Wonderful Ideas

...for teaching, learning, & enjoying Mathematics!

Decimal Festival!

Hurray for Hundredths! Three Cheers for Thousandths!

Recently our resident mathematician, Rachel McAnallen, was approached by a teacher who attended her workshop on decimals using the place value wallet, money & digits featured in this year's volume of WI. He had been skeptical of the lesson in the beginning, but by the end of the week he was convinced. "I have never been able to make the transition from the concrete to the symbolic," he commented. "In this lesson, the symbolic is represented with the digits. It's wonderful, because the students don't have to write it down." He observed that many times his students get so bogged down performing the task of writing the numbers that the math ends up getting lost. "This lesson allows them to focus on the mathematics," he told her.
"It is important not to throw too many things at the students at once," responds Rachel. "I do not worry about having students writing anything down in the beginning. The digits serve that purpose. I want learners to visualize the mathematics through the coins and bills, and communicate what they are doing by speaking out loud."

The place value wallet, money, and digits continue to be the foundation in our latest installment of this year's "Dancing With Decimals" series. In our last issue, students crossed over to the right side of the decimal point, and began working with tenths. This time, students move into the hundredths and thousandths columns. The lesson requires a little more lead up work, because students must know how to read smaller decimal fractions before they can play the money game. "Learners need to understand the difference between eight-tenths and eighty-hundredths," explains Rachel. "It is important that they understand that those numbers are equivalent, but not the same.

Topics Involved: reading and expressing tenths, hundredths, and thousandths in different forms, number sense, place value (number versus digit), addition and subtraction of decimal fractions.

Materials: materials to play the money/digit game--pairs of students share a bank of $1, $10 and $100 bills, a tray of tenth and hundredth coins and colored thousandths discs; an egg carton or tray containing digits & symbols 0 through 9, +, -, . ; an addition/subtraction hexahedron; and six dice marked with the place value for tenths, hundredths, thousandths, ones, tens, and hundreds respectively. Each student also needs a white sheet of paper or a manila folder to represent their wallet. (See WI Volume 16 #3 for a copy of a reproducible place value wallet.)

Type of Activity: large group instruction and small group activity

Grade Level: 3rd and up

Billions, Millions, Thousands, Hundreds, Ones—Reading Big Numbers

In order to teach students to read numbers to the right of the decimal point, Rachel begins the lesson on the left side of the decimal point. "I have found that if learners do not know how to read big numbers, then it is hard to teach them to read little numbers," Rachel reasons. "In other words, if students don't know how to read numbers to the left of the decimal point, then they will have difficulty reading numbers to the right of the decimal point."

On the overhead projector, she writes out a number on the board:

432

"Everyone read that number."

"Four hundred thirty-two," the students chorus.

Next she writes:

57

"Fifty-seven."

Each time she writes a new number on the board the class reads it back to her. "Eight. Three hundred nine. Sixty. One."

"If you can read all those numbers," Rachel tells them, "you can read the highest number in the world." On the overhead, she writes a decimal point on the far right, and moving left, spaces out three commas. "When we read numbers on the left side of the decimal point," she explains, "we group them in threes."

She draws brackets between the sets of commas and labels them—millions, thousands, and ones.

$$\underbrace{\quad}_{\text{millions}},\underbrace{\quad}_{\text{thousands}},\underbrace{\quad}_{\text{ones}}.$$

"When you read a big number, you start with the first three-digit number, and then say it's last name. Then you read the next three-digit number and you say it's last name, and then you read the next three-digit number and you say it's last name."

Above each set of brackets, she writes the same three-digits, so that the example now reads:

$$\underbrace{432}_{\text{millions}},\underbrace{432}_{\text{thousands}},\underbrace{432}_{\text{ones}}.$$

"You all know how to read this big number," Rachel tells the class.

With her hands, she covers up everything but the 432 furthest to the left. "Read the number," she instructs. "Four hundred thirty-two." Moving her hand, Rachel reveals the word "millions" beneath the digits. "Now give it it's last name." "Million." Quickly, she adjusts her hand so that the next 432 is visible. "Four hundred-thirty-two," the class reads. Again, Rachel's hand moves and uncovers the number's last name for the students to read. "Thousand." Next she exposes the last 432. "Four hundred thirty-two." Finally, students read the last remaining word. "Ones."

"Yes, but we don't really the ones' last name out loud," admits Rachel. "It is silent. Let's try to read the whole number again." "Four hundred thirty-two MILLION, four hundred thirty-two THOUSAND, four hundred thirty-two."

"Now you are ready to read a bigger number," she tells the class. In the next example, Rachel adds another comma to the left and writes "billions" in a bracket beneath the space. She decides to put the word "ones" in parentheses, to remind students that it is silent. "Remember, we think it, but we don't say it."

Rachel calls on students to provide the numbers for the example. "Someone give me a three-digit number," she directs. "Nine hundred ninety-nine." She writes this above the billions bracket. "Good—someone else give me another three-digit number." "Seven hundred fifty-two." Above the millions bracket she writes 752. "Who has another?" "Two hundred." She enters the digits above the thousands bracket. "Someone give me one more!" "Eight hundred sixty-four."

$$\underbrace{999}_{\text{billions}},\underbrace{752}_{\text{millions}},\underbrace{200}_{\text{thousands}},\underbrace{864}_{(\text{ones})}.$$

"Anybody can read that number!" exclaims Rachel. "Let me hear you read it for me—read the first three digit number." "Nine-hundred ninety-nine…" the class tells her. "Last name?" "BILLION." "Next three-digit number." "Seven hundred thirty-two MILLION. Two hundred THOUSAND. Eight hundred sixty-four."

As she sets up the decimal point and commas for the next example, Rachel makes a confession to the students. "Teachers love to give you tricky ones--we like to put lots of zeros in the number!"

She urges the classroom teacher to come up and fill in a "tricky" number for the students to read. The students watch eagerly as their teacher devises a zero-laden number to try and stump them:

$$\underbrace{500}_{\text{billions}},\underbrace{050}_{\text{millions}},\underbrace{005}_{\text{thousands}},\underbrace{505}_{(\text{ones})}.$$

Beneath the number, Rachel labels each three-digit group with its last name. "Go for it," she tells the students.

The class reads, "Five hundred BILLION…fifty MILLION… five THOUSAND…five hundred five."

"Oh, wow!" Rachel sounds very impressed. "You couldn't be tricked by all those zeros!"

Tenths, Hundredths, Thousandths
Reading Small Numbers

"When we read numbers to the right of the decimal point, we say things a little differently," explains Rachel. "To the right of the decimal point, we read the entire number, and then we look above the last digit we read and the last name is at the top." On the overhead she writes .4 and then labels the place value $\frac{1}{10}$ above the digit.

$$\begin{array}{c} \frac{1}{10} \\ .4 \end{array}$$

"In this case, we only have one digit to read, so it's just four-tenths. Tenths is the number's last name. Four times the quantity one-tenth."

Next Rachel writes .45, labeling each digit with its respective place value. Drawing an arrow through the digit 5, she directs students to the last name. "So, we read the entire number—forty-five—and then if we look above the digit five we can see it's last name is hundredths. Forty-five hundredths. In distributive form, that is four times the quantity one-tenth plus five times the quantity one one-hundredth. She draws an arrow through the five.

$$\begin{array}{cc} \frac{1}{10} & \frac{1}{100} \\ .4 & 5\uparrow \end{array}$$

In the next example, Rachel writes three digits to the right of the decimal point:

$$\begin{array}{ccc} \frac{1}{10} & \frac{1}{100} & \frac{1}{1000} \\ .4 & 5 & 6\uparrow \end{array}$$

"Read the whole thing," directs Rachel. "Four hundred fifty-six…" She points to the place value above the digit 6. "Thousandths," the students read.

"Let's look at four-tenths again," Rachel suggests, writing it out once more. "Here we have four-tenths, but let's suppose we put a zero in the hundredths place?" To the right of the 4, she writes a 0 with the proper place value above.

$$\begin{array}{cc} \frac{1}{10} & \frac{1}{100} \\ .4 & 0\uparrow \end{array}$$

"As soon as we put the zero in that hundredths place, we can no longer say four-tenths," explains Rachel. "We must read what is there—it is forty-hundredths. The zero in the hundredths place means we want to know how many hundredths we have, not how many tenths. We have forty-hundredths. It is not the same as four-tenths, but it is equal."

She writes:

$$\begin{array}{ccc} \frac{1}{10} & \frac{1}{100} & \frac{1}{1000} \\ .4 & 0 & 0\uparrow \end{array}$$

"Read the entire number, remember." Rachel directs. "Four hundred THOUSANDTHS" the class replies.

On the overhead projector, Rachel writes:

$$.4 = .40 = .400$$

or

$$\frac{4}{10} = \frac{40}{100} = \frac{400}{1000}$$

"Four-tenths is equal to forty-hundredths, which is equal to four hundred-thousandths," she reads. "The numbers are all equal, but they are not the same. If I have a quarter in one hand and two dimes and a nickel in the other hand, the money in each hand is worth equal amounts, but they are not the same coins.

Return of the Money/Digit Game!

Once she is confident that students have a firm understanding of how to read the decimal fractions, Rachel introduces the now-familiar "real-fake money" and digits to the lesson. As always, she starts by leading a large group activity where she directs the numbers. Displayed on the overhead projector is a place value wallet. For this activity she demonstrates with transparent bills and coins rather than base-ten blocks. Each student has an empty place value wallet in front of them, and shares a bank of money and digits with a partner. In this game, they will work with tens, ones, tenths, and hundredths.

"Add eleven-tenths," instructs Rachel.

Some students carefully count out eleven dimes—known here as "tenth coins"—to their wallet.

"Eleven-tenths is what?" Rachel asks. "Remember the trading rules," she reminds them, "ten for one and one for ten!"

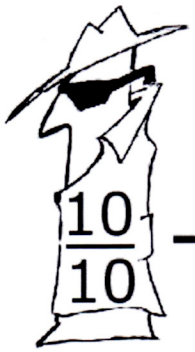
There is another flurry of activity as students return 10 tenth coins to the bank and retrieve a $1 bill.

On the overhead, Rachel writes:

$$\frac{11}{10} = \frac{10}{10} + \frac{1}{10}$$

"Eleven-tenths is equal to ten-tenths plus one-tenth," she reads. "Oh look—there is "Secret Agent One, disguised as ten-tenths." She draws the outline of a one around the $\frac{10}{10}$ to illustrate the trade they have completed.

$$\frac{11}{10} = \frac{10}{10} + \frac{1}{10}$$



The students arrange the digits in their wallet to read: 1.1

"Add nine-tenths," is the next instruction. Provided the class has had enough experience with the game, it is rare that students will need count out 9 tenth coins and put them in the tenths place in their wallets before realizing they can make a trade. Most students will see the trade right away, and simply throw the tenth coin from their wallet into the bank and take a dollar bill in change.

The digits in their wallet now read: 2. or 2.0

"Add eleven-hundredths," Rachel instructs.

Although they just conducted a similar trade using their tenths without any difficulty, the majority of students will count out 11 pennies—known as one-hundredth coins for the purpose of the lesson—and place them in their wallet.

"So now you have eleven one-hundredths," notes Rachel. "What do you see right away?"

"We can make a trade."

"What can you do with those ten one-hundredths?" she asks the student.

"Trade it in for a one-tenth."

"Go for it—make your trade."

As students exchange 10 one-hundredth coins, Rachel writes on the overhead:

$$\frac{11}{100} = \frac{10}{100} + \frac{1}{100}$$

When she has their full attention, she reads, "Eleven one-hundredths is equal to ten one-hundredths plus one one-hundredths. Now, look at that ten one-hundredths. Ten over one hundred is equal to one over ten, isn't it?" She writes:

$$\frac{10}{100} + \frac{1}{100} = \frac{1}{10} + \frac{1}{100}$$

The students fix their digits to read: 2.11

"Let's read our wallet in standard form. Remember, on the right side of the decimal point you read the entire number and then give it's last name. So we have two and eleven-hundredths."

She writes out the amount in expanded form on the overhead:

EF: ~~2.+.01~~

2 + .1 + .01

or

$$2 + \frac{1}{10} + \frac{1}{100}$$

"Two plus one tenth plus one one-hundredth," reads Rachel. The students repeat the number back to her in expanded form.

Next she writes the number in distributive/mulitplicative form:

$$2(1) + 1\left(\frac{1}{10}\right) + 1\left(\frac{1}{100}\right)$$

"Two times the quantity one, plus one times the quantity one-tenth, plus one times the quantity one hundredth."

"Add eighty-four hundredths to your wallet."

Students are quick to recognize that they do not need to count out 84 hundredths coins. Around the room learners add 8 tenth coins and 4 hundredth coins to their wallets. When all the students have fixed their digits, they read their answer in standard, expanded, and distributive forms, and Rachel writes the forms out on the overhead.

$$\text{SF: } 2.95$$

"Two and ninety-five hundredths."

$$\text{EF: } 2.+.9+.05$$

or

$$2+\frac{9}{10}+\frac{5}{100}$$

"Two plus nine-tenths plus five-hundredths."

$$\text{D/MF: } 2(1)+9\left(\frac{1}{10}\right)+5\left(\frac{1}{100}\right)$$

"Two times the quantity one, plus nine times the quantity one-tenth, plus five times the quantity one one-hundredth."

"Now it's time to subtract," Rachel informs the class. "I want you to subtract nine-hundredths." She watches as learners take a tenth coin from their wallet and give themselves a one-hundredth coin in change.

The digits in their wallet read: 2.86

The students read the result in standard form, "Two and eighty-six hundredths."

Rachel writes the number in expanded form on the overhead:

$$\text{EF: } 2.+.8+.06$$

or

$$2+\frac{8}{10}+\frac{6}{100}$$

"Two plus eight-tenths plus six-hundredths," they read together.

$$\text{D/MF: } 2(1)+8\left(\frac{1}{10}\right)+6\left(\frac{1}{100}\right)$$

"Two times the quantity one, plus eight times the quantity one-tenth, plus six times the quantity one-hundredth."

"How about exponential form?" she suggests.

$$\text{XF: } 2(10^{0})+8(10^{-1})+6(10^{-2})$$

"Two times the quantity ten to the zero power, plus eight times the quantity ten to the negative one power, plus six times the quantity ten to the negative two power."

"Next, let's subtract ninety-seven hundredths." Rachel waits for the students to complete the transaction and then asks for a volunteer to explain how they subtracted the amount from their wallet.

"I took the ninety-seven hundredths from one of my ones and got three one-hundredths from the bank," explains one student.

"So what do you have left in your wallet?" asks Rachel.

"I have one and eighty-nine hundredths."

She calls another volunteer who subtracted the same amount a different way. "I want students to be able to show what they did and explain the process," Rachel says.

Group Game

For the next part of the activity, the class plays a large-group cooperative game. They begin with $25.67 in their place value wallets, and Rachel adds the same amount of money to her own wallet on the overhead projector. Individual students are called on to roll three dice—the operation cube, the tenths, and the hundredths—for the whole class. Although Rachel will no longer choose the numbers the students work with, she is able to direct the activity.

"Who wants to roll for the class?" she asks. The first volunteer rolls the dice and reads the result. "Add five-tenths and four-hundredths."

"You're reading what you see on the dice," notes Rachel, "but we are dealing with hundredths," she reminds her. "When we roll the dice we need to put the two numbers together and read it in hundredths. So the way that we read those dice is really, fifty-four hundredths."

"Fifty-four hundredths," repeats the class.

"Good," says Rachel. "Go ahead and add fifty-four hundredths to your twenty-five and sixty-seven hundredths."

Although she has her wallet displayed on the overhead, she does not touch her money until the students have finished.

"How did you add your fifty-four hundredths?" she asks a learner.

"I added five-tenths to six-tenths and got one and one-tenth, so I put five of my tenth coins into the bank and took out a dollar."

On the overhead, Rachel demonstrates this using the transparent money, taking direction from the student. "Then I added four-hundredths to seven-hundredths, which gave me eleven-hundredths."

"When you are in the hundredths column, ten hundredths is equal to what?" Rachel asks.

"One-tenth," he tells her. "Eleven hundredths is equal to one-tenth plus one-hundredth, so I added another tenth coin and a hundredth coin to my wallet."

Rachel adds these two coins to the wallet. "How did you fix your digits?" she asks.

"Twenty-six and twenty-one-hundredths."

"Who has another way?" asks Rachel. She gives a second demonstration with a student who directs her to make one big trade, adding one and two-tenths to the wallet all at once.

"We did it two different ways, but we still have twenty-six and twenty-one-hundredths." Rachel comments.

A new person is chosen to roll the dice and read the result, then the class repeats the number. While the students apply the amount to their wallet, Rachel walks around the room observing. She finds that the students who think they are "math whizzes" will go right to their digits without working with the money, and then quickly discover they are lost. "I have a strong, firm rule," Rachel tells the class. "You must do the money first, then the digits. Later on you will play the game in your head, but right now you must play it with your hands."

When everyone has finished fixing their digits, she calls their attention to back to the wallet on the overhead projector and has two more students talk her through the process, beginning with the money. "I never touch my digits until I have touched all the money," reiterates Rachel firmly. She involves as many students as possible in the activity. "Who will read the answer in standard form?" she asks, writing out the forms on a transparency. "Who will read the answer in expanded form? Who will read it in distributive form? Who will read the answer in exponential form?"

Partner Up!

After the class has worked out a few problems as a group, they are allowed to play a cooperative game with their partner, rolling tenths and hundredths together. "Take turns reading your wallet in all four forms," instructs Rachel. "One person reads it in standard form, then the other person reads it in expanded, then the first person reads it in distributive form, and finally the second person reads it in exponential form."

She walks around the room listening to different pairs. When it comes to getting students to speak the math, money talks. If she hears a group following her directions, she rewards them by allowing them add $10 to their wallet. If she discovers two partners who are not complying, both players must subtract $10 from their wallets.

After about ten minutes of play, a student asks, "Can we roll our ones?" Rachel gives permission for students to roll ones, tenths, and hundredths if they desire. If some students feel ready, they are allowed to roll tens in addition to the other dice.

During the game, Rachel stops the students and calls their attention to a number she has written on the overhead projector: 34.68

She intentionally chooses a number that features the lowest digit in the highest place value, and the highest digit in the lowest place value. Pointing to the first digit in the number, she asks, "What is the name of this digit?" "Three." Still pointing to the three, Rachel inquires, "What is the name of this number?" "Thirty," the class tells her. She repeats these questions for each of the remaining digits. "Digit?" "Four." "Number?" "Four." "Digit?" "Six." "Number?" "Six-tenths." "Digit?" "Eight." "Number?" "Eight-hundredths."

She writes the number out in all four forms and the class reads them aloud.

SF: 34.68

"Thirty-four and sixty-eight hundredths."

EF: 30.+4.+.6+.08

Thirty plus four plus six-tenths plus eight-hundredths."

$$\text{DF: } 3(10)+4(1)+6\left(\frac{1}{10}\right)+8\left(\frac{1}{100}\right)$$

"Three times the quantity ten, plus four times the quantity one, plus six times the quantity one-tenth, plus eight times the quantity one-hundredth."

$$\text{XF: } 3(10^{1})+4(10^{0})+6(10^{-1})+8(10^{-2})$$

"Three times the quantity ten raised to the first power, plus four time the quantity ten raised to the zero power, plus six times the quantity ten raised to the negative one power, plus eight times the quantity ten raised to the negative two power."

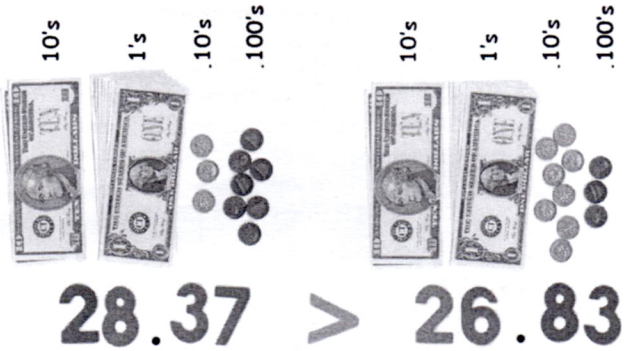
Competitive Game

Once students are comfortable with playing the game cooperatively, they may choose to play a competitive game with their partner. "You must roll the tenths and hundredths with your action cube," directs Rachel, "and then on each turn you can make a decision if you want to roll any of the other dice. You do not have to roll the same amount of dice that your partner rolls."

Players must use a greater-than/less-than symbol positioned between them to accurately reflect who has the greatest and least amount in their wallet, and read the result. Rachel wanders through the classroom, listening. "Twenty-eight and thirty-seven hundredths is greater than twenty-six and eighty-three hundredths." reads one partner. "Twenty-six and eighty-three hundredths is less than twenty-eight and thirty-seven hundredths." reads the other.

"By how much?" Rachel asks.

"One and fifty-four hundredths."



28.37 > 26.83

Tossing In Thousandths!

Eventually, students will ask permission to roll the hundreds die in hopes of making more money. Rachel uses this as an opportunity to introduce thousandths into the game. "I will make a deal with you," she tells them. "You can roll the hundreds, but you must also roll the thousandths."

Each pair of students decides for themselves when they are ready to add thousandths, and whether they want to continue with a competitive game, or return to a cooperative game. "You must roll all the dice together now--hundreds, tens, ones, one-tenths, one-hundredths, and one-thousandths, and then roll the operation cube," instructs Rachel.

She does not find it necessary to do a large group instruction when making the transition to thousandths. "We don't use thousandths much with money in real life," Rachel explains, while many students begin adding the colored thousandth discs to their wallets, "but the trading rules are the same—ten for one, and one for ten. We do see thousandths in sports quite a bit," she adds. "At the Olympics, someone might win a race by just one one-thousandth of a second!"

If a group gets in trouble, she sits down and helps them individually. "I watch and listen very carefully," Rachel concludes. "The students must always speak the mathematics as they play."

Variation

A "wonderful teacher" has come up with a new variation on the money/digits game. Instead of putting students in pairs, she has them work in groups of three. Two of the players play the game as usual. The third player is the banker and is in charge taking and handing out money to the players depending on what they roll. After two rolls, the banker trades places with one of the other players in the group so that each student has a turn at running the bank.