Multiplication!

Facts, Fun, and Awesome Algebra

Topic Involved: grouping, counting on, multiplication facts, the commutative associative, & distributive properties of multiplication, justification of work...

Materials: counters (lima beans, unpopped popcorn kernels, unifix cubes, or similar items), pencil and paper, calculator. Teacher may wish to use an overhead projector.

Type of Activity: individual or groups of two.

Grade Level: 2nd and up

Relation to NCTM Standards: Increased attention on use of manipulative materials, discussion of math, justification of thinking. Understanding meanings of operations and how they relate to one another. Content integration, creating algorithms and procedures. Decreased attention on rote practice, rote memorization of rules, one answer one method, use of worksheets, written practice, teaching by telling.

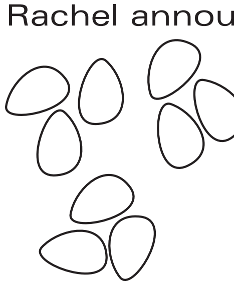
Counting Out Kernels

"Multiplication is nothing but groups of things," explains WI's resident mathematician, Rachel McAnallen, "So when I start a lesson on multiplication—no matter what grade I'm teaching—I begin by giving students a handful of 'stuff' to put into groups."

The criteria for these handfuls of stuff is simple—whatever is used must be plentiful and all the same thing. Items Rachel commonly uses are lima beans, unpopped popcorn kernels, or dotted hexahedra (dice).

For this lesson, Rachel doles out varying quantities of popcorn kernels (she estimates she hands out anywhere from 45 to 75 kernels). Each student is given a different amount, which they place in a small tray at their desk. "Count out how many popcorns you have," instructs Rachel, "counting by ones." After students complete their count, they are told to write down the number on a sheet of paper.

"Now you're going to put the popcorns in groups of twos and count them," Rachel announces. Many students come up with a different number in this second count. "Don't worry about it, just write it down," Rachel assures them. "Now put them in groups of five and count." A few students arrive at a different number for the third time.



"Next I want you to put your popcorns in groups of ten and count them," says Rachel. Although some students have four different numbers written on their paper, other students have the same answer counting by tens that they got for fives.

Rachel directs students to place their popcorn kernels back in their trays. "Which was the easiest way to count your popcorn?" she asks. Most of the class agrees that counting by fives or tens was easiest. "It didn't take as long," explains one student.

"Yes," says Rachel. "When I count things I like to put things in groups of fives or tens. So we know it's easier to count things in groups." As an example, she shows the class how to use tally marks to count by fives. "It is important to teach students that the slash across is number five," Rachel notes, "because they don't realize it on their own. So they end up county by fours."

She surveys the classroom and ponders, "If I wanted to count the people in this class what is the easiest way to do it? You're all seated in groups of four, so I can easily count by fours...four, eight, twelve, sixteen, twenty, twenty-four," she counts, concluding, "I don't count many things in groups of one—it takes too long."

Just the Facts, Ma'am

Throughout the rest of the lesson, Rachel uses an overhead projector to model what students should copy down in their notes. She begins by writing:

"Multiplication is just groups of stuff."

Next Rachel provides each learner with a calculator. "You all have four kinds of calculators," she tells them. "You have the push-button calculator, but you also have your popcorn calculator, your pencil-paper calculator, and your brain calculator. We are going to use all of these calculators to discover and write down groups of things."

On the overhead, she writes:

Groups of Twos

"Has anyone taken out a group of two?" she asks. "No," students tell her. Rachel writes down:

0g x 2p

(Zero groups of 2 popcorn kernels)

"We have 0 groups of 2 popcorn," she explains. "And how many pieces of popcorn do we have? Zero popcorns. So we write":

0g x 2p = 0p

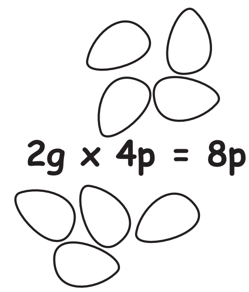
"Take one group of two out," she instructs. "Make the two popcorn kernels touch, so you know it's a group."

"What do you have?" Rachel asks. "One group," the class tells her. "So that's one group of two popcorns—how many popcorns do you have all together?" "Two popcorns." ON the overhead Rachel writes out the equations:

1g x 2p = 2p

Rachel takes students through this same process—making each group with the popcorn, and writing out the corresponding equation—until they reach 12g x 2p = 24p. "There is a reason I take them all the way through to twelve groups of two," she explains. "I want them to see the patterns. I teach all the multiplication tables at one time—I don't do twos and threes in September and fours and fives in October."

When the class has written out all the multiplication facts for groups of twos on their paper, Rachel instructs them to check their work with a calculator.



"Students do not do the multiplication with the calculator," she emphasizes. "They are using the calculator to check the work they have done with the manipulatives once they have it all written down." If the calculator answer matches the answer written on the students' paper, they may put a check mark next to that equation to indicate it has been checked.

When students have completed checking their twos, Rachel asks them, "Okay, what do you think we're going to do next?" "Groups of three!" they guess correctly. "I want the students to see that my teaching follows a pattern also. I want them to see the sequence of it," explains Rachel.

Students can also work with a partner for this activity, Rachel suggests. "It's a good idea because they can talk and argue—if they have a disagreement on an answer they have to work it out."

Students are again directed to check their work with the calculator when they get to 12g x 3p = 36p.

As students work, Rachel asks them to let her know if they see any patterns along the way.

"Who thinks they can work on their own now? What do you think is next?" asks Rachel again. "Groups of four," they tell her.

A student asks, "Do we have to use the popcorn all the time?" "No," Rachel tells her, "you may stop using the manipulatives if you have the pattern."

Rachel walks around the room observing the students. She notes that in every class there will be one learner who does not want to do all the work, and goes directly to the calculator. "That is a no-no," warns Rachel. "You may only use the mechanical calculator to check your work. You must do all 13 problems with your popcorn and pencil and paper first. The calculator does not help you understand the process."

By the time students get to four groups of four, many of them stop using the popcorn and begin "counting on." "Counting on means you have a number and you count on to it," explains Rachel. For example, she watches as one learner counts on from 5g x 4p = 20. "Twenty... twenty-one, twenty-two, twenty-three, twenty-four," he whispers, touching his fingers. He then writes 6g x 4p = 24p.

Around the classroom, students are beginning to see the pattern unfold. One student waves her hand. Rachel walks over and asks, "What did you discover?" "Well, you're just adding four every time," the student tells her. "Wow!" exclaims Rachel, "That is a great discovery!" Rachel invites the student to share her realization with the class.

"I'm always excited when students make a new discovery," Rachel comments. "Did I already know what that student discovered? Of course. But she didn't. She just realized that she doesn't need to use the popcorn anymore—she can count on."

As students stop using the popcorn to calculate their answers, the reason for checking their work becomes evident. "They miscount," says Rachel. "They may count on from eight groups of four is thirty-two, and mistakenly get thirty-five—they then continue the mistake right on down to twelve groups of four."

She tells the class, "If the answer you've written down is different than the one you got on the calculator double check it to make sure which is correct." If a learner is still unsure, they may wave their hand for help.

When students have completed the facts for fours, they may go on and do the fives, but no further. "In a class of twenty-four, you have twenty-four students in twenty-four different places, or twelve groups of two students in twelve different places," says Rachel I want every kid to be done with their fives before I take them any further."

The first two students to complete their fives wave their hands to alert her. "You need to practice your twos," she tells

Groups of Two	
0g x 2p =	0p
1g x 2p =	2p
2g x 2p =	4p
3g x 2p =	6p
4g x 2p =	8p
5g x 2p =	10p
6g x 2p =	12p
7g x 2p =	14p
8g x 2p =	16p
9g x 2p =	18p
10g x 2p =	20p
11g x 2p =	22p
12g x 2p =	24p

Groups of Three	
0g x 3p =	0p
1g x 3p =	3p
2g x 3p =	6p
3g x 3p =	9p
4g x 3p =	12p
5g x 3p =	15p
6g x 3p =	18p
7g x 3p =	21p
8g x 3p =	24p
9g x 3p =	27p
10g x 3p =	30p
11g x 3p =	33p
12g x 3p =	36p

Groups of Four	
0g x 4p =	0p
1g x 4p =	4p
2g x 4p =	8p
3g x 4p =	12p
4g x 4p =	16p
5g x 4p =	20p
6g x 4p =	24p
7g x 4p =	28p
8g x 4p =	32p
9g x 4p =	36p
10g x 4p =	40p
11g x 4p =	44p
12g x 4p =	48p

Groups of Five	
0g x 5p =	0p
1g x 5p =	5p
2g x 5p =	10p
3g x 5p =	15p
4g x 5p =	20p
5g x 5p =	25p
6g x 5p =	30p
7g x 5p =	35p
8g x 5p =	40p
9g x 5p =	45p
10g x 5p =	50p
11g x 5p =	55p
12g x 5p =	60p

them. She has them sit with their paper in front of them and take turns asking each other facts. "It's okay if you have to look at the paper to answer. You're learning to read your notes." If the students really know their twos, they can mix in the facts for threes and so on. "Often it will take students more time to ask a question than to give an answer," smiles Rachel. "They aren't used to asking questions—usually the teacher asks and they answer."

After the entire class has completed their fives, Rachel returns to the overhead projector and writes:

Groups of Sixes

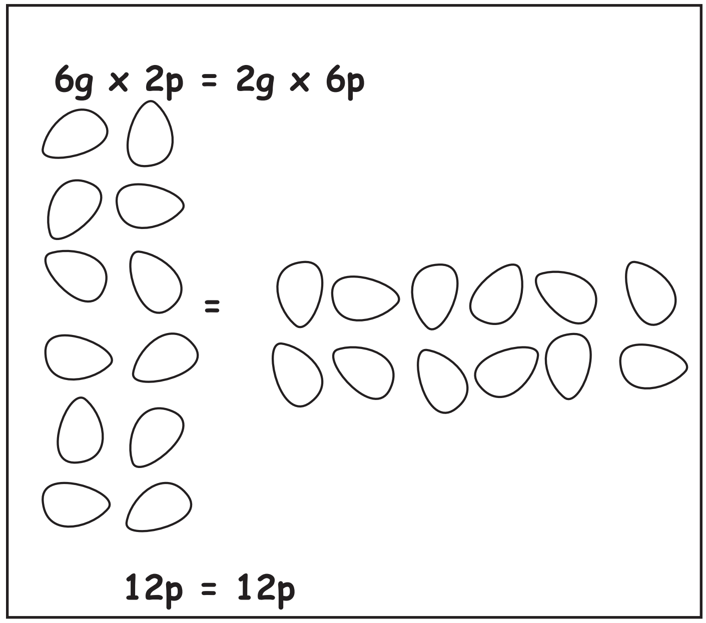
"This time we're not going to write down any facts that we already know, or that we've already written down somewhere else," she informs the class.

"Do we have to write down 0g x 6p?" she asks. "No." "Why not?" "Because we know it," they tell her. "Do we know what one group of six is?" "Yes." "How many?" "Six." "Do we write it down?" "No."

"Now, let's look at two groups of six popcorn," says Rachel. "Is two groups of six already written down someplace else, in a way?" Students point out 6g x 2p from the twos facts. "That's right," Rachel agrees, "If we have two groups of six, it also means that we have six groups of two." She has students quickly count out two groups of six kernels, and does the same on the overhead. "How many popcorn do we have in all?" she asks. "Twelve popcorn." "Okay, now lets suppose I do this," says Rachel, rearranging the kernels. "Look, we can also make six groups of two popcorn—how many in popcorn in all?" "Twelve," answers the class.

"So, two groups of six is not the same as six groups of two," Rachel explains, "but they are equal. We don't have to write down two groups of six popcorn, because that is equal to six groups of two. No matter what, we have how much?" "Twelve."

"Mathematicians call this the commutative property of multiplication,"



Rachel tells them. She writes:

Commutative Property of Multiplication

6g x 2p = 2g x 6p

12p = 12p

Armed with this new information, the class continues to look at groups of six. "Now, what about three groups of six?" asks Rachel. "We don't have to write that out because we already wrote six groups of three popcorn—so we know we've got what?" "Eighteen," one student volunteers. "Do we have to write down four groups of six?" "No." "Five groups of six?" "No." "Why not?" "Because we already know them."

When they reach six groups of six, the class informs Rachel that they must finally write out a fact for six. "So we have to start with six groups of six popcorn," Rachel remarks. "Does anyone know how many that is?" "Thirty-six," replies a student. "How did you get your answer?" "Five groups of six is thirty, so I added another six and got thirty-six," she tells Rachel.

Rachel writes:

6g x 6p = 36p

The class determines that they must write down the facts for seven groups of six and on up to twelve groups of six. "Some students already know ten groups of six and eleven groups of 6," says Rachel. "Those students can figure out nine groups of six by subtracting 6 from 60.

Students are quick to see the patterns that emerge within process. "Where are we going to start writing out groups of seven?" asks Rachel. "Seven groups of seven popcorn," the class replies. "And with groups of eight?" "Eight groups of eight." "What about your nines?" "Nine groups of nine." "And tens?" "Ten groups of ten." "How many facts will you have to write out for 12?" she asks them. "Only one," a student tells her.

Groups of Sixes	
6g x 6p =	36p
7g x 6p =	42p
8g x 6p =	48p
9g x 6p =	54p
10g x 6p =	60p
11g x 6p =	66p
12g x 6p =	72p

Groups of Sevens	
7g x 7p =	49p
8g x 7p =	56p
9g x 7p =	63p
10g x 7p =	70p
11g x 7p =	77p
12g x 7p =	84p

Groups of Eights	
8g x 8p =	64p
9g x 8p =	72p
10g x 8p =	80p
11g x 8p =	88p
12g x 8p =	96p

Groups of Nines	
9g x 9p =	81p
10g x 9p =	90p
11g x 9p =	99p
12g x 9p =	108p

Groups of Tens	
10g x 10p =	100p
11g x 10p =	110p
12g x 10p =	120p

Groups of Elevens	
11g x 11p =	121p
12g x 11p =	132p

"When students learn their multiplication facts this way, it isn't so overpowering," says Rachel. "It isn't like they have 144 to learn."

During this lesson, Rachel encourages the students to share their ideas and discoveries. "I'm working with 24 brains and I have no idea what they are going to say, but I listen to all of them," she says. "If students have had good primary teachers who have emphasized patterning, they will discover fantastic stuff." She suggests teachers make a chart of students' discoveries and put it on the wall so the class can see if the patterns always work.

Fun Factoring! Daring Doubles!

"As much as we hate to admit it—even if we've done the best job teaching multiplication, and we've given learners loads of practice—there will always be students who choose not to learn their multiplication facts," Rachel acknowledges. "It's not that they can't," she explains, "But for a variety of reasons, they simply choose not to memorize." For instance, a gifted student may simply view memorization as a waste of time. When a student forgets the math fact for 9 x 7, Rachel says to that learner, "You forgot nine times seven is sixty-three. I want you to think of five ways that you can find that answer." If the student knows ten times seven is seventy, they can subtract seven to arrive at sixty-three. "It is important to give students other processes—I want learners to investigate ways they can get an answer if they forget a multiplication fact."

Even students who dislike memorization will know their twos, and all their fives and tens. "If you know your twos, I can show you the coolest way to find your fours and eights," Rachel tells them. "But maybe you only know your twos up to a certain point—I want you to be able to double up to fifty, so I'm going to teach you a fun way to double numbers."

Rachel writes on the overhead:

Associative Property of Multiplication

"We've already learned about the commutative property, which means two groups of six is equal to six groups of 2," she reminds students. "Now we're going to look at the associative property. Which means this," she writes:

$$(4 \times 3) \times 2 = 4 \times (3 \times 2)$$

"Order does not matter in multiplication. And it's because of the associative property that we are now going to look at something like this:"

Factor x Factor = Product

Or

Factor x Factor x Factor = Product

"The numbers we multiply together are called factors, and the answer to the multiplication is called the product," Rachel tells students. "So, if two is a factor, and four is a factor, then our product is eight."

2 x 4 = 8

(factor) (factor) (product)

"Now I'm going to give you a number, which is one factor. Then I'm going to tell you to double it—so the other factor is two," she explains. "If I tell you to double 13, here is how you do it." On the overhead she writes:

$$\begin{array}{r} 13 \\ +13 \\ \hline 10 + 10 = \\ 3 + 3 = \underline{6} \\ 26 \end{array}$$

"Ten plus ten is twenty, three plus three is six. Twenty plus six is twenty-six," she narrates. Our answer is twenty-six!"

The students catch on quickly, doubling numbers aloud, as Rachel calls them out. "Double seventeen!" "Thirty-four!" "Double twenty-three!" she tells them. "Twenty plus twenty is forty...three plus three is six...Forty-six!" students reply without hesitation. "You'll never be able to double forty-two!" challenges Rachel. "Eighty-four!" they reply with delight. "How did you do that?" she demands. "Forty plus forty is eighty...two plus two is four...eighty-four!" as student ells her. "Oh wow!" Rachel exclaims. "If you know how to double all your numbers up to fifty, you are going to be great mathematicians!"

Next Rachel writes on the overhead:

4 x 7

"These are two factors," she says. "But look—lets break that four into two times two. See how easy your fours are—our four is nothing but a doubled 2!" Rachel explains as she writes: "Two times seven is fourteen...double 14 and we have 28!"

4 x 7

2 x 2 x 7

2 x 14 = 28

There is an advantage in teaching students this process, claims Rachel. "If you just teach students their math facts, they will only learn their fours up to twelve. If you teach them how to double their twos, a really bright learner will expand on that knowledge." For example, one student announces, "I know four times fourteen." "How did you get that?" she asks him. "Double fourteen is twenty-eight...double twenty-eight is fifty-six," he replies.

You'll never be able to do this one," Rachel teases:

4 x 125

Her students don't bat an eyelash. "Double one hundred twenty-five is two hundred fifty. Double two hundred fifty is five hundred."

4 x 125

2 x 2 x 125

2 x 250 = 500

"That is the advantage of teaching the process over a fact," Rachel concludes. "Teaching this process means that students can learn their fours as far as their own ability will take them."

Next she writes:

8 x 6

"If you double your fours, you have your eights." Rachel talks students through the process as she writes, "Double six," she instructs the class. "Twelve."

"Double twelve?" "Twenty-four." "Double twenty-four?" "Forty-eight."

8 x 6

2 x 2 x 2 x 6

2 x 2 x 12

2 x 24 = 48

"This is called factoring," she tells them. They do several more eights:

8 x 15

2 x 2 x 2 x

2 x 2 x 30

2 x 60 =

"Eight times fifteen...double 15 is 30, double thirty is 60...double 60 is 120..." Students are clearly impressed with themselves.

Some learners begin to see a pattern. "If I double again, I can do sixteens!" a student tells Rachel. "What about if you wanted to do your thirty-twos?" Rachel asks her. "Double again," she replies.

"See how cool multiplication is when you know how to double?" Rachel laughs.

Distributive Property of Multiplication

There is another important property that we use in algebra," says Rachel. She writes:

Distributive Property of Multiplication

The distribution of multiplication over addition and subtraction.

"Suppose I want to multiply a two-digit number times a one-digit number," she says. "Let's multiply 14 x 6 as an example. "We are going to expand fourteen into ten plus four—remember, left-hand side equals right-hand side."

$$\begin{array}{r} 14 \\ \times\ 6 \\ \hline \end{array} = \begin{array}{r} 10 + 4 \\ \times\ 6 \\ \hline \end{array}$$

"The distributive property tells us to distribute six over ten, and six over four," Rachel explains. "What's six times ten?" "Sixty." "What's six times four?" "Twenty-four." "Right—so, sixty plus twenty-four is eighty-four. Fourteen times six is eighty-four.

$$\begin{array}{r} 14 \\ \times\ 6 \\ \hline \end{array} = \begin{array}{r} 10 + 4 \\ \times\ 6 \\ \hline 60 + 24 = \end{array}$$

Students are usually taught to multiply this equation using what Rachel requently refers to as the "nonsense" method. Rachel gives a critical demonstration:

$$\begin{array}{r} 14 \\ \times\ 6 \\ \hline 84 \end{array} =$$

"Six times four is twenty-four...put down the four, take the 'two' and write it out small at the top above the 'one.' Now, six times 'one' is six, plus the little 'two' is eight. Write the eight at the bottom next to the four. Why? B'cuz."

According to Rachel, the most serious problem with this approach is that students have no idea what numbers they are working with, therefore they have little understanding of the mathematics behind the process. "This is really six times *ten*, which equals sixty, not six," she stresses. "And that little two is really a twenty—when a student writes the four down at the bottom and then puts the two at the top, they have no idea they are writing twenty-four."

Rachel writes out the problem again. "What do you want to multiply by first?" she ask. "Do you want to distribute six over four first, or six over ten?" "Ten," replies a student. Rachel takes the class through the multiplication, emphasizing the importance of using the correct place value names for every number. On the overhead, she shows them how to justify their work:

$$\begin{array}{r} 14 \\ \times\ 6 \\ \hline 60 = 6 \times 10 \\ 24 = 6 \times 4 \\ \hline 84 = 60 + 24 \end{array}$$

"I don't care which way they write the problem, but if a teacher insists on starting with four times six, they need to have their students justify their work this way. Writing a little two at the top is not justifying your work." She writes:

$$\begin{array}{r} 14 \\ \times\ 6 \\ \hline 24 = 6 \times 4 \\ 60 = 6 \times 10 \\ \hline 84 = 24 + 60 \end{array}$$

"Here is what we all learned in algebra," says Rachel. She writes out the following algebraic expression:

$$6(x+4) = 6x + 24$$

"We distribute six over x, to get 6x. Six over 4 is 24. Because this is an algebraic expression I can let 10 = x. Now we have:

$$\text{Let } 10 = x$$

$$6\,(10 + 4)$$

"Distribute six and we have:

$$6\,(10 + 4) = 60 + 24$$

"This is the algebraic application," says Rachel. "When a teacher uses the nonsense method to tech multiplication, the algebra teacher must go back and unteach that method and reteach students this way. If we taught this process in the lower grades, then students would understand what is going on in algebra. It's a two-way street though," she adds. "The algebra teacher has to take students back and apply the algebra to arithmetic. Many times algebra students will ask, 'When will I ever use this?' The teacher should exclaim, 'Now!' and show them how it applies to 14 x 6."

Rachel writes out another problem on the overhead:

$$\begin{array}{r} 14 \\ \times\ 6 \\ \hline \end{array} = \begin{array}{r} 10 + 6 \\ \times\ 4 \\ \hline 40 + 24 \end{array} =$$

"This is a really fun activity to do," she tells students. "We already know our answer is sixty-four. What I want you to do is prove ten different ways that four groups of sixteen is sixty-four."

She explains, "Notice that I broke 16 into 10 + 6. But what else could I break it into?"

Examples of creativity:

$$\begin{array}{r} 16 \\ \times\ 4 \\ \hline \end{array} = \begin{array}{r} 8 + 8 \\ \times\ 4 \\ \hline 32 + 32 = 64 \end{array}$$

$$\begin{array}{r} 16 \\ \times\ 4 \\ \hline 64 \end{array} \begin{array}{c} = \\ \\ = \end{array} \begin{array}{r} 4 + 4 + 4 + 4 \\ \times\ 4 \\ \hline 16 + 16 + 16 + 16 \end{array}$$

$$\begin{array}{r} 16 \\ \times\ 4 \\ \hline \end{array} = \begin{array}{r} 20 - 4 \\ \times\ 4 \\ \hline 80 - 16 = 64 \end{array}$$

$$\begin{array}{r} 16 \\ \times\ 4 \\ \hline 64 \end{array} \begin{array}{c} = \\ \\ = \end{array} \begin{array}{r} 15\tfrac{1}{2} + \tfrac{1}{2} \\ \times\ 4 \\ \hline 40 + 20 + 2 + 2 \end{array}$$

Students do lots of work on two-digit times one-digit problems. "They show their work, which means they are using the distributive property," says Rachel.

When students are ready to move on, Rachel tells them "Let's try a three-digit times a one-digit." She writes:

$$\begin{array}{r} 125 \\ \times\ 6 \\ \hline \end{array}$$

They expands 125 as follows:

$$\begin{array}{r} 125 \\ \times\ 6 \\ \hline \end{array} = \begin{array}{r} 100 + 20 + 5 \\ \times\ 6 \\ \hline \end{array}$$

"What do you want to multiply first?" she asks the class. "One hundred," they tell her.

$$\begin{array}{r} 125 \\ \times\ 6 \\ \hline 750 \end{array} \begin{array}{c} = \\ \\ = \end{array} \begin{array}{r} 100 + 20 + 5 \\ \times\ 6 \\ \hline 600 + 120 + 30 \end{array}$$

"I don't care which number they multiply first," says Rachel. On the overhead, she illustrates other ways to look at the problems.

$$\begin{array}{r} 125 \\ \underline{\times\ 6} \end{array}$$

$$30=6\times5$$
$$120=6\times20$$
$$600=6\times100$$
$$750=600+120+$$

or

$$6(x^2 + 2x + 5)$$
$$\text{Let } 10=x\text{: } 6(100+20+5)$$
$$600+120+30+50$$

When students are comfortable multiplying three-digit times two-digit, Rachel moves on to two-digit times two-digit. “The first number I choose to demonstrate is twelve times thirteen,” says Rachel. “I tell them right off the bat that the answer is one hundred fifty-six. I want them to focus on the process.”

“How many times do you have to multiply in order to solve this problem?” she asks the class. “Four,” they tell her.

Rachel first demonstrates solving the problem in the traditional order working right to left beginning with 3.

$$\begin{array}{r} 12 \\ \underline{\times13} \end{array}$$

$$6=3\times2$$
$$30=3\times10$$
$$20=10\times2$$
$$\underline{100}=10\times10$$
$$156=100+30+20+6$$

“This is called double distribution,” she notes. “We distribute three over ten and two, and we distribute ten over ten and two.”

“There is a reason why I want students to write all of this out,” says Rachel. “These students will go into algebra and they will see this problem.” She writes:

$$(x+2)\ (x+3)$$

“Put your thumb over the two,” she tells students. “We distribute x over x plus three. Now we cover the first x and we distribute two over x plus three.”

$$x(x+3)+2(x+3)$$
$$x^2+3x+2x+6$$

“Now, see what happens when we let x = 10.”

$$\text{Let } x=10$$
$$(10 + 2)\ (10 + 3)$$

“We’re going to distribute 10 over 10 + 3, and 2 over 10 + 3”:

$$10(10 + 3) + 2(10 + 3)$$
$$100 + 30 + 20 + 6 = 156$$

“Isn’t that beautiful?” laughs Rachel.

“Other ways to do the problem:

$$\begin{array}{r} 12 \\ \underline{\times13} \end{array}$$

$$36=3\times12$$
$$1\underline{20=}10\times12$$
$$156=36+120$$

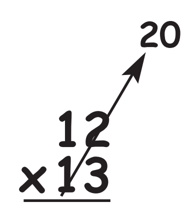
“This is a shortened version,” explains Rachel. “In this method, we’ve distributed 3 over 12 and 10 over 12. But,” she warns, “if this is the only way a teacher teaches the problem, the algebra teacher will still have to go back and re-teach their students. I want students to know that there are four multiplications going on because we are multiplying two binomials.”

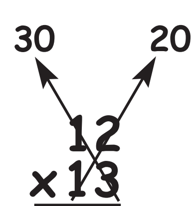
Around the World...

A particularly fun method of multiplying two binomials is something Rachel calls the “Around the World” method. “Leave some space around the problem,” she advises. Once more she writes out:

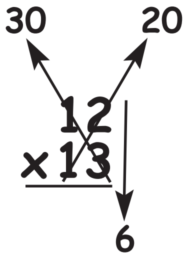
$$\begin{array}{r} 12 \\ \underline{\times13} \end{array}$$

“Ten times two is twenty.” She draws an arrow through the ten and the two and writes twenty in the upper right corner of the problem:



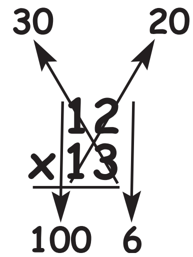


“Three times ten is thirty.” She draws another diagonal arrow, this time through the ten and three. She writes thirty in the upper left corner:



"Two times three is six." Next, Rachel draws and arrow down from two to three, and writes six in the lower right corner:

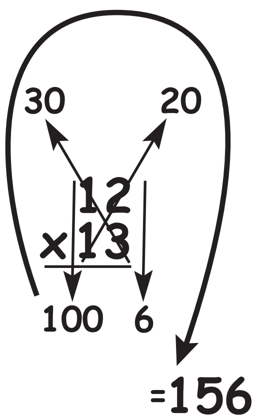
"Ten times ten is one hundred."



Finally she draws an arrow along the tens and writes one hundred in the lower left corner:

"Now let's start at 100 and go around the world," says Rachel. One hundred plus thirty equals one hundred thirty...One hundred thirty plus twenty is one hundred fifty...One hundred fifty plus six is one hundred fifty-six."

"Kids love this method because they can do it in their head—they just go "Criss,



cross, right left, and then around the world!" says Rachel.

Teachers will often ask Rachel about multiplying three-digit numbers by three-digit numbers. "Use a calculator," she jokes. All kidding aside, Rachel asks, "How many times do you have to multiply when you take a three-digit times a three digit?" It is a question that makes most teachers stop and think. "Nine times."

If students are confident in multiplying two-digit by three-digit, they can often figure out the rest.

"Here is the process I expect to see when a student does three-digit by three-digit," says Rachel in conclusion:

```
   156
  x432
   312=      2x156
  4680=     30x156
62,400=    400x156
67,392=312+4680+62,400

400x100=40,000
400x 50=20,000
400x  6= 2,400
         62,400

30x100=3,000
30x 50=1,500
30x  6=  180
       4,680

2x100=200
2x 50=100
2x  6= 12
      312
```