pointing to different tables at random. "Thh-thh-thh!"

"Pretty good!" she tells them. "Now when you say nine tenths, I want you to say it with your body." With body language reminiscent of an exaggerated sneeze, she demonstrates for the class. "Niiiiiiiiine..." Rachel leans far back as she draws out the first word, then pops forward to quickly expel, "tenths!"

Now it is the students' turn. "Make sure you get the THH on the end!" Rachel reminds them. "Teachers, you do it, too! Excellent," she says, "Absolutely wonderful!"

Returning to her notes, Rachel writes another fraction on the overhead: *7/9*

"Now, I am going ask you to tell

me all of the ways we can say this fraction," explains Rachel, "But we are only going to write down one of the ways."

Students work through the first example with assistance from Rachel. "Seven out of nine. Seven divided by nine. Seven is to nine."

"And what's another way?" she asks, "Seven ninths—this is the one that we are going to write down." On the overhead she writes: *Seven nineTHS* "I think they ought to leave the e in there," she comments, acknowledging her unorthodox spelling choice. "And let's write the little ths by the nine." *7/9ths* Rachel pauses. "I think I see a pattern. We say that bottom number with a th on it, don't we?"

While the students are writing in their notes, Rachel speaks to the teachers. "I am doing this activity for a reason," she explains. "There are things that we take for granted—there is a breakdown in the pattern in three places, and we assume that students know about it."

The next fraction she writes is 5/8. "Five out of eight. Five divided by eight. Five is to eight."

"Use your body when you say this last one," urges Rachel, "FIIIIIIIVE eighths!"

They all write:

*5/8ths Five EighTHS*

"Now, let's look," says Rachel. "I did tenths. I did ninths. I did eighths. Raise your hand if you think you know what is coming next."

"Sevenths."

"Right," Rachel agrees. She writes: *2/7*

"What is the first way, teachers?" asks Rachel.

"Two out of seven, two divided by seven, two is to seven, TWOO-sevenths!"

*2/7ths Two SevenTHS*

"Let's take turns," Rachel



suggests. "Students, it's your turn." On the overhead, she writes:  $5/6ths$

"Five out of six, five divided by six, five is to six, FIIIIIVE-sixTHS!"

Now it is the teachers turn. Rachel writes:  $3/5$

"Teachers, I do not want you to break the pattern," she warns. "Ready?"

"Three out of five. Three divided by five. Three is to five. Three..." There is a mixed response made up of fifths and fiveths. The students giggle.

"Oh-oh! Some of you broke the pattern," Rachel tells them. "If they didn't break the pattern, it would be what?" she asks the students.

"Fiveths!"

"We make the assumption that kids know fifths," says Rachel to the teachers, "but they have to be taught that there is a break in the language."

Turning back to the students, she says, "Seven ninths, five eighths, two sevenths, five sixths, and—if we didn't break the pattern—three fiveths! Well," Rachel explains, "in this case, mathematicians don't say fiveths. They say *fifths*."

She writes:

$3/5ths$  *Three Fifths*

(*Three FiveTHS*)

"Fifths breaks the pattern, doesn't it?" she asks. "A good mathematician tries not to break the pattern, but the English language has messed things up here. So outside this classroom, we say *fifths*."

Next, she writes:  $3/4$

It is the students' turn. "Three out of four, three divided by four, three is to four, three fourths."

"Exactly," Rachel agrees, "It fits our pattern." On the overhead, she writes:

$3/4ths$  *Three FourTHS*

The next fraction is written on the overhead:  $2/3$

"Ready, teachers?" asks Rachel. "Remember our pattern."

"Two out of three, two divided

by three, two is to three...two threeths!"

"Yes, two threeths," Rachel confirms. "But we don't say two threeths, do we? We say two thirds."

In their notes, students write:

$2/3ths$  *Two Thirds*

(*Two ThreeTHS*)

Now Rachel writes the final fraction on the overhead:

$1/2$

"One out of two, one divided by two, one is to two...one TWOTH!"

"Do we say one twoth?" asks Rachel skeptically.

"No."

"What do we say?"

"One second?" a student volunteers.

"It makes sense," reasons Rachel, "We come in fifth place, fourth place, third place, and second place. It's not quite the way we say it though. We say one half," she tells the class. "Now, what does the word half got to do with two?" she wonders aloud. "At least fifths begins with f-i, and thirds begins with t-h—the word h-a-l-f doesn't have anything to do with the word t-w-o! It breaks our pattern again, doesn't it?" On the overhead she writes:

$1/2th$  *One Half*

(*One TwoTH*)

"I want you to write one twoth underneath one half, so you will know what that really means," says Rachel.

When their notes are completed, the students are told to hold them up and show their work. "I want to see how well second graders take notes," Rachel tells them. "Show your notes to the teachers in the back."

To the teachers, Rachel says, "Knowing how to take notes in mathematics is really important. They can start early, you just have to go slowly. I tell them what I want them to write and they somehow manage to write it down."

During the lesson Rachel never

once uses the word numerator or denominator with the students. "I don't worry about numerator or denominator at this point," she explains. "I just want them to speak the language. Fractions are everywhere—junk fractions, people fractions, room fractions. I want them to understand that there are fractions all around."

## Fantastic Fractioneze!

$\frac{9}{10}$ ths	Nine TenTHS
$\frac{7}{9}$ ths	Seven NineTHS
$\frac{5}{8}$ ths	Five EightTHS
$\frac{2}{7}$ ths	Two SevenTHS
$\frac{5}{6}$ ths	Five SixTHS
$\frac{3}{5}$ ths	Three FifTHS (Three FiveTHS)
$\frac{3}{4}$ ths	Three FourTHS
$\frac{2}{3}$ ths	Two Thirds (Two ThreeTHS)
$\frac{1}{2}$ ths	One Half (One TwoTHS)