

Pigs in the Pen!

One-to-One Correspondence & Parts of a Whole

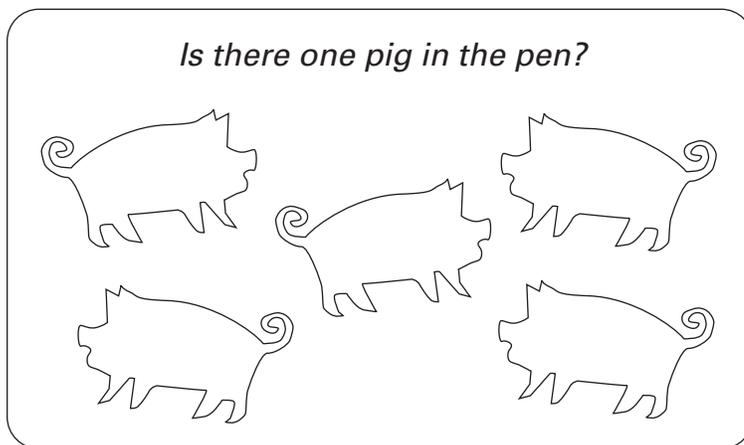
Shortly after Rachel McAnallen left high school teaching and began working in elementary schools she came to a realization. "As adults we take so many things for granted," she says, "We assume that kids understand a concept when they really don't have it." It was a group of second graders in Northern Vermont that helped Rachel make that important discovery, while playing a money game. As a result, Rachel developed the lesson on one-to-one correspondence and parts of the whole featured in this issue.

"In this money game that I had been teaching, when the kids had five pennies they were to trade it for one nickel," Rachel explains. "So kids were always taking their pennies to the bank, and they would say, 'Five pennies trade for one nickel.'" The game continued, and then, during the next part of the activity, Rachel noticed something puzzling. "All the kids had six pennies in front of them, and they were just sitting there," she remembers. "Now, the rule was that if they had five pennies, they were always to trade them for one nickel."

"Can you make a trade?" she asked them. "No," they answered. "Why not?" Rachel asked. "We don't have five pennies," they told her.

"That hit me," says Rachel. "I thought, 'Well, if they've got *six*—don't they know they have five?'"

She asked them, "Do you have one penny in your wallet?" "No." "Two pennies?" "No." "Three pennies?" "No." "Four pennies?" "No." "Five pennies?" "No." "Six pennies?" "Yes."



"They did not know that within the 6 pennies they also had 5 pennies," says Rachel. "I had been teaching for 27 years, and it never occurred to me that they would not have this concept. They didn't have it because we didn't teach it."

At that point, Rachel was not prepared to teach the lesson. In order to continue the game, she simply asked the students to count out five of the pennies from their wallet and put them in their hands. "Now, do you have five pennies in your hand?" she asked.

"Yes." "Take those to the bank and get your nickel."

"As long as those five pennies were sitting in their wallet as part of six, they did not have five pennies," explains Rachel. "There is an old saying, 'When the lesson needs to be learned, the teacher will arrive.' That day those little second graders were my teachers. Thank goodness I had enough sense to know that I was the one that needed to learn the lesson."

Pigs in the Pen

Topics involved: one-to-one correspondence, number sense, parts of the whole, addition, subtraction, groups, fractions.

Materials: large piece of string, 20 pigs (or other shape) in varying colors. Variation: overhead projector, transparent shapes in varying colors.

Type of activity: group.

Relation to NCTM Standards: using the language of mathematics, enabling students to understand numbers, ways of representing numbers, & relationships among numbers.

Grades: K and up (some sections of this lesson can be applied to a variety of age groups).

Getting Started:

Rachel begins by creating a large circle on the floor with the piece of string, and then she and the students sit around it. Once they are seated, Rachel throws “stuff” into the circle. Although any shapes or items can be used with this lesson, in this case she has chosen to use colorful foam pigs.

Rachel tosses in the five pigs to start, making sure that two of the pigs are the same color: 2 yellow pigs, 1 purple pig, 1 blue pig, and 1 grey pig.

“I ask one of the kids to go into the pen and count the pigs,” says Rachel. She tells them to pick up the pigs and put them in their hand, and as they count them, she has them speak. “I want them to say ‘one pig, two pigs, three pigs, four pigs, five pigs,’” she says.

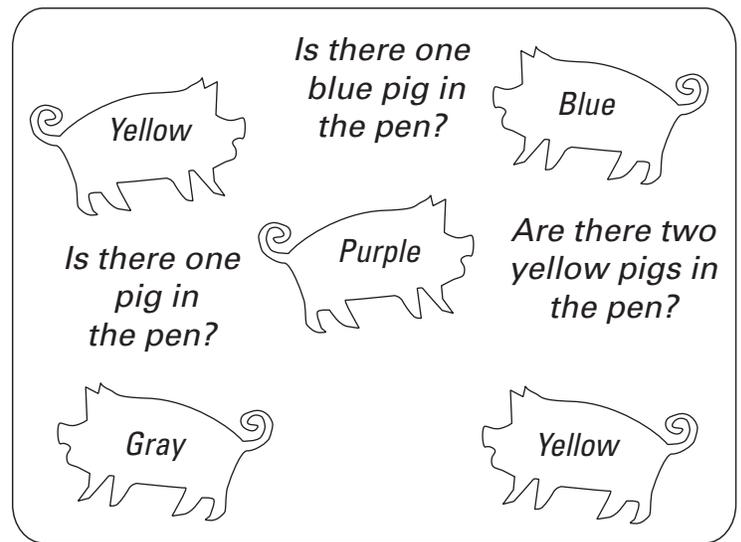
“How can you subtract 3 from 5, if you don’t know that within 5 there is 3?”

Rachel has two or three learners go in and count those same five pigs, asking each of them to start with a different pig. If one child begins with the purple pig, the next students must each start counting with a different color.

“Now they have the idea that it doesn’t matter what pig you start with,” says Rachel, “and they know that the name of the purple pig isn’t ‘One.’”

Parts of the Whole

“When we teach kids to count, we are always concentrating on the whole. We emphasize the total, but we never teach the parts,” says Rachel. “Then we



wonder why the children have trouble with subtraction and fractions. How can you subtract 3 from 5 if you don’t know that within 5 there is 3?” she reasons.

“When kids say ‘One-two-three-four-five,’ we take it for granted that they know they have 3,” says Rachel. “Wrong! They do not know!”

Using the five little foam pigs, Rachel demonstrates the same discovery she made years ago in that 2nd grade classroom. “Teachers are usually shocked when they see this,” Rachel says with a smile. “It has nothing to do with how I ask the question—the students always give the same answers.”

“Is there one pig in the pen?” she asks the class. “No,” they reply.

Next Rachel asks, “Is there one blue pig in the pen?” “Yes,” answers the class.

“So,” she asks again, “is there one pig in the pen?” “No.”

“I just continually ask questions,” notes Rachel, “because I’m not teaching the lesson yet.”

She asks, “Are there two pigs in the pen?” “No,” the students respond.

“Are there two yellow pigs in the pen?” “Yes.” “So, are there two pigs in the pen?” “No.”

“They are focusing on the total, and I want them to focus on the parts as well as the total,” explains Rachel. “Because the total

is the sum of the parts—you can't have the total unless you have the parts."

"Come here a second," Rachel says to one student. "Is there one pig in the pen?" "No," he answers. "Go into the pen and get me one pig," instructs Rachel. He hands her the pig. "What did you just hand me?" Rachel asks. "One pig," answers the student. Rachel holds the pig high up in the air. "How many pigs is this?" she asks the class. "One," they reply. Rachel puts the pig on her head and smiles. "Now how many pigs is this?" "One." She hands the pig back to the student, "Here is one pig," she tells him. "Put it back in the pen, but keep your finger on it—how many pigs is your finger on?" "One," he answers. "Is it in the pen?" she asks. "Yes." "Is there one pig in the pen?" "No."

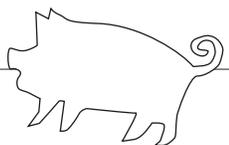
"I have to go through that routine at least three times," Rachel says, "until finally one kid in the group will say, 'Oh, I get it!'"

"I have the child explain it to the rest of the class," she says. The student goes into the pen and holds up each pig, saying, "This is one pig, and this is one pig. There are 5 one pigs in the pen."

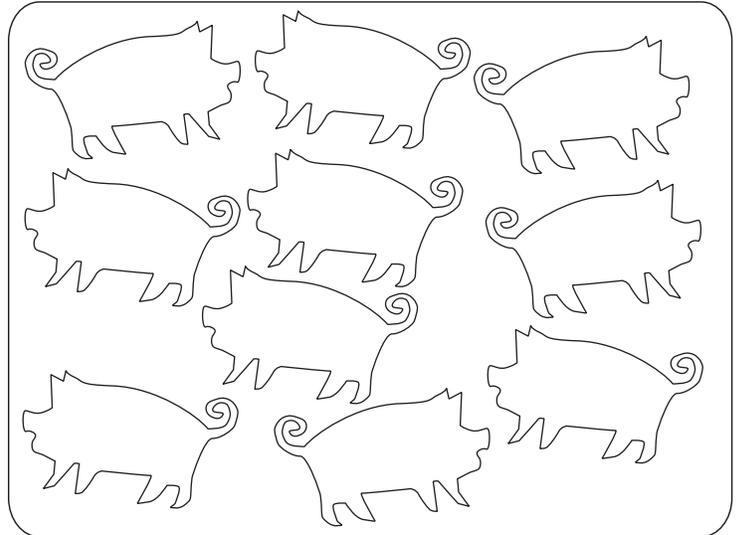
Rachel continues, "Then I will ask the class, 'Are there two pigs in the pen?' And they all say 'Yes.'"

"Someone go show me two pigs" she directs. A child goes in and holds up two pigs. "Who is looking at a different two pigs?" asks Rachel. "How many different groups of two do you see?"

Half a pig in the pen.



"I see ten $\frac{1}{2}$ pigs in the pen, but I don't see $10\frac{1}{2}$ pigs."



Fractions

Next, Rachel removes four of the pigs, leaving one pig remaining in the circle. "Is there half a pig in the pen?" she asks. Rachel asks one of the students to show her half a pig. She is delighted when they go in and fold the foam pig in half. "This is half, and this is half," they say, showing her each folded side.

"So what do you have in your hand?" she asks. "Two half pigs," they tell her.

"At this point, the kids get excited," says Rachel. As she throws two more pigs into the pen, she tells them. "Oh, kindergartners or first graders can't get this! Raise your hand if you know how many half pigs are here total."

"Six. Six half pigs," a student tells her.

"I have kindergartners doing division of fractions," Rachel laughs. "If I was to write this down: $3/1 \times \frac{1}{2}$...most of us would say, 'Oh, my gosh, in 6th grade did we invert the first one or the second one...the second one...okay...the answer is 6. Six what? Six half pigs.'" But the little ones say, 'I have 3 pigs, and I divide each one in half. What do I get? Six half pigs.' It's all right here in these little pigs in the pen."

How Many Pigs Do You See?

Rachel now throws in seven more pigs, for a total of 10 pigs in the pen. Going around the circle, each learner is asked

to say a mathematical sentence about what they see in the pen. For instance: "I see one pig in the pen but I don't see 100 pigs in the pen." "I see three pigs in the pen, but I don't see 12 pigs in the pen" and so on.

"If you want someone to know what something is, it's also really important that you know what it isn't," says Rachel. "To really understand a dog, you have to understand that a cat is not a dog."

"They pick out how many they want to see," explains Rachel. "Every time a child says a mathematical sentence I have the whole class repeat it. This means they must pay attention to their classmates—they can't be daydreaming, because they have to repeat it."

As they go around the circle, the answers will be diverse as the learners themselves. One child may say, "I see 10 half pigs but I don't see 10 $\frac{1}{2}$ pigs. Another student may say, "I see 8 pigs but I don't see infinity pigs in the pen."

"One of the nice things about this lesson is that it is open-ended," says Rachel. "A child will see at their own developmental level. Some kids won't have gotten the concept that there are parts there yet. Those learners will say, "I see 10 pigs in the pen, but I don't see 11." Rachel makes a mental note of students who are seeing 10 pigs, because it may be an indicator that they do not yet have the concept. "That's okay," she says. "They may get it tomorrow, or they may get it

in ten minutes from now, after hearing and repeating the sentences of their classmates."

If a child can't think of a sentence (and they are allowed to repeat one that has already been spoken) Rachel will ask, "Do you need more think time?" If they do, Rachel tells them, "Okay, when you're ready, raise your hand and let me know."

Every learner gets the opportunity to talk and be creative. "You cannot do this on a worksheet," says Rachel. "This is not worksheet driven."

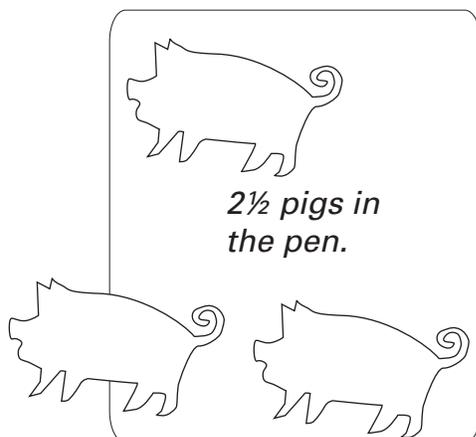
Pig Addition

Once the class has mastered the concept of parts to a whole, Rachel asks them to go into the pen and create an addition problem using the ten pigs. "What

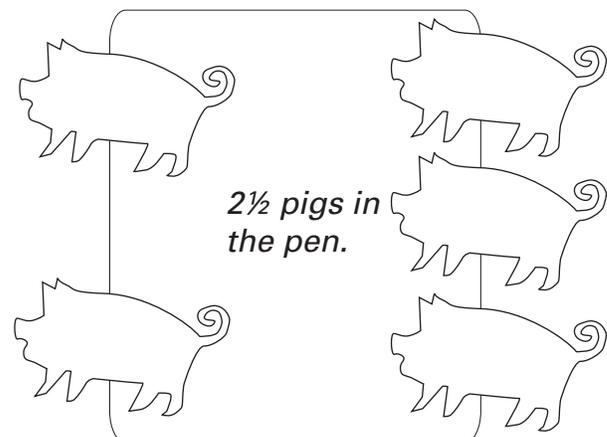
we are doing is saying that addition is putting parts together to make a whole," explains Rachel. "The first couple of kids will make a math sentence using just two facts," she observes. For example: "If I have 8 pigs in the pen and I add 2 pigs, I have 10 pigs." Or "If I have 1 pig and I add 9 pigs, I have 10 pigs." The students continue to listen and repeat one another's math sentences.

Eventually, Rachel knows that a student will come along and say, "If I add 2 pigs to 2 pigs, to 2 pigs, to 2 pigs, to 2 pigs, I'll have 10 pigs." "So I wait," she says knowingly. "I wait until they start to do more advanced things. When the first kid breaks out of the box, we teachers get big smiles on our faces—but I don't

"These pigs in this pen are different for everybody."



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force them to come out of the box, I let them come out on their own," maintains Rachel. "There are an infinite number of combinations to make 10, and soon the kids are being so creative. It's beautiful." She shakes her head, "Usually the only math facts we teach for 10 are $5 + 5$, $4 + 6$, $3 + 7$, $2 + 8$, and $1 + 9$."

Once again, every child will create an addition problem that is at his or her own developmental level. "Some kids are at the '5 pigs plus 5 pigs equals 10 pigs' stage," reasons Rachel. "That is where they are at that moment of their life. Then there is the kid that is at the '9 $\frac{1}{2}$ pigs plus $\frac{1}{2}$ a pig is 10 pigs' stage. The catchword these days is differentiation—and these pigs sitting on this pen are different for everybody."

Pig Subtraction

"After we've done addition, I talk about subtraction," says Rachel. "In other words, I've got 10 pigs in the pen and I'm going to take one pig out—adding is putting in, subtracting is taking out." Rachel asks learners of all ages to become familiar with using the proper mathematical terms. "If a child can say 'tyrannosaurus rex' when they are 5 years old, they can say the word subtract," she says matter-of-factly. "So they might as well learn the correct vocabulary."

Again the students go around in the circle, speaking, listening, and repeating. "At this point, they just tell me how many they can subtract," says Rachel. "They don't tell me how many they have left." For example: "I can subtract 0 pigs" or "I can subtract $\frac{1}{2}$ a pig."

There will often be a student who has not yet grasped the concept and announces, "I can subtract 11 pigs." In that case, Rachel will instruct them to go in and take 11 pigs out of the pen and count them. "By doing this," she says, "they will see that they cannot take out 11 pigs when they only have 10 pigs in the pen."

"The next step," says Rachel, "is to have students tell me what they can subtract

from 10, and what they will have left in the pen."

"I can subtract 2 pigs," says one learner, picking up two pigs and taking them out of the pen. She can then touch and count the remaining 8 pigs. "If I subtract 2 pigs, I have 8 pigs left in the pen."

As they go around in the circle, one student inevitably pipes up, "I can subtract $\frac{1}{2}$ a pig."

"I want you to think about that," Rachel tells them. "Think about how many pigs you will have left in the pen if you subtract $\frac{1}{2}$ a pig." That student instinctively lays the pig right on the line and counts, "1, 2, 3, 4, 5, 6, 7, 8, 9, and $\frac{1}{2}$. 9 $\frac{1}{2}$ pigs left."

"I'm not worried that the half pig isn't exactly equal," notes Rachel. "True, the pig isn't symmetrical, but at that point I'm not concerned. I can take half of four and get two, but if I take a half of a cookie, I can never get it exactly in half. It's the concept of half that's important."

Once that particular child breaks out into subtracting a half, other children will follow. Another learner may say, "I can subtract 2 $\frac{1}{2}$ pigs." "They might say that they have 8 $\frac{1}{2}$ pigs left," cautions Rachel. "I will tell them to come into the pen and subtract 2 $\frac{1}{2}$ pigs. They'll take out 2 and $\frac{1}{2}$, and then they are able to count 7 $\frac{1}{2}$ pigs left in the pen."

One day a child surprised Rachel by using this approach to make 2 $\frac{1}{2}$ pigs: he placed five pigs—half-in and half-out—along the string, subtracting one half from each of the five pigs. "A teacher might be tempted to correct the student once he lays down the third pig," says Rachel, "but it is important to wait."

For the last subtraction activity, students make up the whole problem. "How many pigs do you want to put in the pen?" Rachel asks. "Twelve," answers the student. Rachel tosses twelve pigs into the pen. "How many pigs do you want to subtract?" she asks. "Four." "How many do you have left?" "Eight."

"Now I make them say the whole sentence," says Rachel, "They must speak in sentences."

"If I have 12 pigs and I subtract 4 pigs, I will have 8 pigs left."

Everyone gets a chance to create his or her own problem. If they hesitate, they are allowed to tell Rachel, "Come back to me later, I need more think time."

"That allows them to gain security with it," says Rachel. "And they let me know when they are ready."

Groups (Pigs in Packs)

"Now we can look at groups," Rachel says. "If there are 10 pigs in the pen, how many groups of 2 do we have?"

"Five groups of 2." "Let's make groups of 3. How many groups of 3 do we have?"

"Three groups of 3 with 1 left over."

We call the leftover a remainder," says Rachel. "How many groups of 1?" "Ten."

"How many groups of 4?" "Two groups of 4 with a remainder of 2."

Rachel tosses in 2 more pigs. "How many groups of 4 now?" "Three groups of 4 with no remainder."

"The wonderful thing about these pigs is that I can teach addition, subtraction, multiplication, division, fractions, percents, everything," says Rachel. "And the kids can see the pigs, and touch them, and they can talk and hear the other kids talking. There is a lot of mathematics going on within this little pig lesson!"