Subtraction Is Shopping!

Replacing Nonsense with Number Sense!

Topic Involved: Number sense, subtraction, algebra

Materials: Pencil and paper (students), overhead projector, transparencies (teacher)

Grade Level: 2nd and up

Relation to NCTM Standards: Understanding numbers, ways of representing numbers, relationships among numbers and number systems. Understanding meanings of operations and how they relate to one another.

Other Info: In preparation to this lesson, we recommend "Pigs in the Pen: Parts of the Whole" and "Place Value vs. Face Value" and "Algebraic Addition"

A lesson on subtraction—it's the sort of activity that might seem geared only to primary grade students. But Rachel McAnallen has developed a lesson that defies those traditional boundaries. Drawing on students' innate number sense (money sense), and using their artwork, their imaginations and an algebraic approach, Ms. Math shows learners that there is more than one way to solve a subtraction problem! In fact, they may discover (gasp!) that subtraction can be fun!

"Subtraction Is Shopping" can be taught from second grade, right on up," declares Rachel. "Certainly the lesson gets more involved for the upper grades." Even in upper grades there may be a learner that needs the basics. "if I start a lesson and discover a student doesn't have the conceptual basics for that lesson—and it only takes a matter of minutes to determine—I can't go on. I would only leave that learner further behind," she explains. "And if there is one child that doesn't have it, I know that there are lots of others that don't either. It never hurts the students that do have the concept to review it," she adds, "because they may see it differently this time around."

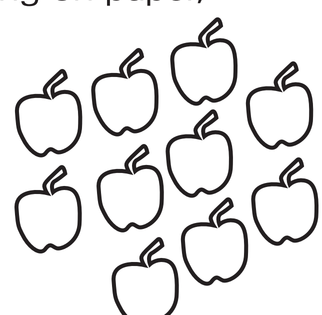
A Part from the Whole

"In order to understand subtraction, learners first have to understand parts of the whole," says Rachel. "They must know that within 10 you have 1 of something, you also have 2, you have 3, you have 8, and so on," she emphasizes. "It's important to begin this lesson by asking learners to think about what subtraction really means. Subtraction is nothing but shopping. Subtraction is taking away part from the whole."

One way for students to explore these ideas is to have them create subtraction problems using items that are in the classroom. "Pencils, chairs, book bags—there are lots of things in the classroom to subtract," suggests Rachel. "If there are 20 students in the classroom, have 7 of them leave the room—how many students are left? Twenty students subtract 7 students leaves 13 students."

Getting It Down on Paper

"Teachers always want to know how to get from subtracting stuff in the classroom to subtracting on paper," says Rachel. "I find the transition is really easy if you allow the time for the conceptual development. I do an activity that is really fun to do with second and third graders in particular, because it combines thinking skills and their own artwork."



On the overhead transparency, Rachel quickly draws a picture of 10 or 12 apples. Underneath her drawing she writes, "10 apples." She tells the class, "I'm going to make up my own subtraction problems. How many apples do you want me to subtract?" "Five," one student suggests.

Rachel writes:

10 apples
- 5 apples

"What do we have left?" she asks. "Five apples."

Rachel works with the class to create several subtraction problems using the 10 apples, writing each one out on the overhead projector:

10 apples
- 9 apples
1 apple

After writing the word "apples" in each problem, Rachel finally exclaims, "Oh, I am so tired of writing out apples all the time! Mathematicians like to use a letter for things," she tells students. "What letter could we use?" The class quickly decides that "a" is the best choice.

On the overhead projector, Rachel writes:

Let "a" Equal Apples

Now when they create a subtraction problem, Rachel writes it this way:

10 a
- 8 a
2 a

or

10 a – 8 a = 2 a

Once they've done a few more problems, Rachel has students take out a sheet of paper and a pencil. "Who loves to draw?" she asks them. Students raise their hand in the air.

"Here is your job," Rachel tells them. "I am going to give you a few minutes to draw 10 of something." Students are told they can draw anything but apples. "Think of what you want to draw, and then draw ten of them."

Rachel gives them about five minutes to work on their pictures, telling them, "When you are done, put your pencil down, look at me and smile." When most of the class is smiling vigorously at her, Rachel says, "Someone tell me what you've drawn."

"Puppies," shares one student. "Do you want to write out puppies all the time?" Rachel asks her.

"No," the student answers. She decides to use the letter "p" to represent the puppies she has drawn.

"Underneath your picture, write 'Let "p" equal puppies,'" directs Rachel. She goes around the room and asks for a few more examples. Each student chooses a letter to represent their pictures and writes out their "Let __ equal ___" sentence.

"You are going to make up your own subtraction problems from your pictures," Rachel continues. "Write down how many you have—10—then write down how many you want to subtract." For example, if a student has drawn flowers:

Let "f" Equal Flowers

"Take your hand and cover up two of the flowers," Rachel urges. "You can count and see how many you have left."

10 f
-2 f
8 f

Rather than giving students a specific amount of problems to create, Rachel sets a time limit. "Let's see how many subtraction problems you can make in 10 minutes," she tells them. This approach allows students to work at their own pace, and at their own level of development. "They can be very creative within that 10 minutes," says Rachel. "A learner may want to take part from a part—they may ask, 'Can I write 8 apples subtract 5 apples?' Another learner may want to use fractions. Of course they can!"

The homework assignment is to pick a number of things to draw—more than

10, but less than 20—and then draw that many things on a sheet of paper. "They do not have to make any subtraction problems," says Rachel. "We will do that in class. They should just draw. They can be as free as they want and they can color it, of course, and have a great time."

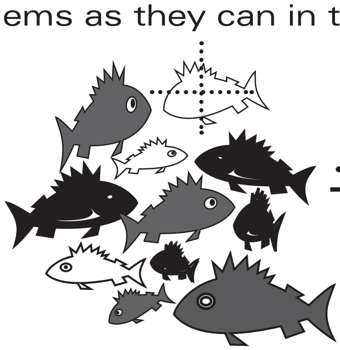
There may be students who want to draw something like fruit, or toys. In that case, they would draw a picture of different things within that category. "They can draw 15 separate toys," says Rachel, but she cautions, "You will want to make sure that they write toys in their subtraction problems. They can't write 15 toys – 1 teddy bear. It has to be 15 toys – 1 toy."

When students come in the next day, Rachel looks at their pictures and has them hold up their work for their classmates to see. "Some of them have started making subtraction problems," notes Rachel. "Some of them have written, "let". She gives the class 15 minutes to write their "let" sentence and make up subtraction problems using their pictures. "While they work, I walk around and see which students are getting the idea" she says.

One student has drawn lines through one of their pictures to make fractions. "When a child does that, I bring the other students around that table so they can see what he or she has done," says Rachel. "They can feed off of each other's creativity."

Let "f" Equal Fish

After students have done as many problems as they can in ten minutes,



$$\begin{array}{r} 11\ f \\ -7\tfrac{3}{4}\ f \\ \hline 8\ f \end{array}$$

they can check their work. Rachel puts them into groups of four and has them exchange papers. "They can see each others drawings and the kinds of problems their classmates have created."

Putting Worksheet Problems in Context

If a teacher is using a textbook or worksheets, Rachel suggests having students copy problems out of the book. "If there is a page with 30 subtraction, tell students to pick out the 15 easiest problems and copy them—in essence they'll do all 30 problems hunting for the 15 easiest," she laughs. In the textbook, the problem will read:

12 – 9

When they copy the problem, however, students must decide what things are being subtracted, for example:

12 trees – 9 trees

"They have to take 9 of something from twelve of something," explains Rachel. "If it's trees, then they write, 'Let "t" equal trees.'"

Their completed problem would then read:

12t – 9t = 3t

Having students copy the problems gives them practice in writing, but more importantly, it allows learners to put every number they use in context.

Subtraction is Shopping

Rachel teaches subtraction from the number sense or "money sense," point of view, using dollar signs to put problems in context. "I teach that subtraction is shopping," she says. "The number on top is the money you have in your wallet. The number on the bottom is how much you are spending."

She writes the problem $12 - $9 on the overhead projector:

$$\begin{array}{r} \$12 \\ -\ \$9 \\ \hline \$3 \end{array}$$

"You have $12 in your wallet—let's say it's in the form of one $10 bill and two $1 bills," she explains. "If you walk in to the store and want to buy something for

$9, you don’t give the clerk the whole $12. You give them the $10 bill. The clerk gives you back a $1 bill, and you add it to the two other $1 bills in your wallet. So you have $3 left.”

If we handed the clerk the entire $12, the clerk would wonder ‘Where did this kid go to school?” she laughs. “You don’t even touch the $2 bill in your wallet. You take the nine dollars out of the $10 bill.”

Using base-ten blocks is a wonderful way to illustrate this concept, says Rachel, giving the blocks dollar values. The small cube represents $1, the rod is worth $10, and the large square equals $100. “How many ways can you have $10?” Rachel asks. Students tell her they can use one $10 rod, or ten $1 cubes.

“Take out $15,” Rachel instructs. Some students may have fifteen $1 cubes in front of them, but most have take out one $10 rod and five $1 cubes. “You’re going shopping,” Rachel tells them. “You have $15 and you are buying something that costs $8. What are you going to hand the clerk?” A student raises her hand. “I’ll hand the clerk the $10,” she says.

“What about the five $1 you have in your wallet?”

“I’m not going to do anything with that,” she tells Rachel.

“Okay,” she says to the student, “You’re going to hand the clerk $10. What does the clerk hand back to you?”

“Two dollars.”

“Put it with your $5 and what do have left?”

“Seven dollars.”

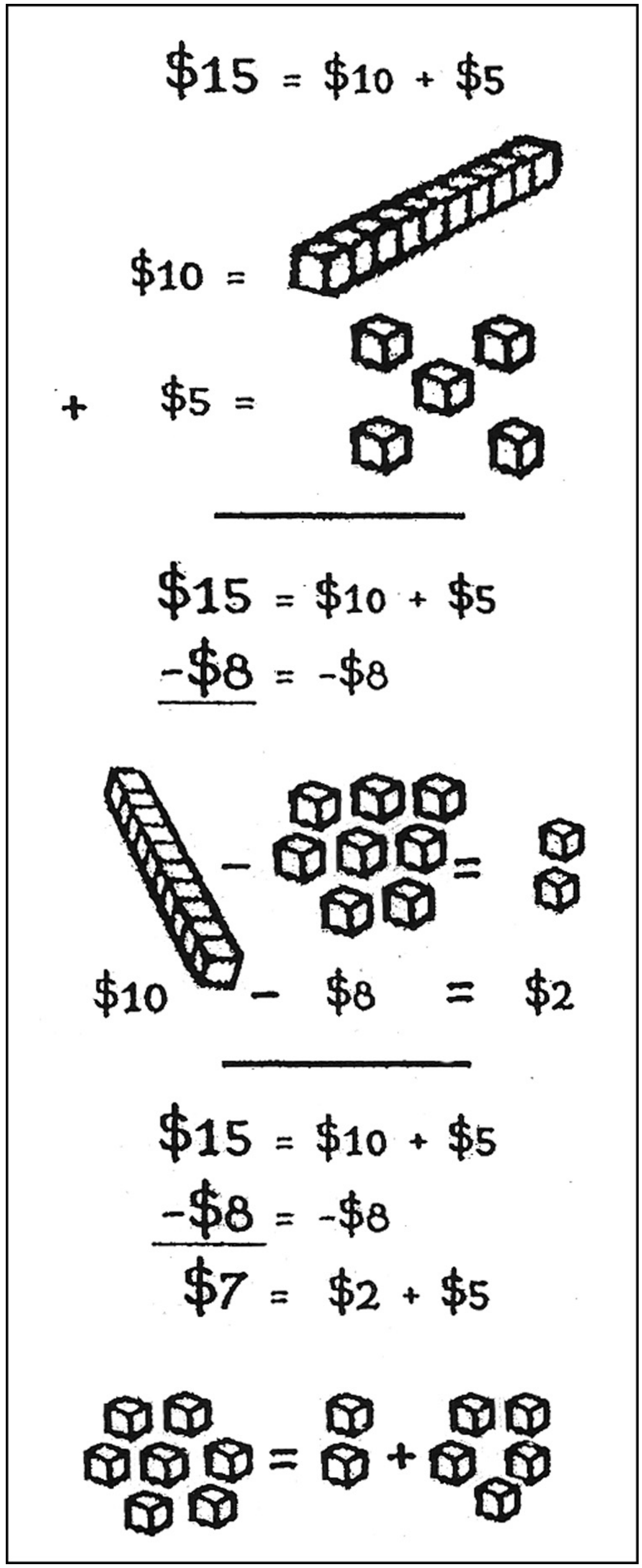
“Kids have great number sense and money sense, but we educate it out of them,” says Rachel. “Now, how do we write down what we’ve just done?” She writes on the overhead projector as she talks students through the process.

$15 = 10 + 5

“What did we do with the $15? We broke it down into $10 and 5.”

$15 = 10 + 5
+$ 8 = -8 - 0

“Then, we subtracted 8 from 10. We didn’t subtract anything form 5.”



"What is \$10 subtract \$8? Two dollars. Two dollars plus \$5 equals \$7."

$$\begin{array}{r} \$15 = 10 + 5 \\ \underline{+ \$\ 8 = -8 - 0} \\ \$\ 7 = \ \ 2 + 5 \end{array}$$

"Our answer is 7."

2-digit Subtract 2-digit

"Let's look at another problem," Rachel suggests. She writes out:

$$\begin{array}{r} \$\ 23 \\ \underline{-\ \$\ 18} \end{array}$$

"Let's look at how we usually teach this problem," she says. "Right off the bat, we tell students that they can't subtract 8 from 3, which is a lie. Next, we go over to the 2 – which isn't a two but a TWENTY – and we make kids slash it out. Then we draw in a little 1 – which is really a TEN – next to the 3.

$$\begin{array}{r} \$\ 23 \\ \underline{-\ \$\ 18} \\ \$\ \ \ 5 \end{array}$$

This is what Rachel derisively refers to as the "crossy-out" method of subtraction. Part of her objection to this approach is the language attached to the process. "I dislike the words 'borrow' and 'regroup'," she explains. "If I borrow something, that means I didn't have it, and now I have to pay it back. Educators have started to use the word regroup, but neither of these words have a mathematical context for the teacher or the learner."

However, Rachel's most serious concern with the crossy-out method is that students do not understand the mathematics behind it. "When I ask a learner what they've just changed 23 into, they honestly do not know," says Rachel grimly. "They do not know that they have done this," she says, writing:

$$\begin{array}{r} \$23 = 10 + 13 \\ \underline{-\ \$18 = -10 - 8} \\ \$\ 5 = \ \ \ 0 + 5 \end{array}$$

"We subtracted \$10 from \$10, which is zero. We subtract \$8 from \$13, which is \$5. Our answer is \$5," she explains. "This is what we are really doing when we use the crossy-out regrouping method."

"The way we teach subtraction is nonsense," says Rachel. "It gives the learner no options. What I am attempting to do is teach number sense. I teach students that they have choices in the way they subtract numbers."

"Imagine going into the store with a \$20 bill and three \$1 bills in your wallet" suggests Rachel. "You want to buy something for \$18. Now, if the clerk made change the way that we teach students subtraction, this is how it would go:

The clerk would look in your wallet and say, "I can't take eight dollars from three dollars-give me your \$20 bill. The clerk would then give you back a \$10 bill and ten \$1 bills. He'd tell you to put the ten \$1 with the three \$1 bills in your wallet. Now you've got thirteen \$1 bills. He tells you to give him eight of your \$1 bills. Then he wants your \$10 bill. Finally, you're left with five \$1 bills in your wallet."

"You would never go into the store if that is what happened—no one in their right mind would make change like that!" she laughs. "When you hand the clerk a twenty-dollar bill you expect to get back two dollars in change to put with the three dollars in your wallet. If the clerk ever looked in your wallet and told you he couldn't take eight dollars from three dollars, you'd tell him to get his nose out of your wallet! *You* decide what to do with your twenty-three dollars."

Rachel recognizes that there will always be teachers who are reluctant to teach alternative ways to subtract. "If a teacher chooses not to be creative, they still need to write out 23 = 10 + 13," she insists. "They need to explain what they are doing, and they need to *show* it. Telling is not good enough," she insists. "Telling is not teaching."

Once students are taught to rewrite 23 as 10 + 13, teachers have opened a door to more creative mathematics. "You've started them thinking," says Rachel with a smile. "They will begin to ask if they can rewrite 23 into other numbers. Maybe they don't want to use 10 + 13."

These are exactly the kinds of choices that Rachel encourages in the classroom. Instead of telling students to "regroup" she simply says, "We are rewriting the number in another form."

"How would *you* like to write 23?" she asks a student. "Can we write it as 20 + 3?" he asks. Rachel writes it on the overhead:

$23 = 20 + 3

"What would you like to do with the $18?" she asks.

The student thinks for a moment. "Subtract $18 from $20," he tells her.

"What do you want to subtract from 3?"

"Nothing."

$ 23 = 20 + 3
- $ 18 = -18 - 0
$ 5 = 2 + 3

Rachel is careful to use the correct mathematical terminology in the classroom. "This isn't twenty dollars minus eighteen dollars," she stresses. "It's twenty dollars subtract eighteen dollars."

During the lesson, Rachel is constantly asking the students questions. "I'm always putting the thinking process back on the child," she says. "How do you want to write $23? How do you want to subtract $18?"

"Can I rewrite $23 as 10 + 10 + 3?" asks a student. "Of course!" says Rachel.

$23 = 10 + 10 + 3

"Now, think about what you want to do with $18," Rachel tells her. She then asks the class, "Raise your hand if you think you know what she's thinking." Many hands go up in the air. "What do you think she's going to do with $18?" Rachel asks one boy. "I think she'll subtract 10 from 10, and 8 from the other 10, and 0 from 3," he guesses.

Rachel writes this out:

$ 23 = 10 + 10 + 3
- $ 18 = -10 -8 -0
$ 5 = 0 + 2 + 3

"Is that what you were thinking?" she asks the first student. "No," she answers. "OH!" says Rachel, "What do you want to do?" "I want to subtract 9 from 10, and subtract another 9 from the other ten, and subtract 0 from 3." "Oh, wow," Rachel says, writing it on the overhead:

$ 23 = 10 + 10 + 3
- $ 18 = -9 -9 - 0
$ 5 = 1 + 1 + 3

"Notice that the left-hand side of the equation is always equal to the right-hand side," points out Rachel. She writes this on the overhead:

Left-Hand Side = Right-Hand Side
(LHS = RHS)

"Let's look at $23 - $18 again," she says.

LHS = RHS
$ 23 = 8 + 8 + 8 -1
- $ 18 = -6 -6 -6 -0
$ 5 = 2 + 2 + 2 -1

Rachel recalls teaching this lesson in a third grade class when a student raised her hand and announced, "I made a discovery." "What did you discover?" Rachel asked her. "You subtracted the same numbers five different ways and you got the same answer every time," the girl told Rachel. Other students nodded in agreement. "What did you think before?" Rachel asked. "Well..." the student paused, "before, I thought that if you subtracted it five different ways you would get five different answers."

Rachel is not surprised by this expectation. "It makes sense from a child's point of view," she reasons, "because we usually only teach them to subtract one way. If you subtract it one way, you get one answer." It may seem obvious to an adult, for example, that combining numbers is much different than combining letters of the alphabet. A child might not have made that distinction yet. A learner knows that when she puts four letters together, she can't mix them up five different ways and spell the same word each time. "We have to teach students that it doesn't make any difference how many ways they subtract a problem, they will always get the same answer."

Teachers often ask Rachel about one particular way of doing the problem 23 – 18. "They want to know if they can write out $23 as 20 + 3, and then subtract 10 and subtract 8," says Rachel. "I tell them to go for it." She writes out the problem:

$$\begin{array}{r} \$\ 23 = \ \ 20 + 3 \\ \underline{-\ \ \$\ 18 = -10 - 8} \end{array}$$

"Twenty dollars subtract $10 equals $10."

$$\begin{array}{r} \$\ 23 = \ \ 20 + 3 \\ \underline{-\ \$\ 18 = -10 - 8} \\ 10 \quad\ \ \end{array}$$

"Now I have $3 subtract $8. That's negative $5 dollars."

$$\begin{array}{r} \$\ 23 = \ \ 20 + 3 \\ \underline{-\ \$\ 18 = -10 - 8} \\ 10 - 5 \end{array}$$

"What is 10 subtract 5?" she asks. "Five! Our answer is still $5."

$$\begin{array}{r} \$\ 23 = \ \ 20 + 3 \\ \underline{-\ \$\ 18 = -10 - 8} \\ \$\ \ \ 5 = \ \ 10 - 5 \end{array}$$

"Here is the algebraic application for this," she adds.

$$\begin{array}{r} \text{Let } 10 = x \\ 2x + 3 \\ \underline{-1x - 8} \\ x - 5 \end{array}$$

or

$$2x + 3 - 1x - 8 = x - 5$$

"This is the generalization for the subtraction," says Rachel. "When I'm teaching algebra and we're looking at x – 5, if a student asks, 'When will I ever use this?' I go right back to 23 – 18," she says.

Practice, Practice, Practice

Students are given just one familiar problem as their homework assignment: $23 - $18.

"The students already know the answer is $5," says Rachel. "Their homework is to come in to class with 10 different ways to subtract it—and they are not allowed to use any of the ways we've done it in class." The assignment allows students to focus on learning the process rather than finding the answer, and then more opportunities to be creative.

It is important that students are given the time to do plenty of two-digit subtract two-digit problems before they move on. "Students need lots and lots of practice so that they understand the process."

When students are comfortable with what they are doing, Rachel moves on to the next step. "Let's subtract a two-digit number from a three-digit number," she says. She writes:

$$\begin{array}{r} \$\ 125 \\ \underline{-\ \ \$\ \ \ 99} \end{array}$$

"You have $125 in your wallet," Rachel tells the class. "Cover that number up with your finger for a minute." Students are now looking at just the $99 on their papers. "What's the easiest number to subtract 99 from?" asks Rachel. "100," the students tell her.

$$\begin{array}{r} \$\ 125 = +\ 100 + 25 \\ \underline{-\ \ \$\ \ \ 99 = -\ \ \ 99 -\ \ 0} \\ \$\ \ \ 26 = \ \ \ \ +\ 1 \ \ + 25 \end{array}$$

"Let's take another number," says Rachel. "Let's try subtracting three-digit from three-digit."

$$\begin{array}{r} \$\ 467 \\ \underline{-\ \ \$\ 289} \end{array}$$

"We want to break down $467 so that we can easily subtract $289 from it," Rachel reminds them. When she works on a problem in the classroom, Rachel often will stop and think about what she wants to do. "I don't rush," she says. "I model the thinking process for students."

"The easiest thing to subtract 200 from is 200," she says, writing as she thinks aloud. "and 89 is easy to subtract from 1800. That leaves 167...." She decides to break the problem down this way:

$ 467 =	+ 200	+ 100	+ 167	
- $ 289 =	- 200	- 89	- 0	
$ 178 =	0	+ 11	+ 167	

"If students can't subtract 89 from 100 without crossing out, they are not ready for subtracting three-digit from three-digit," Rachel notes. "If learners aren't getting this, go back and practice two-digit."

$ 467 =	+ 300	+ 100	+ 67
- $ 289 =	- 289	- 0	- 0
$ 178 =	11	+ 100	+ 67

Rachel models subtracting $467 - $289 in different ways for students:

$ 467 =	+ 200	+ 90	+ 10	+ 167
- $ 289 =	- 200	- 89	- 0	- 0
$ 178 =	0	+ 1	+ 10	+ 167

"Let's look at the Nonsense Method of subtracting 467 – 289":

Nonsense Method

$ 467 =	+ 300	+ 150	+ 17
- $ 289 =	- 200	- 80	- 9
$ 178 =	100	+ 70	+ 8

"In all the classrooms I've taught in, I have never found a student in any grade who can explain to me that they have changed $467 into 300 + 150 + 17," says Rachel.

"If I rewrote 467 as 300 + 150 + 17, I'd go ahead and subtract 289 from the 300," scoffs Rachel. "I wouldn't subtract 200 from 300, and 80 from 150, and then 9 from 17."

Number Sense Method

$ 467 =	+ 300	+ 150	+ 17
- $ 289 =	- 289		
$ 178 =	11	+ 150	+ 17

The Positives & Negatives of Subtraction

"By the time kids get into 5^{th} and 6^{th} grade, kids are so turned off to subtraction it's an educational immorality," says Rachel. "If I walk into a 5^{th}, 6^{th}, or 7^{th} grade classroom and say, 'Raise your hand if you love subtraction,' it's very seldom that anyone puts up their hand."

"How would you like to learn a way to subtract so that you never have to cross out again?" Rachel asks one class. The question earns actual cheers from the students.

"There are some things you have to understand," she tells them. "The first thing you should know is that under certain circumstances you CAN subtract bigger numbers from smaller numbers." She has them write this playful heading at the top of their notepaper:

YES! I CAN Subtract Bigger Numbers From Smaller Numbers Under Certain Circumstances!

"I'm going to teach you about shopping," says Rachel. "Let's say you have $8 in your wallet and y you subtract $5. How much money do you have left? Positive $3. I want you to put a positive sign in front of the 3 in your notes," she stresses. She also writes this out on the overhead projector.

"How did you get the $8 in the first place?" continues Rachel. "Someone must have given it to you to add to your wallet," she reasons. "Put an addition sign there because you added."

+ $ 8
- $ 5
+ $ 3

"But what if you added $5 to your wallet and you spent $8?" she asks. "How could that possibly happen?"

While the students ponder this question, Rachel takes out her own wallet. "Who would like to play with me here?" she asks. Picking a volunteer, she hands him five $1 bills. She keeps another $5 bill out for herself. "You and I are best friends and we're going to the shopping mall" she tells her helper. "How much do I have to spend?" she asks the class. "Five dollars," they tell her. "And how much does my friend have?" "Five dollars."

Ms. Math and her volunteer walk around

the classroom searching desks for something to purchase. "You and I are good friends," she reminds him. "We're best friends, right?" "Sure," he agrees.

Rachel stops at a desk and picks up a pencil. "Oh, this is great!" she says to the desk's proprietor. "How much is it?" Rachel then quickly leans over and whispers, "Tell me it's $8." The owner of the pencil announces the price with a smile, "Eight dollars."

Rachel looks at the money in her hand. "I only have $5!" she says in dismay. "I only have $5 and I just have to have that pencil!" She looks over at her helper with a grin. "Oh, my BEST friend! You're my very best friend! What would my best friend do for me?" she asks.

"He'll loan you $3," says one student. Rachel's new best friend reluctantly hands her three of his $1 bills and she in turn pays for the rather pricey pencil.

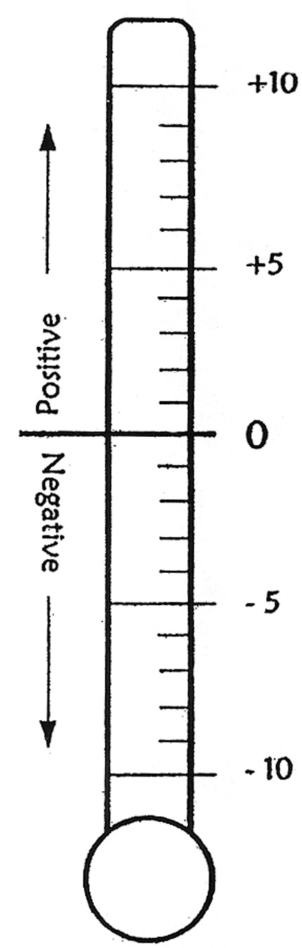
"Where am I financially?" asks Rachel. "You owe me $3," her friend pipes up. "Right," says Rachel. "I'm three dollars in debt. Mathematicians would say I have negative $3."

The class practices a few more of these problems. "The kids talk it out," says Rachel. "I have $9 and I subtract $5. What do I have left? Four dollars. If I have $5 and I subtract $9, I owe $4. I have negative four dollars."

Another way to demonstrate positive and negative numbers is with a thermometer. "This works really well with students who live in colder climates where the temperature gets below zero," says Rachel. "But even kids who live where it's warm can understand this idea. They've seen national weather reports, or maybe they've read Jack London's Call of the Wild, or The Long Winter by Laura Ingalls Wilder."

Rachel begins by drawing a thermometer on the overhead projector.

"I want you to draw your own thermometer on the side of your paper," she instructs students. "In the middle of the thermometer draw a line and write 0."



"Numbers above zero are called positive numbers," Rachel explains. "Numbers below zero are called negative numbers. If we count up five lines from zero, that is positive five. The temperature is 5° above zero."

"Now it's going to really get cold!" Rachel tells them. "Count down five lines from zero. That's negative five! Count down five more—now it's 10° below zero."

Next Rachel creates some subtraction problems. "Let's say the thermometer reads positive 8 degrees," she says. "And the temperature drops 5 degrees. What would the temperature be?" "Positive 3 degrees," students tell her.

$$\begin{array}{r} +\ 8^\circ \\ \underline{-\ 5^\circ} \\ +\ 3^\circ \end{array}$$

"Now let's do it the other way," says Rachel. "If we start at positive 5 degrees, and the temperature drops eight degrees, I end up at the negative 3 degrees."

$$\begin{array}{r} +\ 5^\circ \\ \underline{-\ 8^\circ} \\ -\ 3^\circ \end{array}$$

The class practices several more of these problems using the thermometer as to count down as needed. (Note: Teachers should make sure that students understand that they need to count zero when counting down.)

"This is all you need to know if you want to do algebraic subtraction," says Rachel. "You have to understand about numbers below zero.

Algebraic Subtraction

On the overhead projector Rachel brings back her old standby:

$$\begin{array}{r} \$\ 23 \\ \underline{-\ \$\ 18} \end{array}$$

"What is 20 subtract 10?" she asks. "Ten." "What is 3 subtract 8?" "Negative five," the students tell her, unfazed. "And what is 10 subtract 5?" "Five."

Rachel shows students two ways of writing out the problem in order to justify their work:

$$\begin{array}{r} +23 \\ \underline{-18} \end{array}$$

+10 - 5 = +5

$$\begin{array}{r} +23 \\ \underline{-18} \\ +10 \\ \underline{-\ 5} \\ +\ 5 \end{array} \qquad \begin{array}{l} \\ \\ = +20 - 10 \\ = +\ 3 -\ \ 8 \\ \\ \end{array}$$

They continue to practice with more two-digit subtract two-digit problems. "Have learners do lots of these," advises Rachel. "They are crazy about it."

$$\begin{array}{r} +36 \\ \underline{-29} \end{array}$$

$$\begin{array}{r} +36 \\ \underline{-29} \end{array}$$

+10 - 3 = +7

$$\begin{array}{r} +36 \\ \underline{-29} \\ +10 \\ \underline{-\ 3} \\ +\ 7 \end{array} \qquad \begin{array}{l} \\ \\ = 30 - 20 \\ = \ \ 6 -\ \ 9 \\ \\ \end{array}$$

"Look how you can do this in your head!" says Rachel. She writes 47 – 29 as the next equation.

"What's 40 subtract 20?" "Twenty." "Put 20 in your head," she instructs students. "What is 7 subtract 9?" "Negative two," students reply. "What's 20 subtract 2?" Rachel asks. "Eighteen," they answer. "When I teach them how to do subtraction in their head, they are so excited," she laughs.

$$\begin{array}{r} 47 \\ \underline{-29} \end{array}$$

+20 - 2 = 18

Expressing Yourself Algebraically

In an upper grade classroom, Rachel writes the following expression on the overhead projector:

$$3x^2 + 2x + 6 - 2x^2 - 8x - 7$$

"This is an algebraic expression, not an equation," she explains. "In an expression, you can let x equal anything you want. NO matter what number we choose for x, the answer will work out to x^2-6x-1." "Let x = 10" says Rachel, writing.

If we let x equal ten, this is what our problem is:

Let x=10

$$\begin{array}{r} +326 \\ \underline{-287} \end{array}$$

When you do this problem," she tells the class, "You'll probably get this first part correct. Three hundred subtract two hundred equals 100":

$$\begin{array}{r} +326 \\ \underline{-287} \end{array}$$

+100

"If you're going to make mistakes," she cautions, "You're going to call the next numbers two and eight, instead of twenty and eighty." She has students write in their notes:

This is TWENTY, not two.

$$\begin{array}{r} +326 \\ -287 \end{array}$$

This is EIGHTY, not eight.

"Twenty subtract eighty equals negative sixty":

$$\begin{array}{r} +326 \\ \underline{-287} \end{array}$$

+100 -60

"Six subtract seven is negative one":

$$\begin{array}{r} +326 \\ \underline{-287} \end{array}$$

+100 -60 -1

"One hundred subtract sixty is forty":

+326
-287

+100 -60 -1

+40

"Forty subtract one equals thirty-nine":

+326
-287

+100 -60 -1

+40 -1 = +39

"Our answer is positive thirty-nine."

Rachel writes another problem on the overhead:

+307
-148

"Teachers love giving kids problems with zeros in them," she reveals to students. "Look at where the zero is in this problem." Once again, she makes them write a reminder in their notes:

This is ZERO, not ten.

+307
-148

This is FORTY, not four.

Students may choose to justify their work in whichever manner they prefer:

+307
-148

+200 -40 -1

+160 -1 = +159

or

+307
-148

+200 = +300 -100
-40 = 0 -40
-1 = +7 -8

+159

"Students in the upper grades, and even teachers love doing problems this way," says Rachel. "Algebra teachers can show students how much easier subtraction is using this method."

As they practice, Rachel selects problems where there may be pitfalls:

+568
-392

+200 -30 +6

+170 +6 = +176

or

+568
-392

+200 = +500 -300
-30 = +60 -90
+6 = +8 -2

+176

"Eight subtract two is POSITIVE six," reminds Rachel. "Sometimes students will write negative six because they forget to think."

Even Teachers Need to Practice!

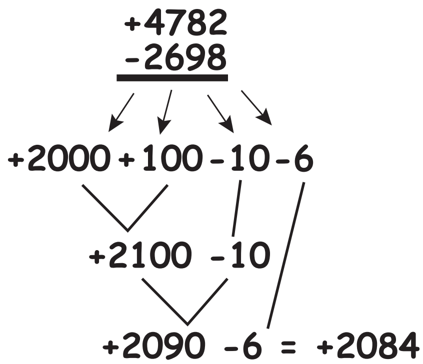
"It is important that teachers who want to teach this lesson don't walk into the class and do this by the seat of their pants," advises Rachel. "I have had teachers tell me that they tried this lesson and it didn't work with their kids," she recalls. "I ask them, 'How much planning did you do?' and there is a long pause, which means they did not prepare."

Teachers should plan out numbers in advance and practice problems before presenting the lesson to their students. "You have to practice it," says Rachel. "If you do not prepare, do not blame the mathematics—the mathematics in this lesson is true, because I can prove it algebraically."

Rachel has standard numbers that she uses in the classroom, as we saw with $23 - $18. "I choose those numbers because they are reasonable," she explains. "It's a twenty-dollar bill, and three one-dollar bills—if you subtract $18 you can give the clerk the whole amount in your wallet, but I know that most adults would use the twenty."

"Practice until you are comfortable," Rachel urges. "You can't teach beyond your own level of competence. You cannot field questions if you are not prepared for them. Unless you are confident with the thinking process, it won't work for you in the classroom."

Try 4-digit subtract 4-digit!

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+4782
-2698
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+2000 =  +4000 - 2000
  +100 =   +700 - 600
   -10 =   +80 - 90
    -6 =   +2 - 8
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+2084
```