ALGEBRA I

mini interventions

QUADRATICS, EXPONENTIAL FUNCTIONS, AND ALGEBRAIC METHODS

REGION 13
EDUCATION SERVICE CENTER

ACKNOWLEDGMENTS

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GETTING STARTED

These explanations of the new state math standards are designed to help you understand what the standards mean and how the models of teaching math help students understand mathematics more deeply. Others may interpret the standards differently and may have different ideas for how to teach them. It is our hope that this deconstruction of the Texas Essential Knowledge and Skills (TEKS) for mathematics makes teaching math more rigorous, more fun, and a little less confusing.

NAVIGATING THE DOCUMENT

This is an interactive PDF. Go to the Table of Contents and click on the Algebra I TEKS you want to view.

WRITING AND SOLVING LINEAR FUNCTIONS, EQUATIONS, AND INEQUALITIES	
A.3	**Linear functions, equations, and inequalities. The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:**
A.3E	determine the effects on the graph of the parent function f(x) = x when f(x) is replaced by af(x), f(x) + d, f(x – c), f (bx) for specific values of a, b, c and d. **RC2, Supporting Standard**

Within the document, there are also links in the Background Information section and the Foundation Section that connect to the cooresponding Click-On TEKS examples. You could also use the tools in Acrobat and type in the page number provided.

SE	Description	Reporting Category	Standard	Page #
8.5C	contrast bivariate sets of data that suggest a linear relationship with bivariate sets of data that do not suggest a linear relationship from a graphical representation.	4	Supporting	221

SE	Description	Reporting Category	Standard	Page #
6.6C	represent a given situation using verbal descriptions, tables, graphs, and equations in the form y = kx or y = x + b.	2	Readiness	190

Another function within Adobe Acrobat is the abilty to Search. You run searches to find specific items in your PDFs by using either the Search window or the Find toolbar (CTRL+F, or CMD+F). In either case, Acrobat searches the PDF body text, layers, and form fields. You can also include bookmarks and comments in the search. Only the Find toolbar includes a Replace With option.

TABLE OF CONTENTS

NUMBER AND ALGEBRAIC METHODS	
A.10	**The student applies the mathematical process standards and algebraic methods to rewrite algebraic expressions into equivalent forms. The student is expected to:**
A.10C	determine the quotient of a polynomial of degree one and polynomial of degree two when divided by a polynomial of degree one and polynomial of degree two when the degree of the divisor does not exceed the degree of the dividend. **RC1, Supporting Standard**
A.11	**The student applies the mathematical process standards and algebraic methods to rewrite algebraic expressions into equivalent forms. The student is expected to:**
A.11A	simplify numerical radical expressions involving square roots. **RC1, Supporting Standard**
A.12	**The student applies the mathematical process standards and algebraic methods to write, solve, analyze, and evaluate equations, relations, functions. The student is expected to:**
A.12C	identify terms of arithmetic and geometric sequences when the sequences are given in function form using recursive processes. **RC1, Readiness Standard**

*A.3E & A.7C are transformations of linear and quadratic functions. A.3E is a building block for A.7C, so they are bundled together.

HOW EACH CHAPTER IS ORGANIZED

These explanations of the new state math standards are designed to help you understand what the standards mean and how the models of teaching math help students understand mathematics more deeply. Others may interpret the standards differently and may have different ideas for how to teach them. It is our hope that this deconstruction of the Texas Essential Knowledge and Skills (TEKS) for mathematics makes teaching math more rigorous, more fun, and a little less confusing.

The goal of these mini interventions is to not only help guide teachers with their understanding of the new TEKS but to provide background information for skills the student might have missed from previous grades and what to do about it. Also included are some intervention skills if your student is having a particularly difficult time learning. Specific examples and/or activities may be adjusted over time as more information becomes available from the state.

There are 17 SE components total. Each SE was selected because it was a "brand new" SE to the 2012 Algebra 1 TEKS. Each SE contains these sections:

1. **Background Information**
    - Introduction to the new SE
    - Scaffolding TEKS to help teachers know when and how the prerequisite skills were taught through references from Click-On TEKS
    - Techniques for spiraling concepts, which will include examples
2. **Pre-Assessment**
    - Formative questions that dig deep into the scaffolding TEKS that lead to that particular new Algebra 1 SE (each question will include an answer key and reference to a prerequisite or scaffolding SE that may be used to help students who did not evidence mastery of each question)
3. **Spiraling**
    - Prerequisite Algebra 1 SEs to spiral, which will include specificity and the big idea for spiraling
    - Examples of spiraling SEs
4. **Teaching**
    - Important ideas
    - Vocabulary and formulas
    - Descriptions and examples of the concepts with step-by-step directions
    - Instructional activities
5. **Foundation**
    - SE skills that may be explored for Tier 2 & 3 Intervention (these are usually "concrete" or "basic" ideas that students have not mastered in prior grades)
    - A description of the SE indicating grade level, reporting category, and whether it was a readiness or supporting standard
    - References from Click-On TEKS
6. **Post-Assessment**
    - Questions that help evaluate whether students have mastered the SE (answer keys will be included)

HOW EACH CHAPTER IS ORGANIZED

		WHAT IS IN EACH SECTION?	HOW DO I USE IT?
PREPARE FOR INSTRUCTION	Background Information	What should students already know? This section includes brief explanations of standards from previous grade levels that students should have mastered through Click-On TEKS examples.	Use this to refresh, build, and extend your knowledge. Remember that K-8 has been implementing new standards. Are you familiar with the changes and how they impact your instruction?
PREPARE FOR INSTRUCTION	Pre-Assessment	Where are student strengths and needs? This section includes questions that help identify prior knowledge including student strengths and gaps.	Use this information to determine what content you need to spiral into instruction. Do a quick re-teach of previous grade concepts if necessary.
PREPARE FOR INSTRUCTION	Spiraling	What are some Algebra I standards I will need to weave into instruction along with the focus standards? This section includes a brief description of additional Algebra I standards that will help students get to mastery of the targeted standard.	Bundle these Algebra I standards with the targeted new standard to support student learning. Use specificity to understand more about the standard. Use examples to help you understand the scope of the standard. Use examples as part of your instruction.
TEACH	Teaching	What is the focus of the new standard? This section includes important ideas, vocabulary, formulas, and examples.	Use important ideas to hone in on the core concepts of the standard. Use vocabulary and formulas to create word walls, sentence stems, and anchor charts. Use examples during instruction with students.
RE-TEACH	Foundation	What standards from previous grade levels do I need to address to close student gaps? This section includes examples and brief explanations of previous grade level standards from Click-On TEKS which give concrete support and points of entry for students with gaps.	Use specific examples from previous grade levels and strategies aligned to these concrete standards.
ASSESS	Post-Assessment	How will I know students have mastered the targeted new standard? This section includes questions to determine whether the student has mastered the targeted new standard.	Cut questions into cards and use for formative assessment. Use questions as an end-of-unit summative assessment. Use results to make continuing instructional grouping and approach decisions.

BASIC STRUCTURE OF THE TEKS

The Texas Essential Knowledge and Skills (TEKS) consists of four parts.

Part A: The Introduction

The state standards for Algebra 1 begin with an introduction. The introduction gives an overview of the focal areas for each grade and provides general information about numerical fluency and processing skills. Although the introduction has not been reprinted in this book, information from the introduction has been included in the explanations of the TEKS, where appropriate.

Part B: Strands

The standards are broken into groups or categories called Strands. The TEKS for Algebra 1 are divided into five strands:

Mathematical Process Standards: This strand contains the process standards for mathematics, which are the same from kindergarten through pre-calculus. The process standards are the ways that students acquire math content through the use of models and tools, communication, problem solving, reasoning and analysis, and making connections. These standards should be woven consistently throughout the content strands (2–6). The dual-coded questions on STAAR will be coded with a content standard and a process standard.

Number and Algebraic Methods

Linear Functions, Equations, and Inequalities

Quadratic Functions and Equations

Exponential Functions and Equations

Example

A.1 Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

(A) apply mathematics to problems arising in everyday life, society, and the workplace.

Part C: Knowledge and Skills Statements

Immediately following the strand is the Knowledge and Skills (K&S) statement. It provides the context for the student expectations that follow it. Numbering: The first letter is the grade level. The second number is the Knowledge and Skills number. The K&S statement shown is from Algebra 1.

Part D: Student Expectations

Immediately following each Knowledge and Skills statement is a list of Student Expectations (SEs). The letters, such as (A), refer to what students are expected to do with regard to a particular Knowledge and Skills statement. We often refer to this example as A.1A. [Grade Level Algebra, Knowledge and Skills statement (1), Student Expectation (A)]

MATHEMATICAL PROCESS STANDARDS

The mathematical processes in the revised math state standards are the same from kindergarten to pre-calculus. Why? Because these are the processes of doing mathematics. These are the ways that mathematicians work every day.

Algebra 1 always had underlying process standards, but they were not clearly delineated until now.
These are the processes that students will use to understand the new math content and show how they know it.

For Algebra 1, these are the processes that will have dual codes on the STAAR EOC with the content standards. This means that students need to look at the content standards through the lens of the process standards. These standards will be dual-coded with the content standards and may include any or all of the process standards.

The following are the process standards:

- A.1A—Apply mathematics to problems arising in everyday life, society, and the workplace.
- A.1B—Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution.
- A.1C—Select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems.
- A.1D—Communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.
- A.1E—Create and use representations to organize, record, and communicate mathematical ideas.
- A.1F—Analyze mathematical relationships to connect and communicate mathematical ideas.
- A.1G—Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication

A.3E and A.7C — 1. BACKGROUND INFORMATION

What's in This Section?

- A description of A.3E and A.7C
- A table that lists all of the SEs that should be considered in preparation of the lesson for the targeted standard
- Specificity from the TEA Side by Side documents for the new K-8 standards
- References to the sections in the appendix that will provide a more thorough description as well as examples for each of the background standards referenced here

The following SE is new to the 2012 TEKS for Algebra 1. The specificity comes from the Algebra 1 Side by Side provided by TEA.

A.3 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:

STUDENT EXPECTATION

A.3E determine the effects on the graph of the parent function $f(x) = x$ when $f(x)$ is replaced by $af(x)$, $f(x) + d$, $f(x - c)$, $f(bx)$ for specific values of a, b, c and d.

RC2, Supporting Standard

SPECIFICITY

The revised SE extends the current SE by determining the effects of parameter changes on the parent function with horizontal transformations.

A.7 Quadratic functions, and equations. The student applies the mathematical process standards when using graphs of quadratic functions and their related transformations to represent in multiple ways and determine, with and without technology, the solutions to equations. The student is expected to:

STUDENT EXPECTATION

A.7C determine the effects on the graph of the parent function $f(x) = x^2$ when $f(x)$ is replaced by $af(x)$, $f(x) + d$, $f(x - c)$, $f(bx)$ for specific values of a, b, c and d. **RC4, Readiness Standard**

SPECIFICITY

The effect of changes in a has been replaced by $af(x)$. The use of function notation and transformations builds the foundation for transformations of other functions in Algebra II. The revised SE continues to address the effects of changes of a and c, and transitions students to examining quadratic functions from the transformational approach, that is used later in Algebra II with other functions. The revised SE is an extension of the current SE related to transformations of quadratic functions. The expectation is that

1. BACKGROUND INFORMATION

students analyze quadratic functions from a transformational perspective, comparing the effects af(x) and f(bx) in the different representations of the function. Graphs may include horizontal and vertical shifts, horizontal and vertical stretches and compressions, and reflections. Transformations with quadratic functions are not explicitly addressed in Algebra II TEKS. Algebra II will extend transformations to other functions.

As a high school teacher, you probably know your TEKS but may want to refresh, build, and extend your knowledge of the vertically aligned K-8 standards that lead into the targeted standard that you are planning to teach to your students.

SE	Description	Reporting Category	Standard	Page #
8.4B	[illegible]tional relationships, interpreting the unit rate as the slope of the line that models the relationship.	2	Readiness	214
8.10B	differentiate between transformations that preserve congruence and those that do not.	3	Supporting	237
8.10C	explain the effect of translations, reflections over the x- or y-axis, and rotations limited to 90°, 180°, 270°, and 360° as applied to two-dimensional shapes on a coordinate plane using an algebraic representation.	3	Readiness	237

8.4B graph proportional relationships, interpreting the unit rate as the slope of the line that models the relationship. **RC 2, Readiness Standard**

SPECIFICITY

These SEs comes from A.6A “Develop the concept of slope as rate of change and determine slopes from graphs, tables, and algebraic representations.” These SEs add specificity regarding the concept of slope through the lens of proportionality. Teaching the formula for slope is not the intent of 8.4A. The intent of 8.4B is to connect the unit rate of a proportional relationship to the slope of the line that models the relationship. Students will not be asked about parameter changes as they would be in Algebra.

8.10B differentiate between transformations that preserve congruence and those that do not. **RC3, Supporting Standard**

1. BACKGROUND INFORMATION

SPECIFICITY

Dilations are limited to positive, rational scale factors. Students may differentiate between the transformations using multiple representations, including algebraic representations. Dilations that result in a reduction or enlargement do not preserve congruence. A scale factor is applied to the coordinate values of the original figure and noted symbolically as $(x,y) \rightarrow (2x,2y)$. This is in contrast to those transformations that preserve congruence such as a reflection in $(x,y) \rightarrow (-x,y)$ or a translation in $(x,y) \rightarrow (x+2,y+1)$.

8.10C explain the effect of translations, reflections over the x- or y-axis, and rotations limited to 90°, 180°, 270°, and 360° as applied to two-dimensional shapes on a coordinate plane using an algebraic representation. **RC 3, Readiness Standard**

SPECIFICITY

This SE provides specificity to the current SE. Students could have been given a rule to apply to the vertices of a geometric figure with the current SE such as $(x,y) \rightarrow (x+2,y+3)$ in order to graph a translation or $(x,y) \rightarrow (-x,y)$ in order to graph a reflection. Students are not expected to explain the effects of composites of transformations. This SE adds rotations about the origin on a coordinate plane. This SE provides specificity as to which rotations students should know. Students would be expected to know that $(x,y) \rightarrow (-x, y)$ represents a 90° rotation. Students would be expected to know that a 180° rotation may have the same result as the composition of a translation and a reflection.

2. PRE-ASSESSMENT

These pre-assessment questions are designed to be given before spiraling and teaching the new Algebra 1 SE in this section. All of the questions may be given as a traditional warm-up or as a pre-assessment where the questions are given individually to each student in advance of teaching the new Algebra 1 SE. This will help identify the prior knowledge that students may or may not have mastered.

The questions are formatted to be cut into cards and used in various formative assessment techniques. Region 13's The Teacher Toolkit, www.theteachertoolkit.com, has excellent opening activities that demonstrate fun and engaging ways to formatively assess students' knowledge. These activities include step-by-step directions as well as classroom videos that will help everyone, from the experienced to novice teacher.

A recommended pre-assessment idea from The Teacher Toolkit is Classroom Mingle:

http://www.theteachertoolkit.com/index.php/tool/classroom-mingle

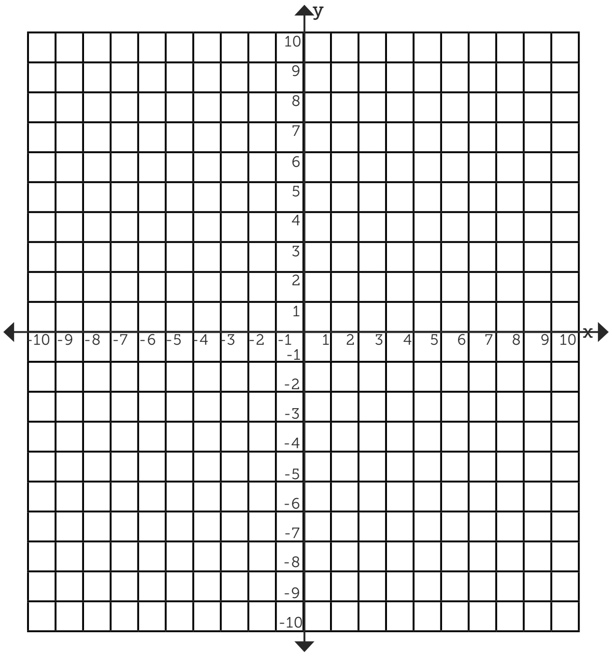
What's in This Section?

- Pre-Assessment Formative Assessment Questions
- A spreadsheet to monitor student mastery on the pre- and post-assessment questions is provided on page 244. This may be used for a more in-depth analysis of each student's performance on the pre-assessment.
- A key to the pre-assessment questions is provided as well as a reference to the SE that the question is assessing.

Pre-Assessment Formative Questions

Question A

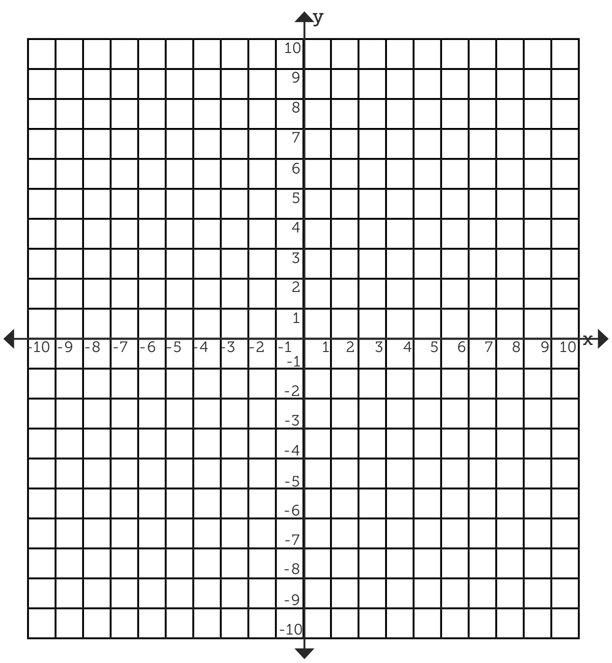
Graph the line $y = 2x - 3$.



For Question A, name the slope and the y-intercept.

Question B

Graph the line $3x - 4y = -4$

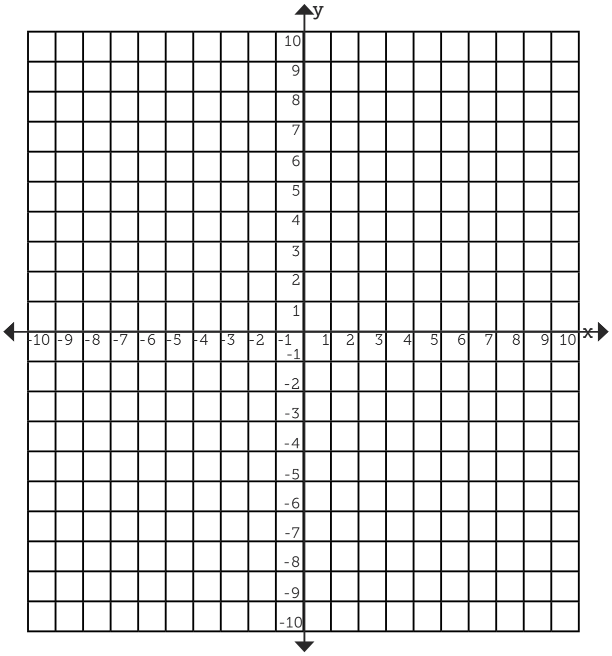


For question B, name the slope and the y-intercept.

2. PRE-ASSESSMENT

Question C

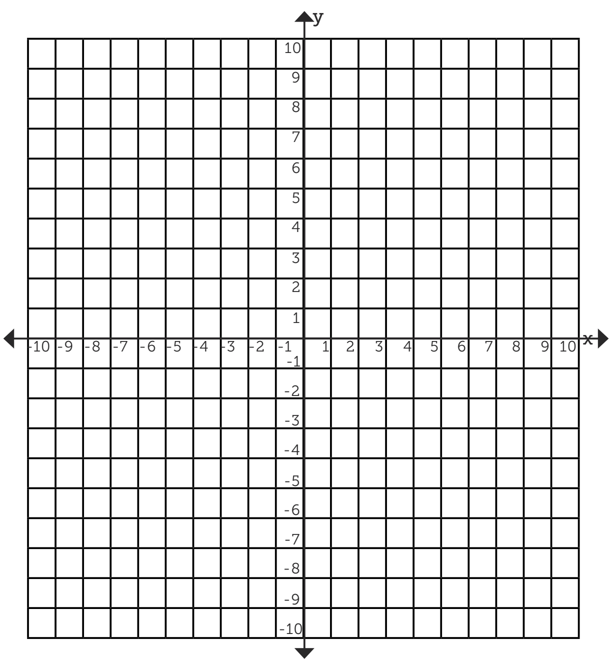
Graph the quadratic $y = -3x^2 + 6x - 1$.



For question C, name the x-intercepts, the y-intercept, the vertex, and the axis of symmetry.

Question D

Graph the quadratic $y = (x - 2)^2 + 3$

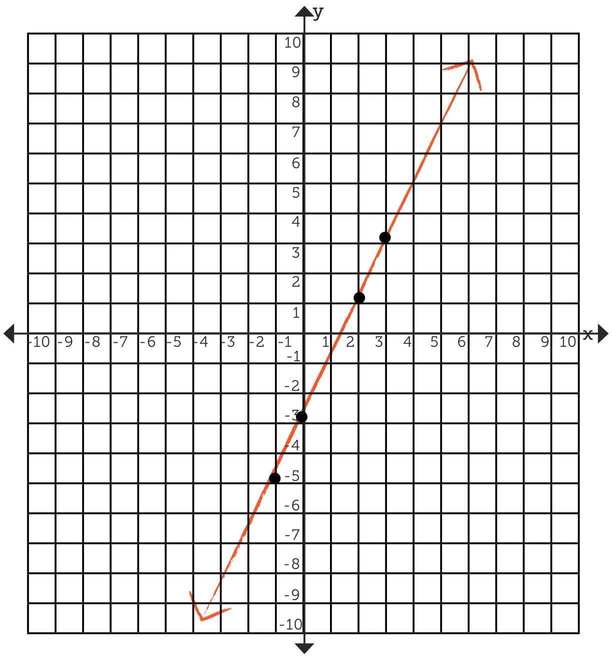


For question D, name the x-intercepts, the y-intercept, the vertex, and the axis of symmetry.

Pre-Assessment Formative Questions

Question A

Graph the line y = 2x – 3. SE: 8.5B



For Question A, name the slope and the y-intercept.

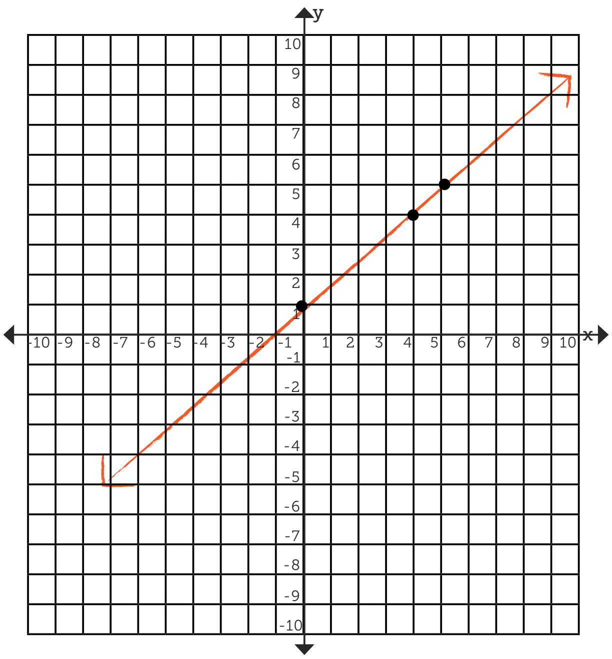
Answer: Slope = 2

y-intercept (0, -3)

SE: 8.4C

Question B

Graph the line 3x – 4y = -4 SE: 8.5B



For question B, name the slope and the y-intercept.

Answer: Slope = $\frac{3}{4}$

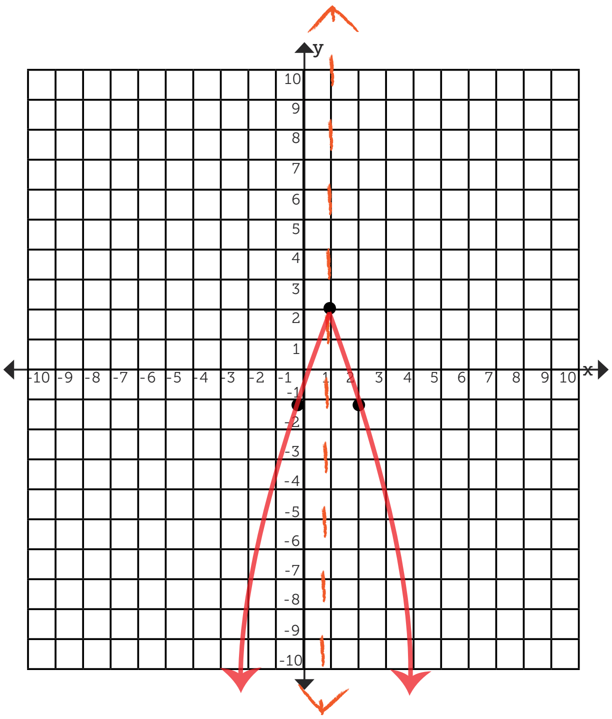
y-intercept (0, 1)

SE: 8.4C

Question C

Graph the quadratic $y = -3x^2 + 6x - 1$.

SE: A.7A



For question C, name the x-intercepts, the y-intercept, the vertex, and the axis of symmetry.

Answer: x-intercepts: (.184, 0) and (1.82, 0)

y-intercept: (0, -1)

vertex: (1, 2)

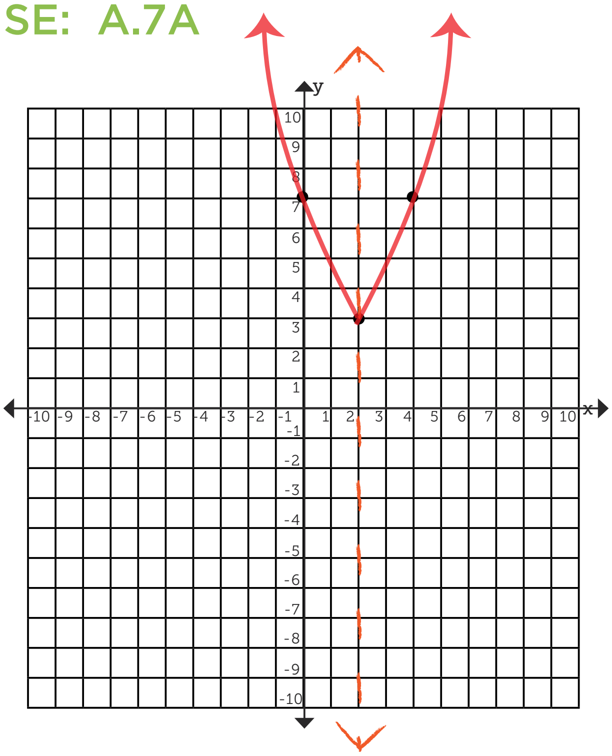
axis of symmetry: x = 1

SE: A.7A

Question D

Graph the quadratic $y = (x - 2)^2 + 3$

SE: A.7A



For question D, name the x-intercepts, the y-intercept, the vertex, and the axis of symmetry.

Answer: x-intercepts: none

y-intercept: (0, 7)

vertex: (2, 3)

axis of symmetry: x = 2

SE: A.7A

3. SPIRALING

These Algebra 1 SEs have been carefully chosen to spiral into your lesson plan where needed to ensure that students are prepared for the new SE in each module. When you make your lesson plans for the targeted standard, plan on weaving in, or spiraling, these SEs prior to and during lessons to reinforce students' prior learning in your class this year. Students need to experience prior learning often in order to build a strong foundation in mathematics.

What's in This Section?

- Table listing SEs that are recommended for spiraling
- The written description of the SE as stated in the 2012 TEKS
- Specificity from the Side By Side documents provided by TEA
- The big idea for spiraling these SEs
- In some cases, examples are provided to go with each spiraling SE

Status	SE	Description	Reporting Category	Standard
CONTINUING SE	A.3A	determine the slope of a line given a table of values, a graph, two points on the line, and an equation written in various forms including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m(x - x_1)$.	2	Supporting
CONTINUING SE	A.3C	graph linear functions on the coordinate plane and identify key features including x-intercept, y-intercept, zeros, and slope in mathematical and real-world problems.	2	Readiness
NEW SE	A.7A	graph quadratic functions on the coordinate plane and use the graph to identify key attributes, if possible, including x-intercept, y-intercept, zeros, maximum value, minimum values, vertex, and the equation of the axis of symmetry.	4	Readiness
CONTINUING SE	A.7B	describe the relationship between the linear factors of quadratic expressions and the zeros of their associated quadratic functions	4	Supporting

What Does Status Stand For?

NEW SE: A targeted standard that is detailed in a different section. Examples may be viewed in that section and are not included here.

CONTINUING SE: Not new to Algebra 1. Remember to closely examine the SE and the specificity that is provided, as subtle changes may be detailed.

A.3A determine the slope of a line given a table of values, a graph, two points on the line, and an equation written in various forms including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m(x - x_1)$. **RC2, Supporting Standard**

SPECIFICITY

Slope is introduced in grade 8 through the use of proportionality using similar triangles, making connections between slope and proportional relationships, and determining slope from tables and graphs in 8.4A, 8.4B, and 8.4C. Specificity has been added through identified specific forms of linear functions. Although specific forms are provided, the expectation is that students be able to manipulate any linear equation to identify key characteristics, such as slope and y-intercept.

Big Idea for Spiraling: Students will be familiar with determining characteristics of a line given an equation and a graph. This will help them determine similarities and differences of two given lines.

A.3C graph linear functions on the coordinate plane and identify key features including x-intercept, y-intercept, zeros, and slope in mathematical and real-world problems. **RC2, Readiness Standard**

SPECIFICITY

When paired with mathematical process standards A.1D and A.1E, the expectation is that students graph linear functions from each of the multiple representations, including tables and algebraic representations, and to connect the key features to the multiple representations.

Big Idea for Spiraling: Students will be familiar with graphing lines and finding key components.

A.7A graph quadratic functions on the coordinate plane and use the graph to identify key attributes, if possible, including x-intercept, y-intercept, zeros, maximum value, minimum values, vertex, and the equation of the axis of symmetry. **RC4, Readiness Standard**

SPECIFICITY

Specificity has been added by listing some of the attributes of a quadratic function. When mathematical process standards A.1E and A.1F are paired wiw zth other content standards, students are expected to analyze graphs of quadratic functions to draw conclusions. The graphing of a quadratic function may occur as a result of a transformation. Students could be given points or attributes to define a quadratic function and be asked to identify additional attributes of that quadratic function.

3. SPIRALING

Big Idea for Spiraling: Students will be familiar with graphing quadratic equations and determining key components of the quadratic.

A.7B describe the relationship between the linear factors of quadratic expressions and the zeros of their associated quadratic functions. RC4, Supporting Standard

SPECIFICITY

Academic language now includes "linear factors." When paired with mathematical process standards A.1D, A.1E, and A.1F, the expectation is that students use multiple representations to describe the connections between the linear factors and the zeros of the associated quadratic functions.

Big Idea for Spiraling: Students will be able to determine the zeros of a quadratic and will be able to use it to compare two or more quadratic functions..

4. TEACHING

This section contains the core concepts of the targeted standard. Explore this section to broaden and refresh your knowledge of the targeted standard. This information will help in developing your lesson plan.

What's in This Section?

- A description of the targeted SE
- Targeted vocabulary
- Formulas
- Examples that help broaden and refresh an Algebra 1 teacher's knowledge about the targeted standard
- Ideas and activities that will enrich your lesson plans for the targeted standard
- Suggested prerequisite skills

A.3 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:

STUDENT EXPECTATION

A.3E determine the effects on the graph of the parent function $f(x) = x$ when $f(x)$ is replaced by $af(x)$, $f(x) + d$, $f(x - c)$, $f(bx)$ for specific values of a, b, c and d.

RC2, Supporting Standard

A.7 Quadratic functions, and equations. The student applies the mathematical process standards when using graphs of quadratic functions and their related transformations to represent in multiple ways and determine, with and without technology, the solutions to equations. The student is expected to:

STUDENT EXPECTATION

A.7C determine the effects on the graph of the parent function $f(x) = x^2$ when $f(x)$ is replaced by $af(x)$, $f(x) + d$, $f(x - c)$, $f(bx)$ for specific values of a, b, c and d. **RC4, Readiness Standard**

IMPORTANT IDEAS

Students need to understand the characteristics of linear equations and quadratic equations. Graphing as well as understanding the components of each will be necessary to accomplish A.3E and A.7C.

VOCABULARY

Transformation: an operation that maps or moves the points of a figure in a plane.

FORMULAS

Parent function of a linear equations: $f(x) = x$

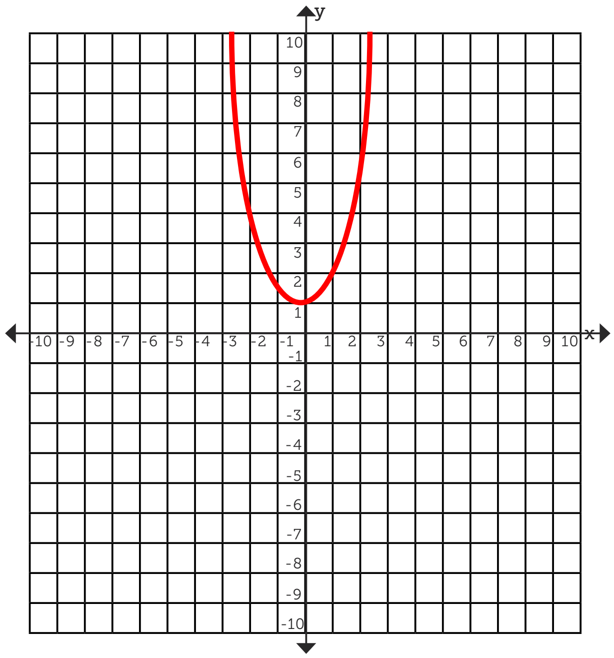
Parent function of a quadratic equation: $f(x) = x^2$

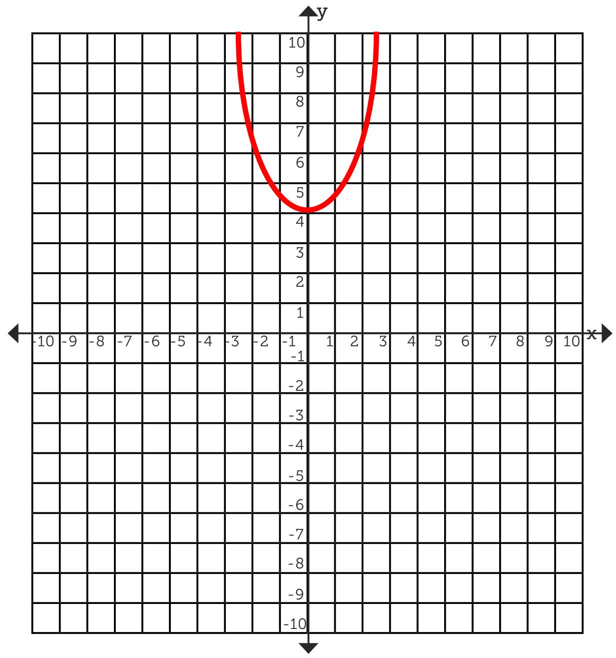
EXAMPLES

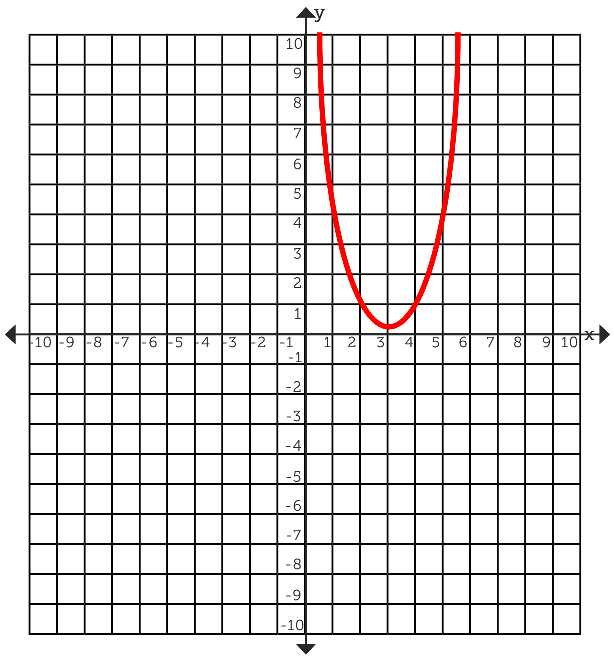
Student Stations Sample Activity

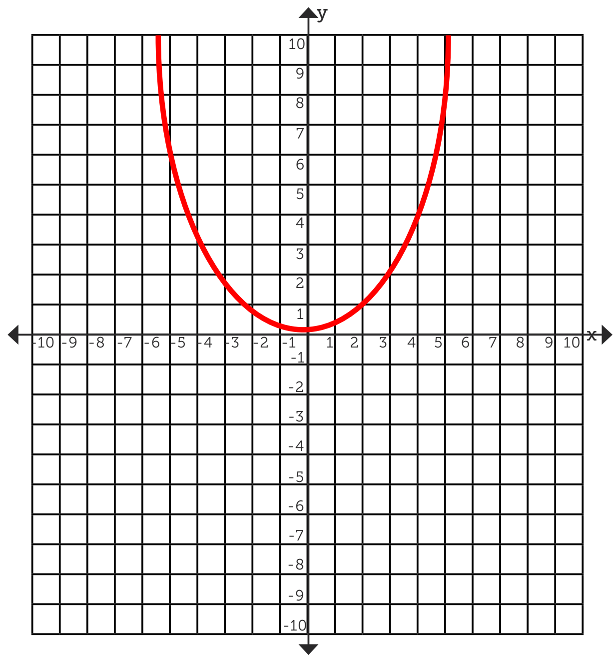
Have students determine equation of parabola shown and similarities/differences to the parent function.

When students have finished, have the students share with their groups anything they discovered. Have students share with whole class.









5. FOUNDATION

The foundation section contains SEs from prior grade levels and provides ideas and activities to help students who have gaps and/or need a more concrete approach. These are students who need to master the basics to understand the targeted standard and may benefit from instruction using manipulatives. High school intervention can vary greatly compared to what might be found in elementary and middle school. The ideas and strategies here may help you provide intervention after school or during a remediation class, as well as provide the Response to Intervention (RtI) or intervention teacher with fresh ideas to help students who are struggling.

What's in This Section?

- Examples and brief explanations of previous grade-level standards
- References to the sections in the appendix that will provide a more thorough description as well as examples for each of the background standards referenced here

Students in Response to Intervention, Tiers 2 or 3, may have problems with the following concepts. These suggestions are not all inclusive. Students in Tiers 2 & 3 will need a more customized intervention. The suggested SEs are provided with a brief description.

Interventionists may start with graphing points, graphing lines and then graphing quadratics.

SE	Description	Reporting Category	Standard	Page #
5.8A	describe the key attributes of the coordinate plane, including perpendicular number [illegible] here the intersection (origin) of the two lines coincides with zero on each number line and the given point (0, 0); the x-coordinate, the first number in an ordered pair, indicates movement parallel to the x-axis starting at the origin, and the y-coordinate, the second number, indicates movement parallel to the y-axis starting at the origin.	3	Supporting	174
6.11A	graph points in all four quadrants using ordered pairs of rational numbers.	3	Readiness	203

6. POST-ASSESSMENT

These post-assessment questions are designed to be given after teaching the new Algebra 1 SE in this section. The whole page may be given as a traditional quiz after teaching the new Algebra 1 SE. This will help identify the knowledge that students may or may not have mastered.

The questions are formatted to be cut into cards and used in various formative assessment techniques. Region 13's The Teacher Toolkit, www.theteachertoolkit.com/, has excellent closing activities that demonstrate fun and engaging ways to formatively assess students' knowledge. These activities include step-by-step directions as well as classroom videos that will help the experienced as well as the novice teacher.

A recommended post-assessment idea from The Teacher Toolkit is Quiz Quiz Trade:

http://www.theteachertoolkit.com/index.php/tool/quiz-quiz-trade

What's in This Section?

- Post-assessment formative assessment questions
- A spreadsheet to monitor student mastery on the pre- and post-assessment questions is provided on page 244. This may be used for a more in-depth analysis of each student's performance on the pre-assessment.
- A key to the post-assessment questions is provided as well as a reference to the SE that the question is assessing.

Post-Assessment Formative Questions	
Describe how the graph of $y = x - 3$ is similar and different from $y = x$.	Describe how the graph of $y = 2x$ is similar and different from $y = x$.
Describe how the graph of $y = (x - 1)^2$ is similar and different from $y = x^2$.	Describe how the graph of $y = \frac{1}{2}x^2$ is similar and different from $y = x^2$.
What transformation from the graph of the parent function would be caused by $y = \frac{3}{4}x$?	What transformation from the graph of the parent function would be caused by $y = x+1$?
What transformation from the graph of the parent function would be caused by $y = x^2 + 2$?	What transformation from the graph of the parent function would be caused by $y = (2x)^2$?

Post-Assessment Formative Questions	
Describe how the graph of $y = x - 3$ is similar and different from $y = x$. Answer: Same slope, Line shifted down 3 units SE: A3E	Describe how the graph of $y = 2x$ is similar and different from $y = x$. Answer: Same y-intercept, steeper slope or stretched by a factor of 2 SE: A3E
Describe how the graph of $y = (x - 1)^2$ is similar and different from $y = x^2$. Answer: Same width, Parabola shifted one unit to the right. SE: A.7C	Describe how the graph of $y = \frac{1}{2}x^2$ is similar and different from $y = x^2$. Answer: Same vertex, Wider parabola or compressed by a factor of 1/2 SE: A.7C
What transformation from the graph of the parent function would be caused by $y = \frac{3}{4}x$? Answer: Same y-intercept, less steep slope SE: A3E	What transformation from the graph of the parent function would be caused by $y = x+1$? Answer: Same slope, Line shifted up 1 unit SE: A3E
What transformation from the graph of the parent function would be caused by $y = x^2 + 2$? Answer: Same width, Parabola shifted up 2 units SE: A.7C	What transformation from the graph of the parent function would be caused by $y = (2x)^2$? Answer: Same vertex, Narrower parabola or stretched by a factor of 4 since the coefficient of 2 is squared SE: A.7C

5.8 Geometry and measurement. The student applies mathematical process standards to identify locations on a coordinate plate. The student is expected to:

5.8A describe the key attributes of the coordinate plane, including perpendicular number lines (axes) where the intersection (origin) of the two lines coincides with zero on each number line and the given point (0,0); the x-coordinate, the first number in an ordered pair, indicates movement parallel to the x-axis starting at the origin; and the y-coordinate, the second number, indicates movement parallel to the y-axis starting at the origin. (RC3, Supporting Standard)

5.8A introduces students to the structure of the coordinate plane. The verb in the Student Expectation (SE) is *describe*. Students in 5th grade need to be able to describe the key attributes of the coordinate plane. Students are only expected to graph points in the first quadrant, that is, where the x-coordinate and y-coordinate are both positive numbers.

Example/Activity

The *coordinate plane* shown below is made up of two number lines that are perpendicular to each other and that cross at the zeros. The number lines are called *axes*. The horizontal axis is the *x-axis*. The vertical axis is the *y-axis*.

An *ordered pair* is made up of two *coordinates*. The first number is called the *x-coordinate* and the second is called the *y-coordinate*. The ordered pair makes a point that is graphed on the coordinate plane.

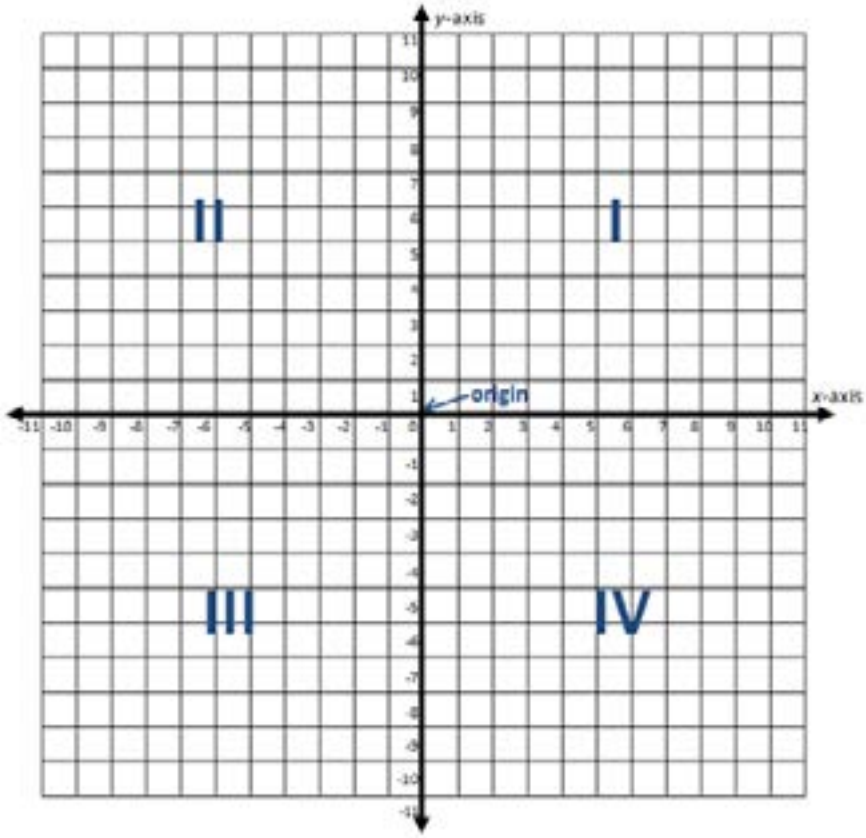
$$(x, y)$$

The x-coordinate corresponds to the x-axis. Since the x-axis is horizontal, the x-coordinate indicates horizontal movement which is parallel to the x-axis. The y-coordinate corresponds to the y-axis. Since the y-axis is vertical, the y-coordinate indicates vertical movement which is parallel to the y-axis. These are sometimes called *Cartesian* (car-tée-shun) *coordinates* after a mathematician named Descartes who created/discovered/invented them.

One special point where the two axes (number lines) meet is called the *origin*. It has the ordered pair (0,0).

Continued on next page

Although it is not part of 5th grade expectations, the information in this paragraph is to deepen your own content knowledge. Points, made up of coordinates, are graphed on a *coordinate plane*. The coordinate plane is divided into *quadrants* which are named with Roman numerals beginning with quadrant I that contains points whose coordinates are both positive. Quadrant II has a negative x coordinate and a positive y coordinate. Points that lie on the x or y axis are not part of any quadrant. The quadrants (I, II, III, and IV) are named going counter-clockwise making a "C" for DesCartes.



6.11 Measurement and data. The student applies mathematical process standards to use coordinate geometry to identify locations on a plane. The student is expected to graph points in all four quadrants using ordered pairs of rational numbers.

(RC3, Readiness Standard)

For 6.11, students will graph points on the coordinate plane whose coordinates are rational numbers (integers and positive and negative fractions and decimals). Although students begin learning the academic language for graphing points in 5th grade, a review of this vocabulary appears below.

Example/Activity

The *coordinate plane* shown below is made up of two number lines that cross at 0. When the number lines cross, they are called *axes*. The horizontal axis is the *x-axis*. The vertical axis is the *y-axis*.

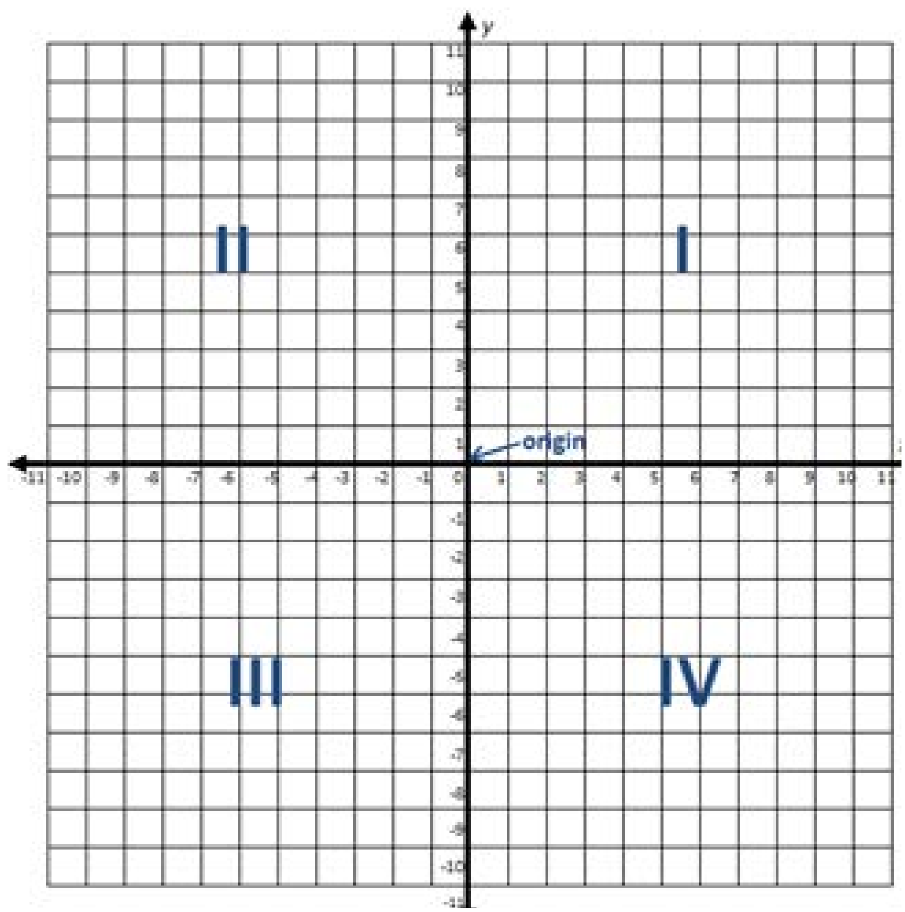
An *ordered pair* is made up of two *coordinates*. The first number is called the *x-coordinate* and the second is called the *y-coordinate*. The ordered pair makes a *point* that is graphed on the coordinate plane.

(x, y)

These are sometimes called *Cartesian* (car-téa-shun) *coordinates,* after a mathematician named Descartes who created/discovered/invented them.

One special point where the two axes meet is called the *origin*. It has the ordered pair (0,0).

Points, made up of coordinates, are graphed on a *coordinate plane*. The coordinate plane is divided into *quadrants,* which are named with Roman numerals, beginning with the quadrant that contains points whose coordinates are both positive. They are named going counter clockwise, making a "C" for Descartes.



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The scale for both axes on the coordinate plane above is 1 unit. However, the scale can be set at whatever is convenient for the problem. If the scale is something other than 1, the numbers should be written onto the graph to make the scale apparent.

To graph an ordered pair, first examine the two coordinates. Students may benefit from using two fingers, one for each coordinate. The right finger slides across the horizontal axis until it reaches the number that is the x-coordinate. The left finger slides up or down the vertical axis until it reaches the number that is the y-coordinate. Then they slide along the intersecting line to meet at a point. The point can be named with the ordered pair itself or can be named with a capital letter.

8.4 Proportionality. The student applies mathematical process standards to explain proportional and non-proportional relationships involving slope. The student is expected to:

8.4B graph proportional relationships, interpreting the unit rate as the slope of the line that models the relationship. (RC2, Readiness Standard)

This Student Expectation (SE) continues to build understanding of slope by relating it to a familiar concept to 8th grade students—*unit rate*. A unit rate is a special type of ratio where two quantities are being compared and the second quantity is 1. Since the slope of the line is presented as a unit rate, the slopes of the lines for this SE should be integers so that they can be written with a denominator of 1. Also, to keep the relationship proportional, the equations need to be written in the form y = mx.

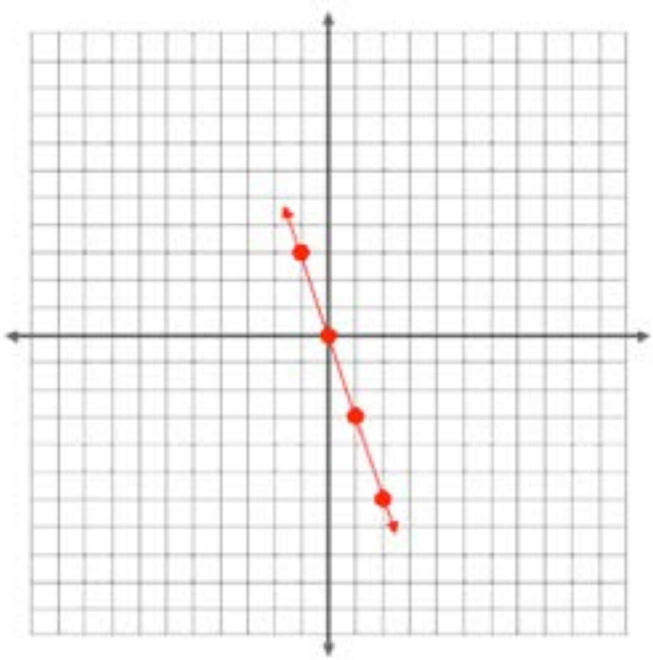
Although the SE does not state this, students will need to create a table from the equation and then graph the equation using the ordered pairs in the table.

Example/Activity

The following example shows a line that has a unit rate for the slope, $\frac{^-3}{1}$. The unit rate is visible in the table in the yellow highlighted line of the table below. In the graph, the unit rate is visible because, as you move from left to right on the graph, one grid line at a time, there is another point on the line.

y = ⁻3x

x	y
0	0
1	⁻3
2	⁻6
⁻1	3



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In 8.4A, students work with a formula for slope. This is quite abstract for even high school Algebra students. This SE approaches slope using something that is much more concrete for students. It uses the graph.

This SE takes a more practical and useful, in the writer's opinion, approach to finding slope on the graph, as the unit rate (or slope) is clearly visible in the diagram. Described below is the process for finding the slope by looking at a graphed line. It has been targeted at seeing a slope that is also a unit rate, but can be used to find the slope of any line that has been graphed.

Find the slope using a graph:

1. Follow the line from left to right and choose a point that has coordinates that are integers. (Don't choose the point that is furthest to the right if you want to see the unit rate immediately when you find the slope.)
2. Continue to follow the line to the next point on the line that has integer coordinates.
3. Go back to the first point and count the lines on the grid vertically to get from one point to the other. This number is the numerator of the slope. In terms of unit rate, it is the amount per one item (e.g., cost per gallon of milk).
4. Go back to the first point. Count the line(s) going across to get to the second point. If the points are consecutive, the number across should be 1, which shows the unit rate. (If you skipped a point when you chose the points, the denominator will be more than 1 and the slope will have to be simplified to see the unit rate.)
5. Make a fraction with the vertical change and horizontal change. This is the slope and it is also a unit rate. It shows the rise or fall of the line for every horizontal unit.

8.4 Proportionality. The student applies mathematical process standards to explain proportional and non-proportional relationships involving slope. The student is expected to:

8.4C use data from a table or graph to determine the rate of change or slope and y-intercept in mathematical and real-world problems. (RC2, Readiness Standard)

8.5 Proportionality. The student applies mathematical process standards to use proportional and non-proportional relationships to develop foundational concepts of functions. The student is expected to:

8.5F distinguish between proportional and non-proportional situations using tables, graphs, and equations in the form $y = kx$ or $y = mx + b$, where $b \neq 0$.
(RC2, Supporting Standard)

In 8.4C, students focus on two attributes of a line: the slope and the y-intercept. They learn to find the slope and the y-intercept from tables or graphs. The problems that students will use are real-world or mathematical problems, not straight calculation.

Why should the problems be based in a problem situation? The problem situation gives meaning to the slope and the y-intercept. The problem situation aids in understanding rather than being a word problem at the end of the section or chapter that gets skipped. Instruction should begin with the problem and understanding the problem. Then the meaning of the slope and y-intercept will be clear.

In 8.5F, students tell the differences and learn to identify a proportional or non-proportional situation from a table, graph, and equation. The discussion here will focus on tables and graphs. Equations are discussed with 8.5A and B.

An informal definition of slope is the rate of change from one point to another point on the line. It is a numerical representation of how much the line rises from one point to another and how far it moves across the grid from one point to another. Notice how much more meaning that has than the old $\frac{\text{rise}}{\text{run}}$, which has meaning for math teachers but not so much meaning for students.

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8.10 Two-dimensional shapes. The student applies mathematical process standards to develop transformational geometry concepts. The student is expected to:

8.10A generalize the properties of orientation and congruence of rotations, reflections, translations, and dilations of two-dimensional shapes on a coordinate plane. (RC3, Supporting Standard)

8.10B differentiate between transformations that preserve congruence and those that do not. (RC3, Supporting Standard)

8.10C explain the effect of translations, reflections over the x- and y-axis and rotations limited to 90°, 180°, 270°, and 360° as applied to two-dimensional shapes on a coordinate place using algebraic representation. (RC3, Readiness Standard)

Eighth grade is students' introduction to transformational geometry. While with the 2008 TEKS students began learning about rotations, reflections, translations, and dilations in 5th grade, these concepts have been moved to 8th grade with the 2012 TEKS.

8.3A, B, and C introduce dilations and their properties. 8.10A involves generalizing properties of orientation and congruence. 8.10B focuses on which of the transformations preserve congruence and which do not. In 8.10C students explain the effects of the transformations related to the coordinate grid. These three Student Expectations (SEs) will be explained together since they are so closely related, and any discussion about one will overlap with discussion about another.

Continued on next page

Example/Activity

<table>
<tr><th>Transformation</th><th>Diagrams and Discussion</th></tr>
<tr><td>

Rotations

8.10A
In 8th grade, students will rotate a figure around the origin. The figures can rotate clockwise or counterclockwise.
A rotation and its image are congruent to each other and their orientation is the same. To determine orientation, compare the pre-image to the image. As shown on the example to the right, the vertices for the pre-image are read clockwise ABCD. The vertices for the image are also read clockwise A'B'C'D', thus the orientation is preserved.

8.10B
The image of a rotation is congruent to the original figure.

8.10C
Rotations can be represented algebraically.
90° clockwise – (x, y) → (y, -x)
90° counterclockwise – (x, y) → (-y, x)
270° clockwise – (x, y) → (-y, x)
270° counterclockwise – (x, y) → (y, -x)
Notice that a 90° clockwise rotation is the same as a 270° counterclockwise rotation.
Also, a 90° counterclockwise rotation is same as a 270° clockwise rotation.

180° clockwise – (x, y) → (-x, -y)
180° counterclockwise – (x, y) → (-x, -y)
Notice that a 180° clockwise rotation is the same as a 180° counterclockwise rotation. Also, a 180° counterclockwise rotation is the same as a 180° clockwise rotation.

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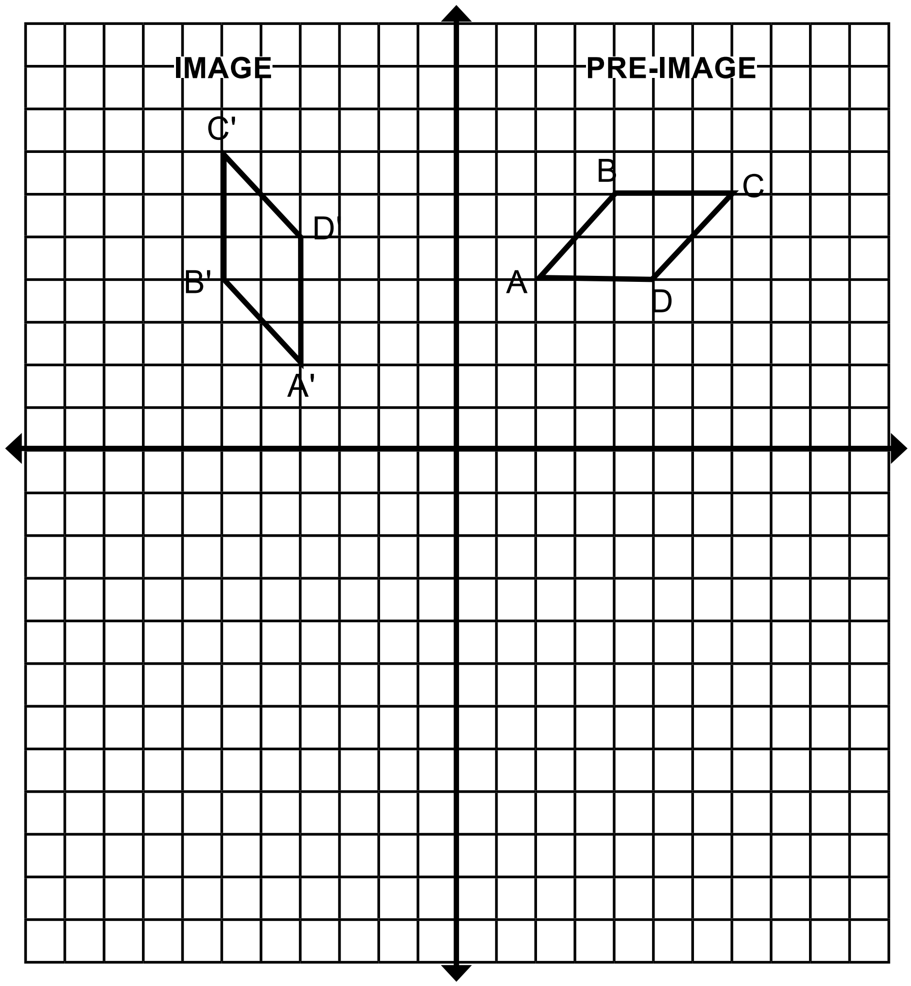

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Reflections

8.10A

A reflection and its figure are mirror images of each other. In other words, the figures look exactly the same, but one is backwards from the other. As shown on the example to the right, the vertices for the pre-image are read clockwise ABCD. The vertices for the image are read counter clockwise A'B'C'D'. When read in a counter clockwise motion, orientation is not preserved. Reflections are the only transformation where orientation is not preserved.

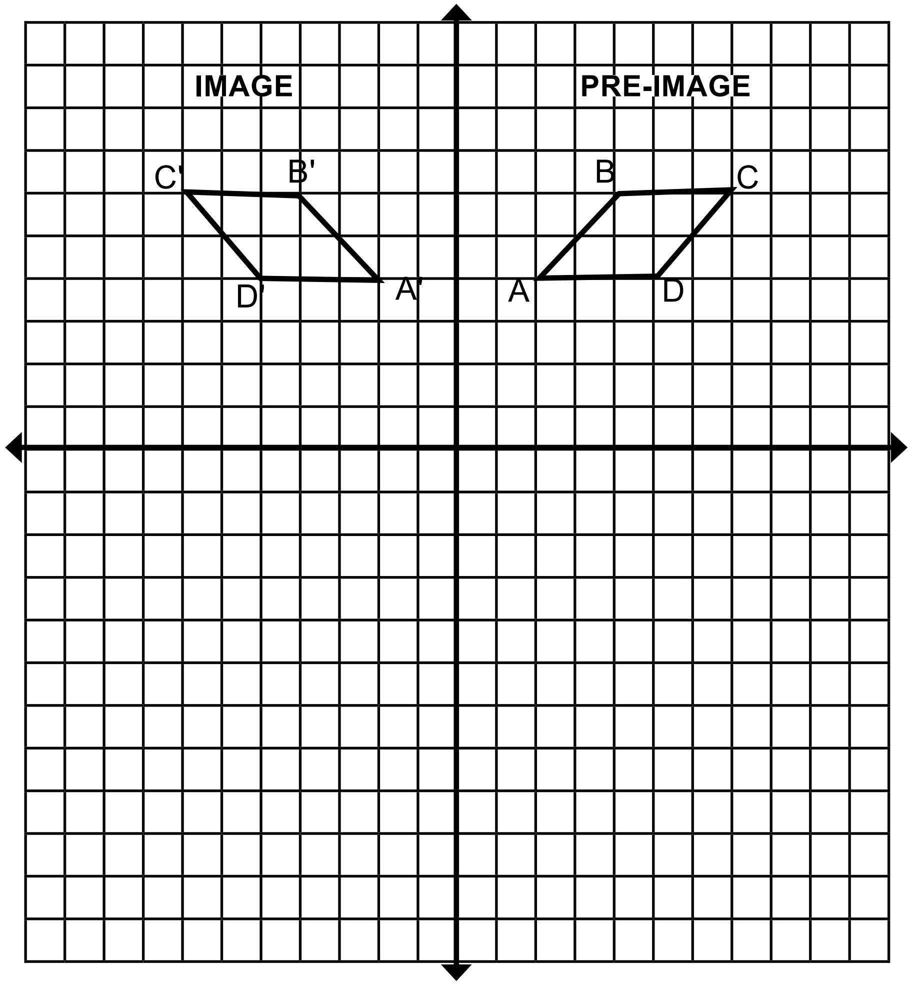
8.10B

The reflection of a figure is congruent to the original figure.

8.10C

When a figure is reflected over the x- or y-axis, the signs on the coordinates change. The axis is the line of symmetry.

Consider the diagram to the right.
The green figure is the original image.
The blue figure is an image that has been reflected over the x-axis.
Reflected over the x-axis – (x, y) → (x, -y)
The red figure is an image that has been reflected over the y-axis.
Reflected over the y-axis – (x, y) → (-x, y)



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Translations

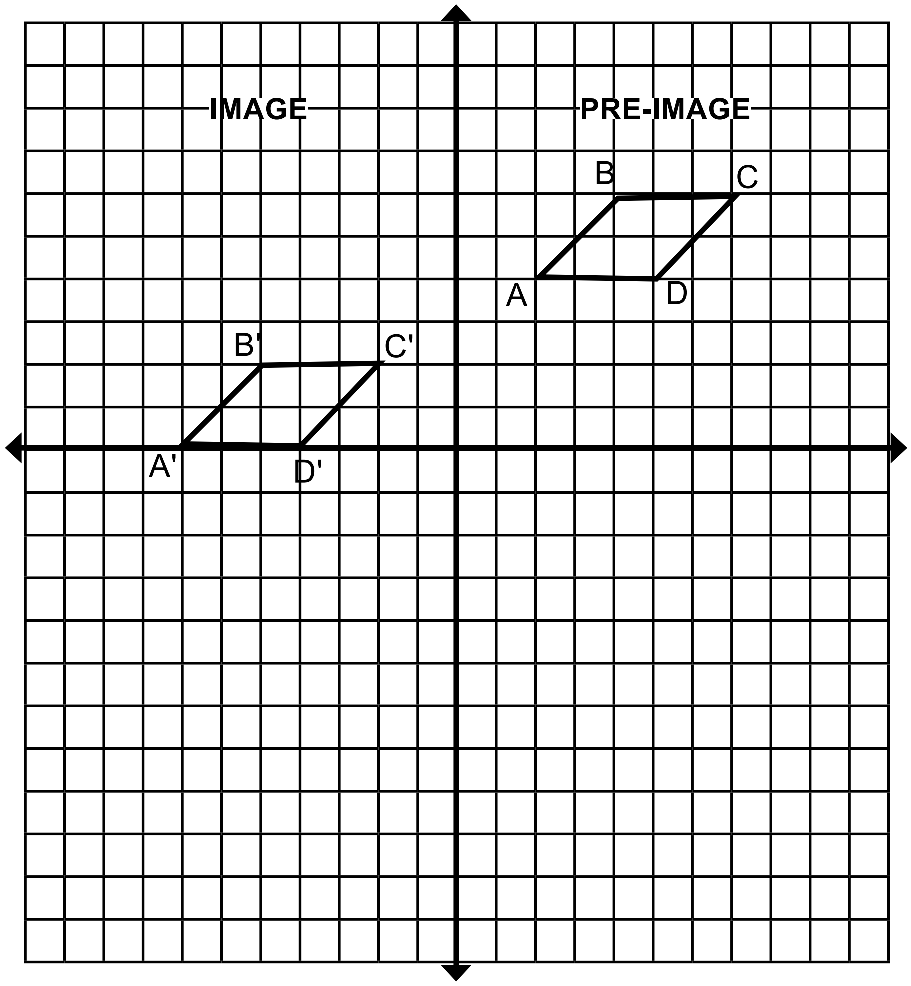
8.10A
A translation can be described as a horizontal slide, vertical slide, or combination of vertical and horizontal slide from one location to another. As shown on the example to the right, the vertices for the pre-image are read clockwise ABCD. The vertices for the image are also read clockwise A'B'C'D', thus the orientation is preserved.

8.10B
The translation of a figure is congruent to the original figure.

8.10C
Translations are described by their movement. Going up or to the right (toward the positives) means a positive change or addition. Going down or to the left (toward the negatives) means a negative change or subtraction. In the diagram to the right, the green figure has been translated down 2 and to the left by 6. The translated image is red. Looking at the coordinates algebraically,

$(x, y) \rightarrow (x - 6, y - 2)$

This is true for all the vertices in the image.



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Dilations
For more information on dilations, see 8.3A, 8.3B, and 8.3C.

8.10A
A dilation is a figure that is an enlargement or a reduction of another figure. Orientation is preserved in dilations. As shown on the example to the right, the vertices for the pre-image are read clockwise ABC. The vertices for the image are also read clockwise A'B'C'.

8.10B
The dilation of a figure is similar, not congruent, to the original figure.

8.10C
When a figure is dilated, a new figure is created that is either smaller or larger than the original figure. In the diagram, the blue figure is the original and the red figure is the dilation. To find the coordinates of the dilation, multiply the original coordinates by the scale factor.
In the example, the scale factor is $\frac{2}{3}$.

(-3, -3)→(-2, -2)
(3, -3)→(2, -2)
(0, 6) →(0, 4)

A scale factor that is greater than 1 will result in an image that is larger than the original figure. A scale factor that is less than 1 will result in an image that is smaller than the original figure.