ALGEBRA I

mini interventions

LINEAR EQUATIONS AND INEQUALITIES

REGION 13
EDUCATION SERVICE CENTER

ACKNOWLEDGMENTS

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GETTING STARTED

These explanations of the new state math standards are designed to help you understand what the standards mean and how the models of teaching math help students understand mathematics more deeply. Others may interpret the standards differently and may have different ideas for how to teach them. It is our hope that this deconstruction of the Texas Essential Knowledge and Skills (TEKS) for mathematics makes teaching math more rigorous, more fun, and a little less confusing.

NAVIGATING THE DOCUMENT

This is an interactive PDF. Go to the Table of Contents and click on the Algebra I TEKS you want to view.

WRITING AND SOLVING LINEAR FUNCTIONS, EQUATIONS, AND INEQUALITIES	
A.2	**The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:**
A.2E	graph the solution set of linear inequalities in two variable on the coordinate plane. **RC3**

Within the document, there are also links in the tables that connect to the corresponding Click-On TEKS examples. You could also use the tools in Acrobat and type in the page number provided.

SE	Description	Reporting Category	Standard	Page #
8.5C	contrast bivariate sets of data that suggest a linear relationship with bivariate sets of data that do not suggest a linear relationship from a graphical representation.	4	Supporting	202

SE	Description	Reporting Category	Standard	Page #
4.6A	identify points, lines, line segments, rays, angles, and perpendicular and parallel lines.	3	Supporting	172

Another function within Adobe Acrobat is the ability to Search. You run searches to find specific items in your PDFs by using either the Search window or the Find toolbar (CTRL+F, or CMD+F). In either case, Acrobat searches the PDF body text, layers, and form fields. You can also include bookmarks and comments in the search. Only the Find toolbar includes a Replace With option.

TABLE OF CONTENTS

*A.3E & A.7C are transformations of linear and quadratic functions. A.3E is a building block for A.7C, so they are bundled together.

HOW EACH CHAPTER IS ORGANIZED

The goal of these mini interventions is to not only help guide teachers with their understanding of the new TEKS but to provide background information for skills the student might have missed from previous grades and what to do about it. Also included are some intervention skills if your student is having a particularly difficult time learning. Specific examples and/or activities may be adjusted over time as more information becomes available from the state.

There are 17 SE components total. Each SE was selected because it was a "brand new" SE to the 2012 Algebra 1 TEKS. Each SE contains these sections:

1. Background Information
    - o Introduction to the new SE
    - o Scaffolding TEKS to help teachers know when and how the prerequisite skills were taught
    - o Techniques for spiraling concepts, which will include Click-On TEKS examples
2. Pre-Assessment
    - o Formative questions that dig deep into the scaffolding TEKS that lead to that particular new Algebra 1 SE
    - o Questions will include an answer key and reference to a prerequisite or scaffolding SE that may be used to help students who did not evidence mastery of each question
3. Spiraling
    - o Prerequisite Algebra 1 SEs to spiral, which will include specificity and the big idea for spiraling
    - o Examples of spiraling SEs
4. Teaching
    - o Important ideas
    - o Vocabulary and formulas
    - o Descriptions and examples of the concepts with step-by-step directions
    - o Instructional activities
5. Foundation
    - o SE skills that may be explored for Tier 2 & 3 Intervention (these are usually "concrete" or "basic" ideas that students have not mastered in prior grades)
    - o A description of the SE indicating grade level, reporting category, and whether it was a readiness or supporting standard
    - o Click-On TEKS examples from previous grade levels
6. Post-Assessment
    - o Questions that help evaluate whether students have mastered the Algebra I SE
    - o Teacher's answer key

HOW EACH CHAPTER IS ORGANIZED

		WHAT IS IN EACH SECTION?	HOW DO I USE IT?
PREPARE FOR INSTRUCTION	Background Information	What should students already know? This section includes brief explanations of standards from previous grade levels that students should have mastered through Click-On TEKS examples.	Use this to refresh, build, and extend your knowledge. Remember that K-8 has been implementing new standards. Are you familiar with the changes and how they impact your instruction?
PREPARE FOR INSTRUCTION	Pre-Assessment	Where are student strengths and needs? This section includes questions that help identify prior knowledge including student strengths and gaps.	Use this information to determine what content you need to spiral into instruction. Do a quick re-teach of previous grade concepts if necessary.
PREPARE FOR INSTRUCTION	Spiraling	What are some Algebra I standards I will need to weave into instruction along with the focus standards? This section includes a brief description of additional Algebra I standards that will help students get to mastery of the targeted standard.	Bundle these Algebra I standards with the targeted new standard to support student learning. Use specificity to understand more about the standard. Use examples to help you understand the scope of the standard. Use examples as part of your instruction.
TEACH	Teaching	What is the focus of the new standard? This section includes important ideas, vocabulary, formulas, and examples.	Use important ideas to hone in on the core concepts of the standard. Use vocabulary and formulas to create word walls, sentence stems, and anchor charts. Use examples during instruction with students.
RE-TEACH	Foundation	What standards from previous grade levels do I need to address to close student gaps? This section includes examples and brief explanations of previous grade level standards from Click-On TEKS which give concrete support and points of entry for students with gaps.	Use specific examples from previous grade levels and strategies aligned to these concrete standards.
ASSESS	Post-Assessment	How will I know students have mastered the targeted new standard? This section includes questions to determine whether the student has mastered the targeted new standard.	Cut questions into cards and use for formative assessment. Use questions as an end-of-unit summative assessment. Use results to make continuing instructional grouping and approach decisions.

BASIC STRUCTURE OF THE TEKS

The Texas Essential Knowledge and Skills (TEKS) consists of four parts.

Part A: The Introduction

The state standards for Algebra 1 begin with an introduction. The introduction gives an overview of the focal areas for each grade and provides general information about numerical fluency and processing skills. Although the introduction has not been reprinted in this book, information from the introduction has been included in the explanations of the TEKS, where appropriate.

Part B: Strands

The standards are broken into groups or categories called Strands. The TEKS for Algebra 1 are divided into five strands:

Mathematical Process Standards: This strand contains the process standards for mathematics, which are the same from kindergarten through pre-calculus. The process standards are the ways that students acquire math content through the use of models and tools, communication, problem solving, reasoning and analysis, and making connections. These standards should be woven consistently throughout the content strands (2–6). The dual-coded questions on STAAR will be coded with a content standard and a process standard.

Number and Algebraic Methods

Linear Functions, Equations, and Inequalities

Quadratic Functions and Equations

Exponential Functions and Equations

Example

A.1 Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

(A) apply mathematics to problems arising in everyday life, society, and the workplace.

Part C: Knowledge and Skills Statements

Immediately following the strand is the Knowledge and Skills (K&S) statement. It provides the context for the student expectations that follow it. Numbering: The first letter is the grade level. The second number is the Knowledge and Skills number. The K&S statement shown is from Algebra 1.

Part D: Student Expectations

Immediately following each Knowledge and Skills statement is a list of Student Expectations (SEs). The letters, such as (A), refer to what students are expected to do with regard to a particular Knowledge and Skills statement. We often refer to this example as A.1A. [Grade Level Algebra, Knowledge and Skills statement (1), Student Expectation (A)]

MATHEMATICAL PROCESS STANDARDS

The mathematical processes in the revised math state standards are the same from kindergarten to pre-calculus. Why? Because these are the processes of doing mathematics. These are the ways that mathematicians work every day.

Algebra 1 always had underlying process standards, but they were not clearly delineated until now.
These are the processes that students will use to understand the new math content and show how they know it.

For Algebra 1, these are the processes that will have dual codes on the STAAR EOC with the content standards. This means that students need to look at the content standards through the lens of the process standards. These standards will be dual-coded with the content standards and may include any or all of the process standards.

The following are the process standards:

- A.1A—Apply mathematics to problems arising in everyday life, society, and the workplace.
- A.1B—Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution.
- A.1C—Select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems.
- A.1D—Communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.
- A.1E—Create and use representations to organize, record, and communicate mathematical ideas.
- A.1F—Analyze mathematical relationships to connect and communicate mathematical ideas.
- A.1G—Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication

A.2E

1. BACKGROUND INFORMATION

What's in This Section?

- A description of A.2E
- A table that lists all of the SEs that should be considered in preparation of the lesson for the targeted standard
- Specificity from the TEA Side by Side documents for the new K-8 standards
- References to the sections in the appendix that will provide a more thorough description as well as examples for each of the background standards referenced here

The following SE is new to the 2012 TEKS for Algebra 1. The specificity comes from the Algebra 1 Side by Side provided by TEA.

A.2 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:

STUDENT EXPECTATION

A.2E write the equation of a line that contains a given point and is parallel to a given line.

RC3, Supporting Standard

SPECIFICITY

This SE extends to include the use of parallel and perpendicular relationships between the slopes as a means for determining the slope of a line. In previous grades, students were introduced to slope and the meaning of parallel and perpendicular. The relationship between the slopes of two lines that are parallel or perpendicular will be new.

The revised SEs builds to G2.B, where students verify geometric relationships, including parallel and perpendicular lines of geometric figures on a coordinate plane.

The revised SE explicitly states to include lines parallel and perpendicular to lines with a slope of zero or an undefined slope.

1. BACKGROUND INFORMATION

As a high school teacher, you probably know your TEKS but may want to refresh, build, and extend your knowledge of the vertically aligned K-8 standards that lead into the targeted standard that you are planning to teach to your students.

SE	Description	Reporting Category	Standard	Page #
6.4A	ules verbally, numerically, graphically, and symbolically in the form of y = ax or y = x + a in order to differentiate between additive and multiplicative relationships	2	Supporting	180
7.4A	represent constant rates of change in mathematical and real-world problems given pictorial, tabular, verbal, numeric, graphical, and algebraic representations, including $d = rt$	2	Readiness	188
8.4A	use similar right triangles to develop an understanding that slope, *m*, given as the rate comparing the change in *y*-values to the change in *x*-values, $\frac{y_2 - y_1}{x_2 - x_1}$, is the same for any two points (x_1, y_1) and (x_2, y_2) on the same line	2	Supporting	193
8.5B	represent linear non-proportional situations with tables, graphs, and equations in the form of y = mx + b, where b ≠ 0	2	Supporting	200
8.5F	distinguish between proportional and non-proportional situations using tables, graphs, and equations in the form y = kx or y = mx + b, where b ≠ 0	2	Supporting	195

6.4A compare two rules verbally, numerically, graphically, and symbolically in the form of y = ax or y = x + a in order to differentiate between additive and multiplicative relationships; RC 2, Supporting Standard

SPECIFICITY

Specificity has been added regarding symbols and their use. The algebraic representations should be in the form y=ax or y= x + a. Students are expected to graph these relationships. Students are expected to compare two rules to differentiate between additive and multiplicative representations. This is a building block for work with proportional and non- proportional situations in grades 7 and 8.

1. BACKGROUND INFORMATION

7.4A represent constant rates of change in mathematical and real-world problems given pictorial, tabular, verbal, numeric, graphical, and algebraic representations, including d = rt; **RC 2, Readiness Standard**

SPECIFICITY

Content of these SEs related to unit conversions, perimeter, area, and volume was moved to grade 6. Proportionality 6.4H; Expressions, equations, and relationships, 6.8B, 6.8C, 6.8D.

8.4A use similar right triangles to develop an understanding that slope, m, given as the rate comparing the change in y-values to the change in x-values, $\frac{y_2 - y_1}{x_2 - x_1}$, is the same for any two points (x_1, y_1) and (x_2, y_2) on the same line; **RC 2, Supporting Standard**

SPECIFICITY

These SEs comes from A.6A: "Develop the concept of slope as rate of change and determine slopes from graphs, tables, and algebraic representations." These SEs add specificity regarding the concept of slope through the lens of proportionality. Teaching the formula for slope is not the intent of 8.4A. The intent of 8.4Ais to note that the rate comparing the change in y-and x-values is the same for any two points on the same line

8.5B represent linear non-proportional situations with tables, graphs, and equations in the form of y = mx + b, where b ≠ 0; **RC 2, Supporting Standard**

SPECIFICITY

8.5B adds specificity and separate proportional (y=kx) from non-proportional (y = mx + b, b≠ 0) situations to support learning related to foundations of linear functions and distinguishing between m(or k) and b. The contexts may now include data from real-world applications or mathematical solutions with paired values. Equations should include rational number coefficients and constants.

8.5F distinguish between proportional and non-proportional situations using tables, graphs, and equations in the form y = kx or y = mx + b, where b ≠ 0; **RC 2, Supporting Standard**

SPECIFICITY

8.5F adds specificity to the current SE with the representational forms.
The current SE has been separated into two SEs. 8.5F focuses on distinguishing between proportional and non-proportional situations using multiple representations.

2. PRE-ASSESSMENT

These pre-assessment questions are designed to be given before spiraling and teaching the new Algebra 1 SE in this section. All of the questions may be given as a traditional warm-up or as a pre-assessment where the questions are given individually to each student in advance of teaching the new Algebra 1 SE. This will help identify the prior knowledge that students may or may not have mastered.

The questions are formatted to be cut into cards and used in various formative assessment techniques. Region 13's The Teacher Toolkit, www.theteachertoolkit.com, has excellent opening activities that demonstrate fun and engaging ways to formatively assess students' knowledge. These activities include step-by-step directions as well as classroom videos that will help everyone, from the experienced to novice teacher.

A recommended pre-assessment idea from The Teacher Toolkit is Nothing Ventured:

http://www.theteachertoolkit.com/index.php/tool/nothing-ventured/nothing-ventured

What's in This Section?

- Pre-Assessment Formative Assessment Questions
- A spreadsheet to monitor student mastery on the pre- and post-assessment questions is provided on page 217. This may be used for a more in-depth analysis of each student's performance on the pre-assessment.
- A key to the pre-assessment questions is provided as well as a reference to the SE that the question is assessing.

2. PRE-ASSESSMENT

Pre-Assessment Formative Questions

What is the equation of the line $(y-3) = 4(x+2)$ in standard form?

What is the equation of the line $-3x + 4y = -8$ in slope-intercept form?

Write the equation $y = \frac{2}{5}x - 4$ in standard form.

What is the slope of the line $x = -3$?

Mr. Wallace wrote this sequence of numbers on the board.

Term	Number
1	1
2	3
3	5
4	7
5	9
n	1

What is the expression that could be used to determine the nth number in this sequence?

What is the slope of the line that passes through the points (-2, 5) and (1, 10)?

2. PRE-ASSESSMENT

Hannah was 3 years old when Cole was born. The table below shows the difference in their ages. Write an equation that describes the relationship of Cole's age in terms of Hannah's age?

Cole's Age	0	1	2	3
Hannah's Age	3	4	5	6

What is the definition of parallel lines?

Pre-Assessment Formative Questions

What is the equation of the line (y-3) = 4(x+2) in standard form?

Answer: 4x - y = -11 **SE: A.2B**

What is the equation of the line -3x + 4y = - 8 in slope-intercept form?

Answer: $y = \frac{3}{4}x - 2$ **SE: A.2B**

Write the equation $y = \frac{2}{5}x - 4$ in standard form.

Answer: 2x - 5y = 20 **SE: A.2B**

What is the slope of the line x = -3?

Answer: Undefined **SE: A.3A**

Mr. Wallace wrote this sequence of numbers on the board.

Term	Number
1	1
2	3
3	5
4	7
5	9
n	1

What is the expression that could be used to determine the nth number in this sequence?

Answer: 2n - 1 **SE: A.2C**

What is the slope of the line that passes through the points (-2, 5) and (1, 10)?

Answer: $\frac{5}{3}$ **SE: 8.4C**

Hannah was 3 years old when Cole was born. The table below shows the difference in their ages. Write an equation that describes the relationship of Cole's age in terms of Hannah's age?

Cole's Age	0	1	2	3
Hannah's Age	3	4	5	6

Answer: c = h -3 SE: 8.5I

What is the definition of parallel lines?

Answer: Lines that never intersect
SE: 4.6A

3. SPIRALING

These Algebra 1 SEs have been carefully chosen to spiral into your lesson plan where needed to ensure that students are prepared for the new SE in each module. When you make your lesson plans for the targeted standard, plan on weaving in, or spiraling, these SEs prior to and during lessons to reinforce students' prior learning in your class this year. Students need to experience prior learning often in order to build a strong foundation in mathematics.

What's in This Section?

- Table listing SEs that are recommended for spiraling
- The written description of the SE as stated in the 2012 TEKS
- Specificity from the Side By Side documents provided by TEA
- The big idea for spiraling these SEs
- In some cases, examples are provided to go with each spiraling SE

Status	SE	Description	Reporting Category	Standard
CONTINUING SE	A.2B	write linear equations in two variables in various forms, including $y = mx+ b$, $Ax + By= C$, and $y - y_1 = m(x - x_1)$, given one point and the slope and given two points.	3	Supporting
CONTINUING SE	A.2C	write linear equations in two variables given a table of values, a graph, and a verbal description.	3	Readiness
CONTINUING SE	A.3A	determine the slope of a line given a table of values, a graph, two points on the line, and an equation written in various forms including $y= mx + b$, $Ax + By = C$, and $y- y_1= m(x - x_1)$	2	Supporting

What Does Status Stand For?

NEW SE: A targeted standard that is detailed in a different section. Examples may be viewed in that section and are not included here.

CONTINUING SE: Not new to Algebra 1. Remember to closely examine the SE and the specificity that is provided, as subtle changes may be detailed.

A.2B write linear equations in two variables in various forms, including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m(x - x_1)$, given one point and the slope and given two points.
RC3, Supporting Standard

SPECIFICITY

When writing linear equations, students are expected to use the form which makes the most sense for the given information. Students are also expected to be able to manipulate the results to the other forms. For example, when provided a point and the slope of the line, using the form $y - y_1 = m(x - x_1)$ may be more efficient. The slope of the line and a point on the line could be provided explicitly or implicitly.

Big Idea for Spiraling: Students should be familiar with point-slope form, slope-intercept form, and standard form. When a student is given a point and the slope or two points without the slope, students will be able to write the equation of the line in all three forms.

EXAMPLES

Given the Slope and a Point

Line through (4, -2) and has a slope of 2.

point –slope form

$y - y_1 = m(x - x_1)$	*point-slope form*
$y - (-2) = 2(x - 4)$	*substitute point and slope*
$y + 2 = 2(x - 4)$	*simplify*

slope-intercept form

$y + 2 = 2(x - 4)$	
$y + 2 = 2x - 8$	*distribute*
$y = 2x - 10$	*simplify*

standard form (using the prior answer)

$y = 2x - 10$	
$2x - y = 10$	*subtract y from both sides*
	add 10 to both sides

EXAMPLES

Given Two Points

Line through (2 , 1) and (-3, 4).

$m = \frac{y_2 - y_1}{x_2 - x_1}$ *slope formula*

$m = \frac{4 - 1}{-3 - 2}$ *substitute*

$m = \frac{3}{-5}$ *simplify*

point-slope form (using the slope calculated above and using point (2, 1))

$y - y_1 =- m(x - x_1)$ *point-slope form*

$y - 1 = -\frac{3}{5}(x - 2)$ *substitution*

slope-intercept form

$y - 1 = -\frac{3}{5}x + \frac{6}{5}$ *distribute*

$y = -\frac{3}{5}x + \frac{11}{5}$ *add 1 to both sides*

standard form

$\frac{3}{5}x + y = \frac{11}{5}$ *add $\frac{3}{5}$ x to both sides*

$3x + 5y = 11$ *multiply both sides by 5*

A.2C The student is expected to write linear equations in two variables given a table of values, a graph, and a verbal description. **RC3, Readiness Standard**

SPECIFICITY

For example, providing both the x-and y-intercept is sufficient to write the equation of the line, as the values needed to determine slope and a point on the line are implicit within the intercepts.

EXAMPLES

Line with x-intercept of (2, 0) and y-intercept (0, -4).

$m = \frac{y_2 - y_1}{x_2 - x_1}$ *slope formula*

$m = \frac{-4 - 0}{0 - 2}$ *substitution*

$m = \frac{-4}{-2} = 2$ *simplify*

slope-intercept form

$y = 2x - 4$ *substitution*

A.3A The student is expected to determine the slope of a line given a table of values, a graph, two points on the line, and an equation written in various forms including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m(x - x_1)$. **RC2, Supporting Standard**

SPECIFICITY

Slope is introduced in grade 8 through the use of proportionality using similar triangles, making connections between slope and proportional relationships, and determining slope from tables and graphs in 8.4A, 8.4B, and 8.4C. Specificity has been added through identifying specific forms of linear functions. Although specific forms are provided, the expectation is that students be able to manipulate any linear equation to identify key characteristics, such as slope and y-intercept.

4. TEACHING

This section contains the core concepts of the targeted standard. Explore this section to broaden and refresh your knowledge of the targeted standard. This information will help in developing your lesson plan.

What's in This Section?

- A description of the targeted SE
- Targeted vocabulary
- Formulas
- Examples that help broaden and refresh an Algebra 1 teacher's knowledge about the targeted standard
- Ideas and activities that will enrich your lesson plans for the targeted standard
- Suggested prerequisite skills

A.2 Linear functions, equations, and inequalities. The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations.

STUDENT EXPECTATION

A.2E write the equation of a line that contains a given point and is parallel to a given line.

RC 3, Supporting Standard

SUGGESTED PREREQUISITE SKILLS

A.2B write linear equations in two variables in various forms, including $y = mx + b$, $Ax + By = C$, and $y - y_1 = m(x - x_1)$, given one point and the slope and given two points.

RC 3, Supporting Standard

IMPORTANT IDEAS

Students may be expected to transform different forms of equations to find the parallel slope. Students may need to find the slope from a table, graph, and verbal description.

VOCABULARY

Parallel Lines: Two lines in a plane are parallel if they have both:

- the same slope
- different y-intercepts

Slope: the steepness of a line

FORMULAS

Slope formula	$m = \frac{y_2 - y_1}{x_2 - x_1}$
Point-slope form	$y - y_1 = m(x - x_1)$
Slope-intercept form	$y = mx + b$
Standard form of a line	$Ax + By = C$

The basic concept of finding the new equation for a line parallel to another line is that parallel lines have the same slope and different y-intercepts. If the equation of the original line is in slope-intercept form ($y = mx + b$) or in point-slope form ($(y - y_1) = m(x - x_1)$), the slope of the new line will be the m value. If the equation of the line is in standard form, the student will have to transform the equation into slope-intercept form to identify the slope.

Original line in slope-intercept form

Find the equation of the line that is parallel to the line y = -2x +3 and goes through the point (-1, 6).

The slope of the parallel lines will be the same: m = -2.	Substitute the point and the slope into slope-intercept form.
y=mx + b ⇨ 6 = -2(-1) + b ⇨ 6 = 2 + b ⇨ 4 = b	y=-2x + 4

Original line in point-slope form

Find the equation of the line that is parallel to the line y - 4= 2(x + 1) and goes through the point (-1, 6).

The slope of the parallel lines will be the same: m = 2	Substitute the point and the slope into slope-intercept form.
y = mx + b ⇨ 6 = 2(-1) + b ⇨ 6 = -2 + b ⇨ 8 = b	y = -2x + 8

Original line in standard form

Find the equation of the line that is parallel to the line 2x + 3y = -6 and goes through the point (2, 4). The equation must be transformed into slope-intercept form.

2x + 3y = -6

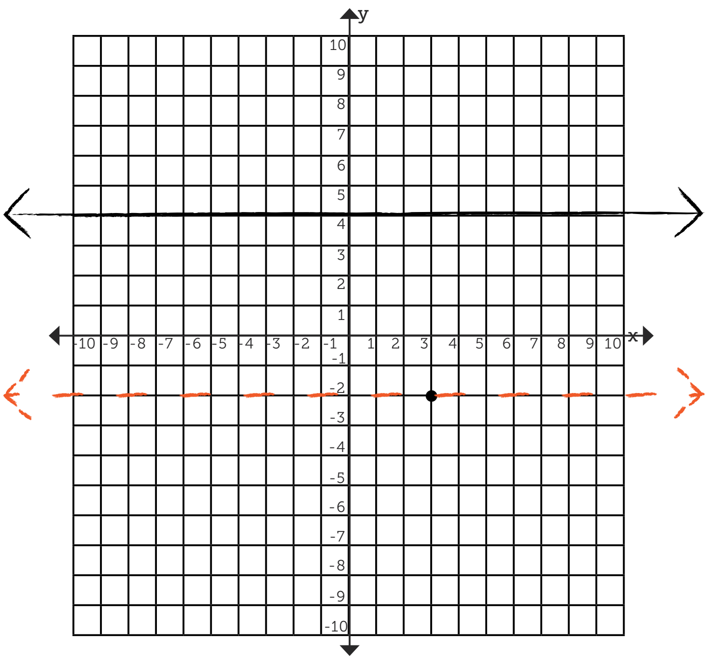
3y = -2x -6 *subtract 2x from both sides*

$y = \frac{-2}{3}x - 2$ *divide both sides by 3*

The slope of the parallel lines will be the same: m = $-\frac{2}{3}$	Substitute the point and the slope into slope-intercept form.
y = mx + b ⇨ $4 = -\frac{2}{3}(2) + b$ ⇨ $4 = -\frac{4}{3} + b$ ⇨ $\frac{16}{3} = b$	$y = \frac{2}{3}x + \frac{16}{3}$

Original line in slope-intercept form

Find the equation of the line that is parallel to the line y = 4 and goes through the point (3, -2). Students may need to see this graphed for them.



The red line will be the line that is parallel. y = - 2

Original line is vertical

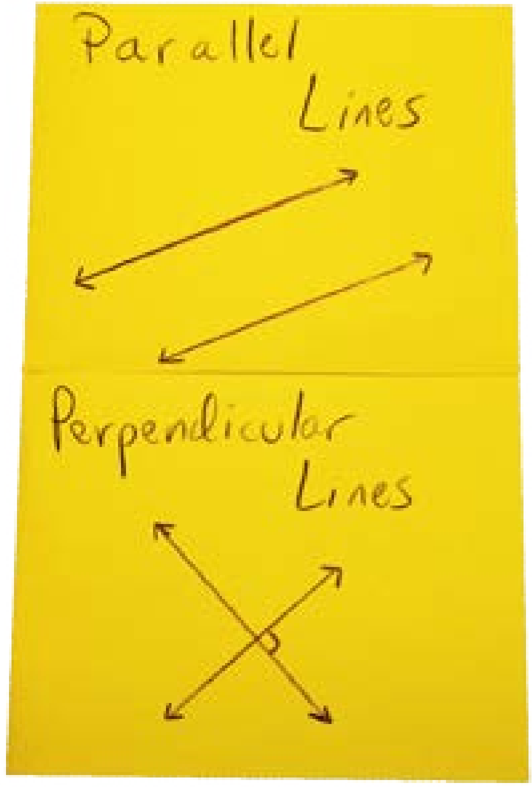
Find the equation of the line that is parallel to the line x = 4 and goes through the point (-2, 2)

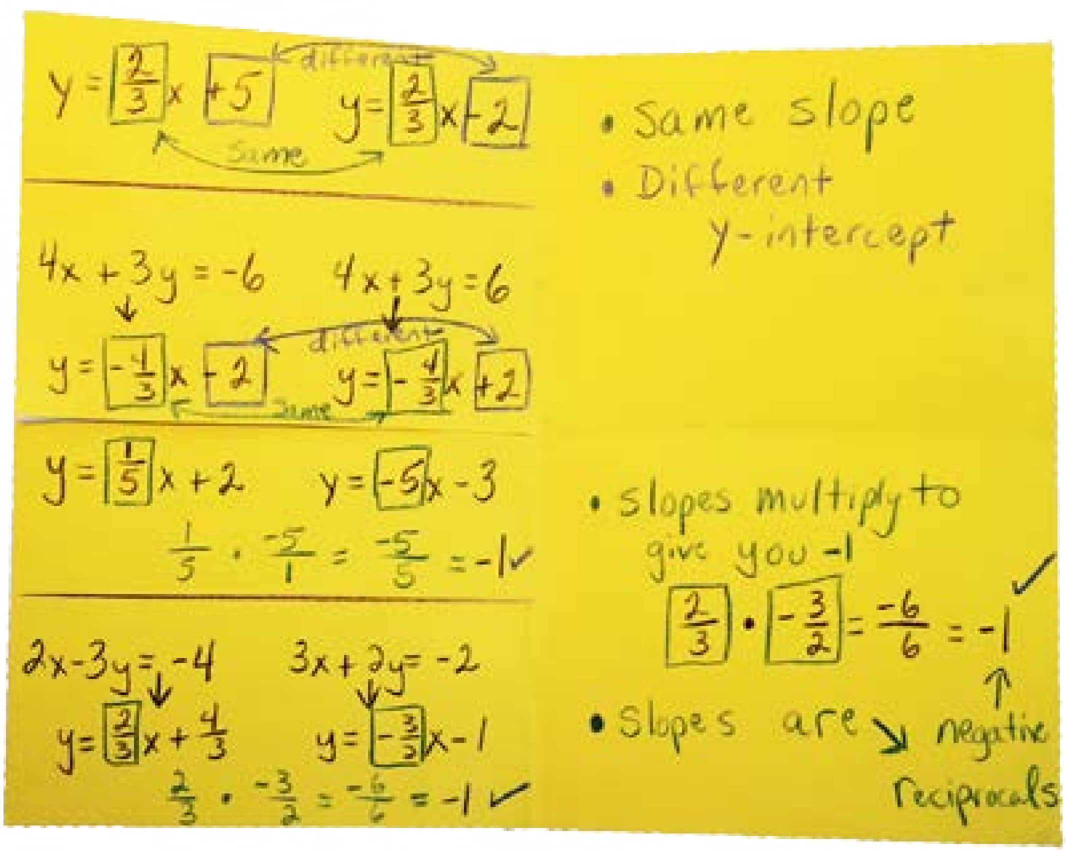
The new line would be x = -2.

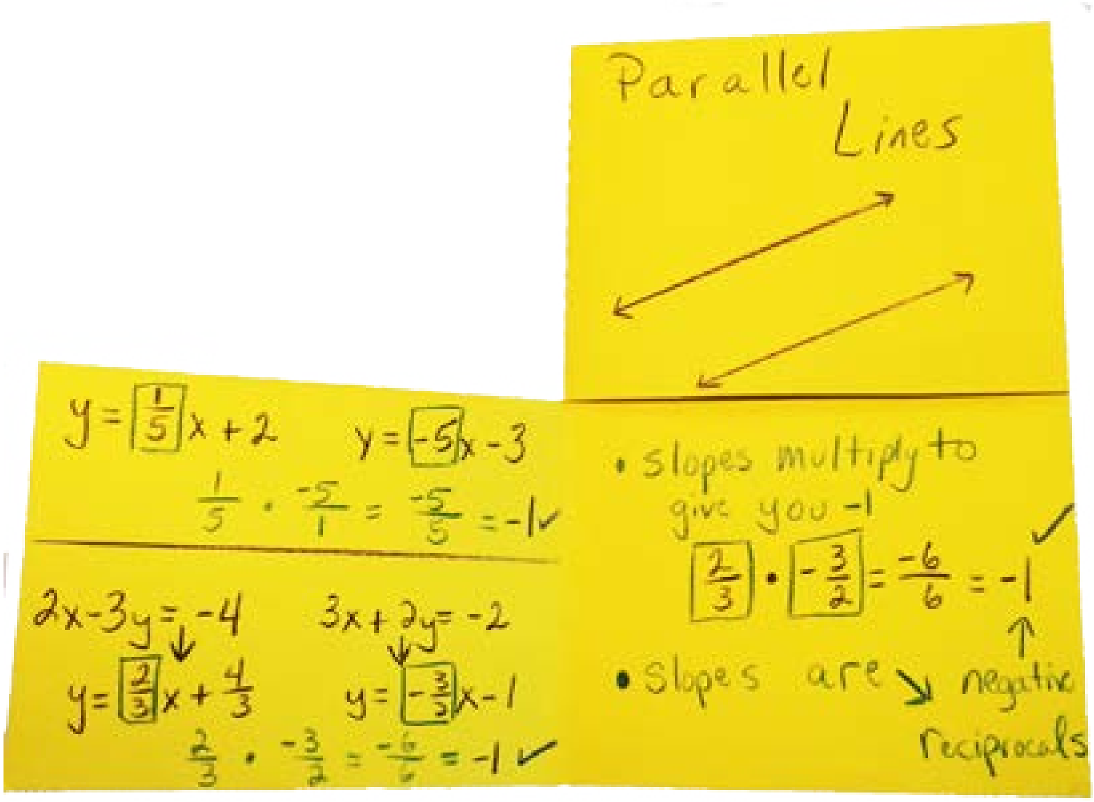
The following pages contain a graphic organizer that combines A.2E and A.2F.

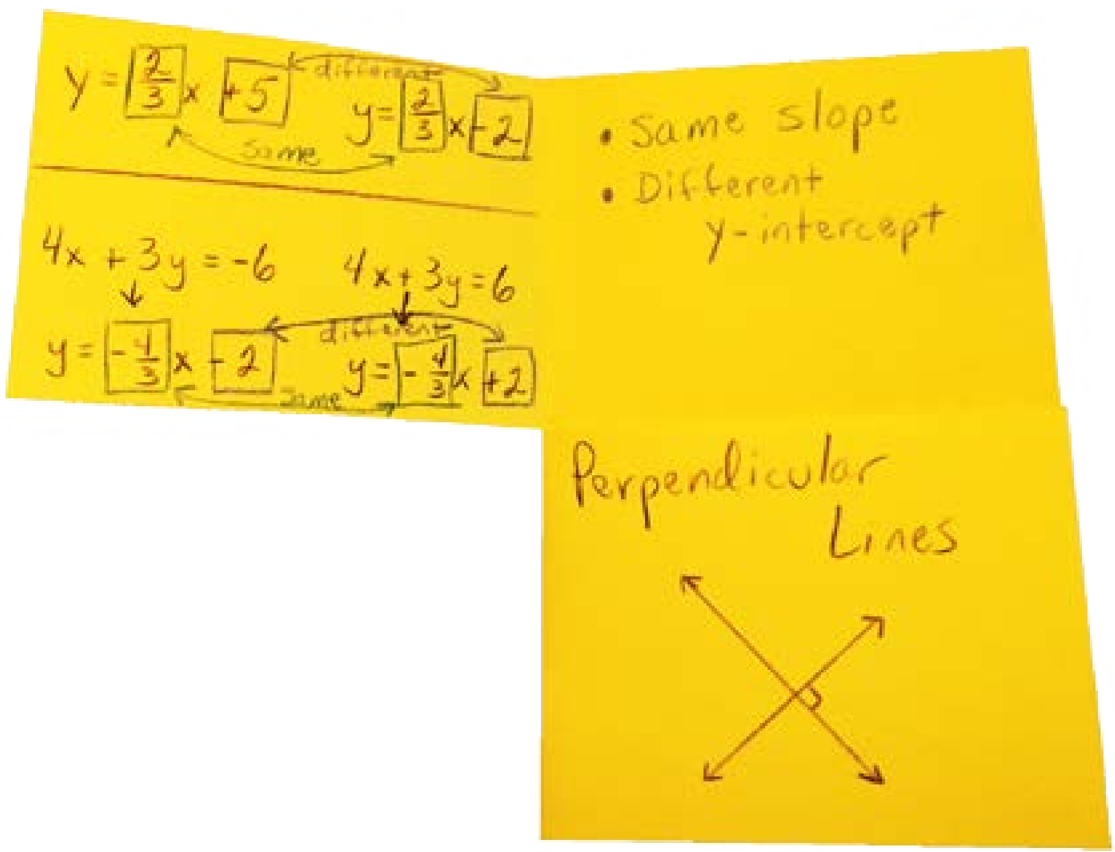
Directions on how to make your own graphic organizer are:

1. Taking a piece of colored paper or card stock, have student fold their paper in half and then in half again.
2. Open the paper out and cut from the outside right edge all the way into the middle. This will form the outside flaps.
3. Write the labels on the outside of the flaps and draw a visual representation of parallel and perpendicular lines.
4. Fill in the inside according to the picture provided here.


Parallel Lines
Perpendicular Lines


y = 2/3 x +5
y = 2/3 x +2
different
same
• Same slope
• Different y-intercept
4x + 3y = -6
4x + 3y = 6
y = -4/3 x +2
y = -4/3 x +2
y = 1/5 x + 2
y = -5x - 3
1/5 • -5/1 = -5/5 = -1
2x - 3y = -4
3x + 2y = -2
y = 2/3 x + 4/3
y = -3/2 x - 1
2/3 • -3/2 = -6/6 = -1
• slopes multiply to give you -1
2/3 • -3/2 = -6/6 = -1
• Slopes are negative reciprocals


Parallel Lines
y = 1/5 x + 2
y = -5x - 3
1/5 · -5/1 = -5/5 = -1
2x-3y = -4
3x + 2y = -2
y = 2/3 x + 4/3
y = -3/2 x - 1
2/3 · -3/2 = -6/6 = -1
• slopes multiply to give you -1
2/3 · -3/2 = -6/6 = -1
• Slopes are negative reciprocals


y = 2/3 x +5
different
y = 2/3 x -2
same
4x + 3y = -6
4x + 3y = 6
y = -4/3 x -2
different
y = -4/3 x +2
same
• Same slope
• Different y-intercept
Perpendicular Lines

5. FOUNDATION

The foundation section contains SEs from prior grade levels and provides ideas and activities to help students who have gaps and/or need a more concrete approach. These are students who need to master the basics to understand the targeted standard and may benefit from instruction using manipulatives. High school intervention can vary greatly compared to what might be found in elementary and middle school. The ideas and strategies here may help you provide intervention after school or during a remediation class, as well as provide the Response to Intervention (RtI) or intervention teacher with fresh ideas to help students who are struggling.

What's in This Section?

- Examples and brief explanations of previous grade-level standards
- References to the sections in the appendix that will provide a more thorough description as well as examples for each of the background standards referenced here

Students in Response to Intervention, Tiers 2 or 3, may have problems with the following concepts. These suggestions are not all inclusive. Students in Tiers 2 & 3 will need a more customized intervention. The suggested SEs are provided with a brief description.

Interventionists may start with integer and rational number operations to transform equations correctly before moving on to graphing lines.

SE	Description	Reporting Category	Standard	Page #
4.6A	identify points, lines, line segments, rays, angles, and perpendicular and parallel lines.	3	Supporting	172
8.4B	graph proportional relationships, interpreting the unit rate as the slope of the line that models the relationship	2	Readiness	194
8.4C	use data from a table or graph to determine the rate of change or slope and y-intercept in mathematical and real-world problems	2	Readiness	195

6. POST-ASSESSMENT

These post-assessment questions are designed to be given after teaching the new Algebra 1 SE in this section. The whole page may be given as a traditional quiz after teaching the new Algebra 1 SE. This will help identify the knowledge that students may or may not have mastered.

The questions are formatted to be cut into cards and used in various formative assessment techniques. Region 13's The Teacher Toolkit, www.theteachertoolkit.com/, has excellent closing activities that demonstrate fun and engaging ways to formatively assess students' knowledge. These activities include step-by-step directions as well as classroom videos that will help the experienced as well as the novice teacher.

A recommended post-assessment idea from The Teacher Toolkit is The One Minute

Problem: http://www.theteachertoolkit.com/index.php/tool/one-minute-problem

What's in This Section?

- Post-assessment formative assessment questions
- A spreadsheet to monitor student mastery on the pre- and post-assessment questions is provided on page 217. This may be used for a more in-depth analysis of each student's performance on the pre-assessment.
- A key to the post-assessment questions is provided as well as a reference to the SE that the question is assessing.

Post-Assessment Formative Questions	
What is the equation of the line $(y+2) = \frac{1}{2}(x+2)$ in standard form?	What is the equation of the line -2x - 4y = 6 in slope-intercept form?
Given the line 2x − 3y = 9, what is the equation of the line that is parallel to the given line and passes through the point (3, 4)?	What is the slope of a line parallel to $y= -\frac{2}{5}x + 2$ that passes through the point (5, 3)?
What is the slope of the line y = 2?	Write an equation in slope-intercept form for the line that passes through point (3,-1) and is parallel to x = -2.
Write an equation in slope-intercept form for the line that passes through point (-1, 5) and is parallel to the line y = 2x -3.	What is the equation of the line that passes through (-6, 4) and is parallel to y = 2?

Post-Assessment Formative Questions	
What is the equation of the line $(y+2) = \frac{1}{2}(x+2)$ in standard form? Answer: x-2y=2 **SE: A.2E**	What is the equation of the line -2x - 4y = 6 in slope-intercept form? Answer: $y=\frac{1}{2}x-\frac{3}{2}$ **SE: A.2B**
Given the line 2x − 3y = 9, what is the equation of the line that is parallel to the given line and passes through the point (3, 4)? Answer: $y-4=\frac{2}{3}(x-3)$ or $y=\frac{2}{3}x+2$ **SE: A.2E**	What is the slope of a line parallel to $y= -\frac{2}{5}x +2$ that passes through the point (5, 3)? Answer: $-\frac{2}{5}$ **SE: A.2E**
What is the slope of the line y = 2? Answer: 0 **SE: 8.5I**	Write an equation in slope-intercept form for the line that passes through point (3,-1) and is parallel to x = -2. Answer: x =3 **SE: A.2E**
Write an equation in slope-intercept form for the line that passes through point (-1, 5) and is parallel to the line y = 2x -3. Answer: y = 2x + 7 **SE: A.2E**	What is the equation of the line that passes through (-6, 4) and is parallel to y = 2? Answer: y = 4 **SE: A.2E**

4.6 Geometry and measurement. The student applies mathematical process standards to analyze geometric attributes in order to develop generalizations about their properties. The student is expected to:

4.6A identify points, lines, line segments, rays, angles, and perpendicular and parallel lines. (RC3, Supporting Standard)

Formal geometry can be thought of as a way to categorize and make abstract characteristics of figures in the real world. This Student Expectation (SE) typifies this. In 4.6A, students begin to give formal geometric names to characteristics of figures (2D and 3D) that they have encountered their whole lives. Through this SE, students begin to see the world through geometric eyes.

According to TEA, this SE should be coupled with the concrete and pictorial models from 4.1D. In other words, students are to find points, lines, line segments, rays, angles, parallel lines, and perpendicular lines in concrete and pictorial models.

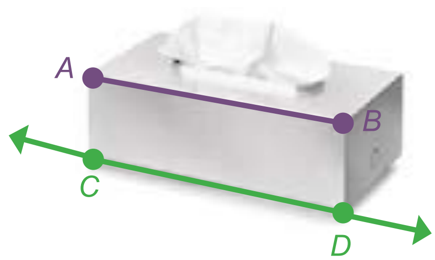
Example/Activity

This explanation is going to balance formal geometry definitions with real-world models. The SE does not say that students need to know the formal geometric definitions. However, some details are included for your own content knowledge.

Points: The word “point” is difficult to define. However, it is simple to understand. Start with the understanding that the tip of a pencil is a point, or the spot on the corner of a box of tissues is a point. To label a point, use a capital letter, like this—A. If several points are named on the same figure, they each need a different name.

Thinking about it in formal geometric terms, a point is a location. It has no width, length, or depth.

Lines and Line Segments: Although a line is difficult to define, a line segment is not. A line is specified by two points, including all the points in between the two points and extending beyond the two points. A line segment is any two of those points and the points in between them.



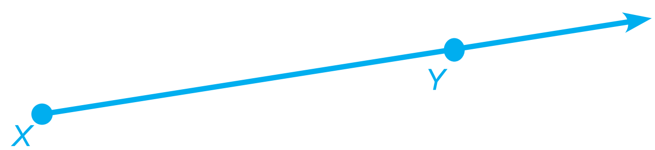
A line segment is shown in **purple** on the tissue box. The line segment $\overline{AB}$ begins and ends with the points at the vertices on the box. To write the name of the segment, write the endpoints together and draw a line segment over them like this: $\overline{AB}$. There is no need to put the points on the line segment above the letters. The order that the points are written does not matter.

Continued on next page

Sometimes in a real-world context, this is called a line. It's not wrong, but it's not as right as it could be. A line, $\overleftrightarrow{CD}$, is shown in **green** on the tissue box, sort of. Actually a line goes on forever and ever in two directions. So it can't be contained on a tissue box or really on any other physical object. (Another real-world picture that is often called a line is one rail of a railroad track. While it doesn't go on forever, if you stand by a railroad track and look both ways, it seems to the human brain that it is going on forever.)

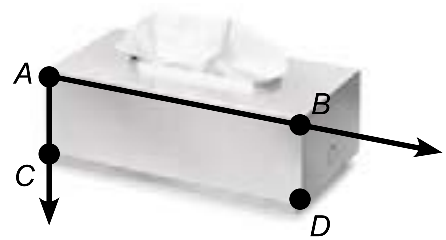
Going back to the tissue box, the line is denoted by the arrows on both ends of $\overleftrightarrow{CD}$. This is how we show a line in a drawn geometric figure. To name a line, choose any two points on the line, write the endpoints next to each other, and draw a line above them like this: $\overleftrightarrow{CD}$. Be sure to use the arrows to distinguish it from a line segment. The order that the points are written does not matter.

Ray: A ray is a part of a line. It has one endpoint but goes forever and ever in the other direction. For a real-world example, think about a laser pointer. The beam starts at the bulb (that's the point) and then shoots out in one direction (that's the "forever and ever in the other direction" part).

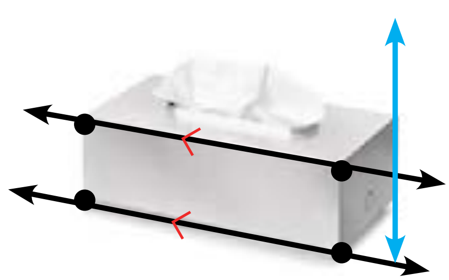


To name a ray, you need two points: the endpoint and any other point on the ray. This ray is called $\overrightarrow{XY}$. The endpoint, *X*, is listed first, and then the other point. Note that $\overrightarrow{XY}$ is not the same figure as $\overrightarrow{YX}$.

Angles: Angles are two rays that share the same endpoint. They are named by putting the shared endpoint (called the *vertex of the angle*) in between two points, one point on each ray. A symbol for angle is placed in front of the three letters. For example, this figure shows two rays that share endpoint *A*. The two rays make $\angle CAB$. Point *A* is written in between points *C* and *B* because it is the vertex of the angle.



Parallel Lines: Parallel lines are two lines that are in the same plane (not a 4th grade vocabulary word) but that never intersect. *Intersect* is not an official vocabulary word for 4th grade. However, you will need to use it when discussing parallel and perpendicular lines. *Intersect* means "to cross." When lines intersect each other, they cross each other at a point (which is a 4th grade vocabulary word).



$\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are parallel lines. They never intersect.

There is a symbol that can be used to show parallel lines that looks like parallel lines. Here is what is looks like: $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$. On a geometric figure, parallel lines may have little arrows on them to show they are parallel, as shown in black above.

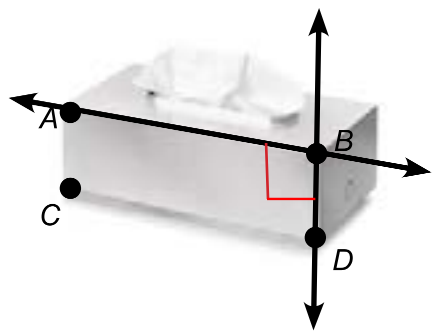
Continued on next page

Examine the blue line. Although the blue line appears to intersect $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ in this two-dimensional figure, if this were a three-dimensional tissue box that you were holding in your hand, it would not intersect the lines because the blue line is actually at the back of the tissue box and therefore not in the same plane as the other lines. Because the blue line is in a different plane it cannot intersect $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ and, therefore, cannot be perpendicular to them. The blue line is called a skew line. When you discuss parallel and perpendicular lines with students, be sure to say that the lines must be **in the same plane.** Please note that students do not have to know the term *skew lines* in 4th grade. However, it is important that they understand that not all lines that do not intersect are parallel.

Real-world examples of parallel lines are railroad tracks or the double yellow line between traffic lanes.

Note: We can also talk about segments being parallel or a line and a segment being parallel. There is no need to limit discussion of parallel to lines only.

Perpendicular Lines: Perpendicular lines are lines that make a right angle when they intersect. (In 4.6C students study right angles.)



$\overleftrightarrow{AB}$ and $\overleftrightarrow{BD}$ intersect each other at a right angle, although they don't appear to be right angles on this two-dimensional drawing. If you were holding the tissue box in your hand, the lines would intersect at a right angle. See the right angle mark shown in red? On a drawn figure, a right angle will be marked with a right angle like this one.

Note: We can also talk about segments being perpendicular or a line and a segment. There is no need to limit discussion of perpendicular to lines only.

There is a symbol that can be used to show perpendicular lines that looks like an upside down T. Here is what it looks like: $\overleftrightarrow{AB} \perp \overleftrightarrow{BD}$.

While it is nice to have geometric solids to study, there are plenty of models that contain all of these geometric figures in your classroom and on the playground. Find the geometry in real life and let the students take pictures of it. Then they can share the geometry that they saw. What a great way to change students' view of the world!

6.4 Proportionality. The student applies mathematical process standards to develop an understanding of proportional relationships in problem situations. The student is expected to:

6.4A compare two rules verbally, numerically, graphically, and symbolically in the form of y = ax or y = x + a in order to differentiate between additive and multiplicative relationships. (RC2, Supporting Standard)

Prerequisite skill: graphing ordered pairs on the coordinate plane (SE 6.11)
This Student Expectation (SE) provides students an introduction to the difference between multiplicative thinking and additive thinking. Although students may have an informal understanding of this, this SE provides an explicit, detailed view of multiplicative and additive thinking.
Equations that represent an additive situation are in the form $y = x + a$.
Equations that represent a multiplicative situation are in the form $y = ax$.
Students will examine each of these by looking at:

- verbal descriptions of situations that are either additive or multiplicative
- numerical expressions that require students to add repeatedly or multiply repeatedly
- the difference between a graph that shows an additive relationship compared to a graph that shows a multiplicative relationship
- equations that are either in the form of $y = ax$, which shows a multiplicative relationship, or $y = x + a$, which shows an additive relationship

This SE may be taught with 6.6A, 6.6B, and 6.6C.

Example/Activity

Comparison of Additive and Multiplicative Relationships

The two rules that will be compared are y = x + 2 and y = 2x.

	Additive Relationship Example and Explanations	**Multiplicative Relationship Example and Explanations**
Verbal Description	A verbal description of an additive relationship will contain words that sound like addition or subtraction. Here are examples: • David bought T-shirts that were on sale at Old Navy. For every purchase, he got 2 pairs of socks free. • For each order you place, you get an order of fries for free. Look at the wording of the verbal description. The independent quantities are the T-shirts and number of Big Macs. The dependent quantity can be found only by adding or subtracting from the original amount. (See 6.6A.)	A verbal description of a multiplicative relationship will contain words that sound like a rate or comparison. Here are examples: • Joe has twice as many baseball hats as Sam. • There are half as many pears as bananas. Look at the wording of the verbal description. In each case, there is a comparison or rate between the two variables that can be found through multiplication. The rate can be written as a fraction. The independent quantity is the number of baseball hats that Sam has. The dependent quantity is the number of baseball hats that Joe has. It is dependent because we can find the number of Joe's hats only if we know the number of Sam's hats. (See 6.6A.)

Continued on next page

Numerical Expression

The numerical expression for an additive relationship will have addition as the operation. The table below shows the T-shirt example above.

T-shirts Purchased	Numerical Expression	Total # of Items
1	1 + 2	3
2	2 + 2	4
3	3 + 2	5
4	4 + 2	6

Note: When looking at the additive relationship, you must compare from variable to variable. In other words, you must look ACROSS the table, not up and down the columns. The relationship that we are looking for is the relationship between T-shirts and items, not from T-shirt to T-shirt or item to item. (See the blue arrow above the table.)

Possible error alert: If students think that this situation is (T-shirts + 1) and not (T-shirts + 2), then they are looking down the columns rather than looking at the relationship between the variables. You might try color coding the rows, as shown above. This might help them look across rather than look up and down.

Writing a numerical expression for a situation as simple as this may still be a challenge for some students, as it requires some metacognition. In other words, we are asking them not to make the leap to the answer first. We are asking them to think about what they do to get the answer.

The numerical expression for a multiplicative relationship will have multiplication as the operation. The table below shows the baseball hat example above.

# of Hats for Sam	Numerical Expression	# of Hats for Joe
1	1 × 2	2
2	2 × 2	4
3	4 × 2	8
4	5 × 2	10

Note: When students read this table, they need to focus on the relationships that go ACROSS the table, not up and down the columns. The multiplicative relationship that we are looking for is between the number of Sam's hats and the number of Joe's hats, not from Sam to Sam and Joe to Joe. (See the blue arrow above the table.)

Possible error alert: If students think that a multiplicative situation is additive, they are probably looking at the columns of the table individually rather than at the relationship between the variables. You might try color coding the rows, as shown above. This might help them look across rather than look up and down.

Writing a numerical expression for a situation as simple as this may still be a challenge for some students, as it requires some metacognition. In other words, we are asking them not to make the leap to the answer first. We are asking them to think about what they do to get the answer.

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<table>
<tr>
<td>Symbolic Expression</td>
<td>
The symbolic expression generalizes the numeric expression. In other words, instead of writing numbers for the independent quantity, we use a variable. This creates a symbolic expression that can be used to find any amount. Again, the symbolic expression for an additive relationship will include addition.
<table>
<tr><th>T-shirts Purchased</th><th>Numerical Expression</th><th>Total # of Items</th></tr>
<tr><td>1</td><td>1 + 2</td><td>3</td></tr>
<tr><td>2</td><td>2 + 2</td><td>4</td></tr>
<tr><td>3</td><td>3 + 2</td><td>5</td></tr>
<tr><td>4</td><td>4 + 2</td><td>6</td></tr>
<tr><td>t</td><td>t + 2</td><td>t + 2</td></tr>
</table>
t + 2 = i
T-shirts + 2 = # of items
The symbolic expression exactly matches the numerical expression except that it includes variables for the quantities that change. (See 6.6B.)
</td>
<td>
The symbolic expression generalizes the numeric expression. In other words, instead of writing numbers for the independent quantity, we use a variable. This creates a symbolic expression that can be used to find any amount. Again, the symbolic expression for a multiplicative relationship will include multiplication.
<table>
<tr><th># of Hats for Sam</th><th>Numerical Expression</th><th># of Hats for Joe</th></tr>
<tr><td>1</td><td>1 x 2</td><td>2</td></tr>
<tr><td>2</td><td>2 x 2</td><td>4</td></tr>
<tr><td>3</td><td>3 x 2</td><td>6</td></tr>
<tr><td>4</td><td>4 x 2</td><td>8</td></tr>
<tr><td>S</td><td>S × 2</td><td>S × 2</td></tr>
</table>
S × 2 = J
Sam’s hats × 2 = Joe’s hats
The symbolic expression exactly matches the numerical expression except that it includes variables for the quantities that change. (See 6.6B.)
</td>
</tr>
</table>

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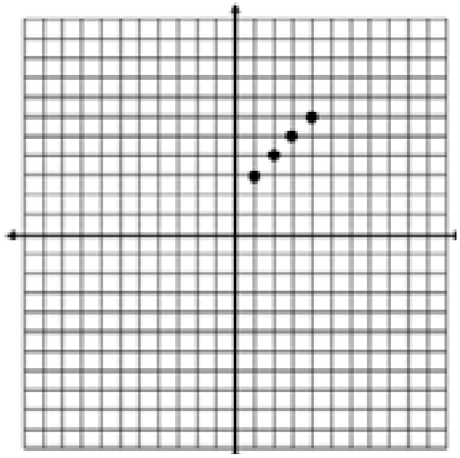
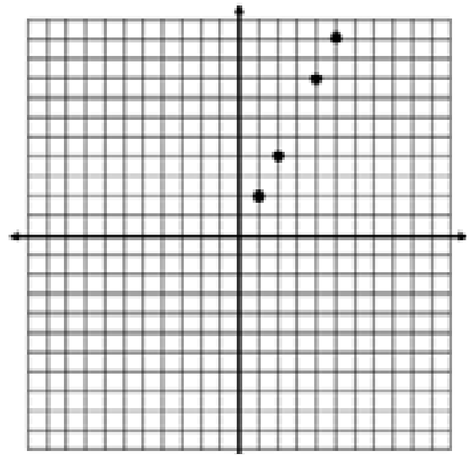
<table>
<tr>
<td>Graph</td>
<td>This is a graph of the table shown above.

Both additive and multiplicative relationships are linear when the points are graphed. It is also fairly easy to find other points on the graph that fit the verbal description, just by following the pattern of the points. In 7th grade, we will call this pattern a “rate of change” and in 8th grade, we will call it “slope.”

There is one difference between the graphs. Look back at the graph and follow the pattern in the points so that x = 0. It seems like the ordered pair should be (0, 2). Think about the problem situation now. Would Old Navy give David 2 pairs of socks if he didn’t buy any T-shirts? No way. If David buys 0 T-shirts, he will not get any for free. So the ordered pair for 0 T-shirts is (0,0).
What does this do to the graph? If this point is graphed, the rate of change between points is not constant anymore. This means that additive relationships are not proportional, even though they are linear. Sixth grade students do not need to know whether a relationship is proportional or non-proportional, but it is appropriate to discuss what happens at x = 0 with 6th grade students to help build meaning.</td>
<td>This is a graph of the table shown above.

Both additive and multiplicative relationships are linear when the points are graphed. It is also fairly easy to find other points on the graph that fit the verbal description, just by following the pattern of the points. In 7th grade, we will call this pattern a “rate of change” and in 8th grade, we will call it “slope.”

There is one difference between the graphs. Think about the problem situation for a moment. If Joe has 0 hats, then Sam also has 0 hats. So the ordered pair for 0 T-shirts is (0,0). If this point is graphed, the rate of change is still constant. This means that multiplicative relationships are proportional. Sixth grade students do not need to know whether a relationship is proportional or non-proportional, but it is appropriate to point out this difference between additive and multiplicative graphs.</td>
</tr>
</table>

7.4 Proportionality. The student applies mathematical process standards to represent and solve problems involving proportional relationships. The student is expected to:

7.4A represent constant rates of change in mathematical and real-world problems given pictorial, tabular, verbal, numeric, graphical, and algebraic representations, including d = rt. (RC2, Readiness Standard)

In 7th grade, students are working constant rates of change involving multiplicative relationships that are proportional. This means equations will be in the form y = kx, which means the equations are linear. Students must be able to represent a real-world or mathematical problem in five ways:

- by creating a table that matches the situation
- by choosing or creating a verbal description that matches the situation
- by writing numerical expressions that fit the constraints of the situation
- by creating a graph
- by writing an algebraic equation or expression that generates the points on the graph

<table>
<tr><th></th><th>Example and Explanations</th></tr>
<tr><td>Verbal Description</td><td>A verbal description of a multiplicative relationship will contain words that sound like a rate or comparison. Here are examples:
Joe has twice as many baseball hats as Sam.
There are half as many pears as bananas.
Look at the wording of the verbal description. In each case, there is a comparison or rate between the two variables that can be found through multiplication. The rate can be written as a fraction. The independent quantity is the number of baseball hats that Sam has. The dependent quantity is the number of baseball hats that Joe has. It is dependent because we can find the number of Joe's hats only if we know the number of Sam's hats. (Students began to study independent and dependent quantities in 6.6A.)</td></tr>
<tr><td>Numerical Expression</td><td>The numerical expression for a multiplicative relationship will have multiplication as the operation. The table below shows the baseball hat example above.
<table>
<tr><th># of Hats for Sam</th><th>Numerical Expression</th><th># of Hats for Joe</th></tr>
<tr><td>1</td><td>1 × 2</td><td>2</td></tr>
<tr><td>2</td><td>2 × 2</td><td>4</td></tr>
<tr><td>4</td><td>4 × 2</td><td>8</td></tr>
<tr><td>5</td><td>5 × 2</td><td>10</td></tr>
</table>
IMPORTANT NOTE: When students read this table, they need to focus on the relationships that go ACROSS the table, not up and down the columns. The multiplicative relationship that we are looking for is between the Number of Sam's hats and the Number of Joe's hats, not from Sam to Sam and Joe to Joe. (See the blue arrow above the table.)

Possible error alert: If students think that a multiplicative situation is additive, they are probably looking at the columns of the table individually rather than at the relationship between the variables. You might try color coding the rows as shown above. This might help them look across rather than look up and down.

Writing a numerical expression for a situation as simple as this may still be a challenge for some students because it requires some metacognition. In other words, we are asking them not to make the leap to the answer first. We are asking them to think about what they do to get the answer.</td></tr>
</table>

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Symbolic Expression

The symbolic expression generalizes the numeric expression. In other words, instead of writing numbers for the independent quantity, we use a variable. This creates a symbolic expression that can be used to find any amount. Again, the symbolic expression for a multiplicative relationship will include multiplication.

# of Hats for Sam	Numerical Expression	# of Hats for Joe
1	1 × 2	2
2	2 × 2	4
4	4 × 2	8
5	5 × 2	10
S	S x 2	S x 2

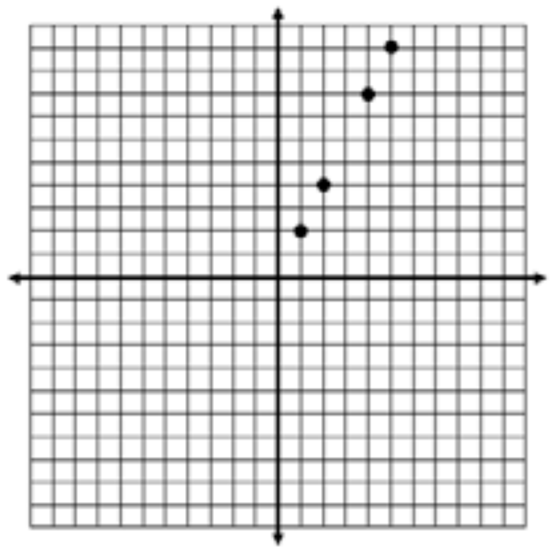
S × 2 = J

Sam's hats × 2 = Joe's hats

The symbolic expression exactly matches the numerical expression except that it includes variables for the quantities that change.

Graph

This is the graph of the table shown above.



Proportional relationships are linear when the points are graphed. It is also fairly easy to find other points on the graph that fit the verbal description just by following the rate of change. In 8th grade, we will call the rate of change its "slope."

Think about the problem situation for moment when Joe has 0 hats. If Joe has 0 hats, then Sam also has 0 hats. So the ordered pair for 0 t-shirts is (0,0). If this point is graphed, the rate of change is still constant. This means that multiplicative relationships are proportional.

8.4 Proportionality. The student applies mathematical process standards to explain proportional and non-proportional relationships involving slope. The student is expected to:

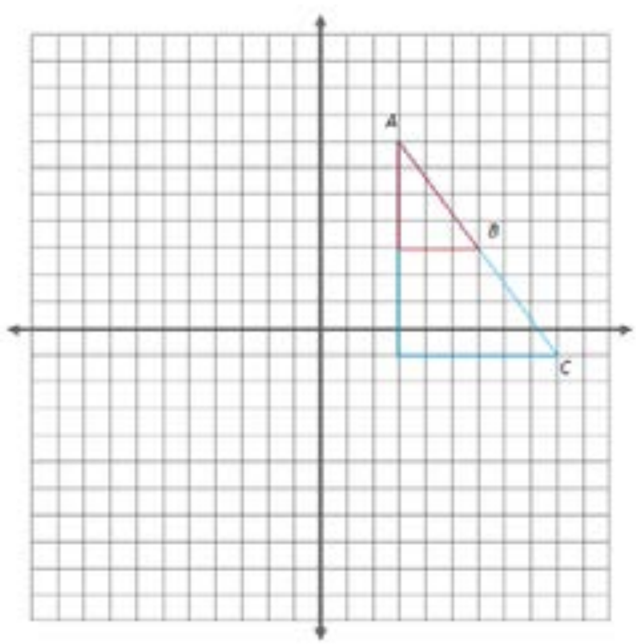
8.4A use similar right triangles to develop an understanding that slope, m, given as the rate comparing the change in y-values to the change in x-values, $(y_2 - y_1 / x_2 - x_1)$ is the same for any two points (x_1, y_1) and (x_2, y_2) on the same line. (RC2, Supporting Standard)

In the 2009 TEKS, slope was the exclusive domain of Algebra I teachers. For the revised TEKS, understanding and calculation of slope has been moved to 8th grade with an interesting development. The concept of slope is developed using similar right triangles set on a coordinate plane. Because slope appears in the Proportionality strand, it is taught as a rate that compares the change in y-values with the changes in x-values. Students also learn that the slope stays the same for any two points on corresponding sides of the triangles.

Example/Activity

Traditional teaching of slope includes choosing two points on a line and using a formula to find the slope. The treatment of slope shown below is significantly different from the traditional approach. Note that the verb in this Student Expectation (SE) says "*use* similar right triangles to develop an understanding." It does not say "calculate."

The graph below shows two similar triangles. There are three named points.



The SE says that students should understand slope as a rate that compares the change in y-value with the change in x-value. It also says that this rate is the same for any two points.

$$\text{Slope of AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 7}{6 - 3} = \frac{^-4}{3}$$

$$\text{Slope of BC} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{^-1 - 3}{9 - 6} = \frac{^-4}{3}$$

$$\text{Slope of CA} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - (^-1)}{3 - 9} = \frac{8}{^-6} = \frac{4}{^-3}$$

The three different slopes found are the same along the proportional sides of the similar triangles.

8.4 Proportionality. The student applies mathematical process standards to explain proportional and non-proportional relationships involving slope. The student is expected to:

8.4B graph proportional relationships, interpreting the unit rate as the slope of the line that models the relationship. (RC2, Readiness Standard)

This Student Expectation (SE) continues to build understanding of slope by relating it to a familiar concept to 8th grade students—*unit rate*. A unit rate is a special type of ratio where two quantities are being compared and the second quantity is 1. Since the slope of the line is presented as a unit rate, the slopes of the lines for this SE should be integers so that they can be written with a denominator of 1. Also, to keep the relationship proportional, the equations need to be written in the form y = mx.

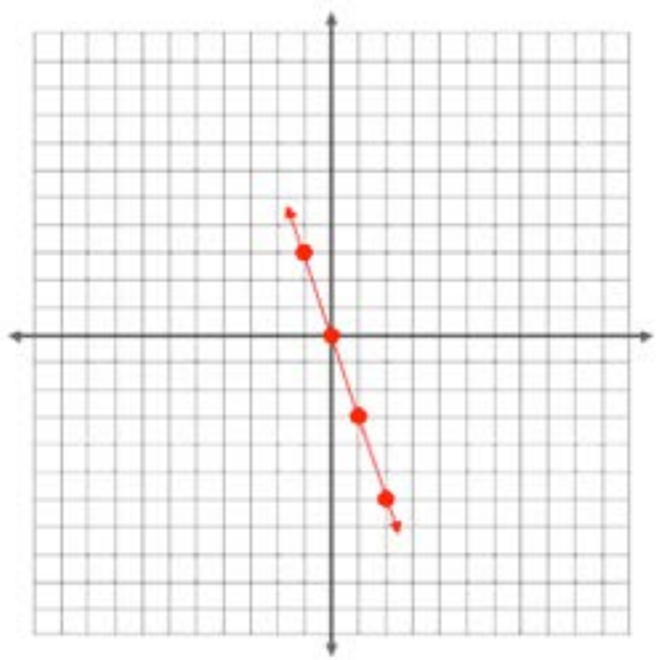
Although the SE does not state this, students will need to create a table from the equation and then graph the equation using the ordered pairs in the table.

Example/Activity

The following example shows a line that has a unit rate for the slope, $\frac{^-3}{1}$. The unit rate is visible in the table in the yellow highlighted line of the table below. In the graph, the unit rate is visible because, as you move from left to right on the graph, one grid line at a time, there is another point on the line.

y = ⁻3x

x	y
0	0
1	⁻3
2	⁻6
⁻1	3



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In 8.4A, students work with a formula for slope. This is quite abstract for even high school Algebra students. This SE approaches slope using something that is much more concrete for students. It uses the graph.

This SE takes a more practical and useful, in the writer's opinion, approach to finding slope on the graph, as the unit rate (or slope) is clearly visible in the diagram. Described below is the process for finding the slope by looking at a graphed line. It has been targeted at seeing a slope that is also a unit rate, but can be used to find the slope of any line that has been graphed.

Find the slope using a graph:

1. Follow the line from left to right and choose a point that has coordinates that are integers. (Don't choose the point that is furthest to the right if you want to see the unit rate immediately when you find the slope.)
2. Continue to follow the line to the next point on the line that has integer coordinates.
3. Go back to the first point and count the lines on the grid vertically to get from one point to the other. This number is the numerator of the slope. In terms of unit rate, it is the amount per one item (e.g., cost per gallon of milk).
4. Go back to the first point. Count the line(s) going across to get to the second point. If the points are consecutive, the number across should be 1, which shows the unit rate. (If you skipped a point when you chose the points, the denominator will be more than 1 and the slope will have to be simplified to see the unit rate.)
5. Make a fraction with the vertical change and horizontal change. This is the slope and it is also a unit rate. It shows the rise or fall of the line for every horizontal unit.

8.4 Proportionality. The student applies mathematical process standards to explain proportional and non-proportional relationships involving slope. The student is expected to:

8.4C use data from a table or graph to determine the rate of change or slope and y-intercept in mathematical and real-world problems. (RC2, Readiness Standard)

8.5 Proportionality. The student applies mathematical process standards to use proportional and non-proportional relationships to develop foundational concepts of functions. The student is expected to:

8.5F distinguish between proportional and non-proportional situations using tables, graphs, and equations in the form y = kx or y = mx + b, where b ≠ 0.
(RC2, Supporting Standard)

In 8.4C, students focus on two attributes of a line: the slope and the y-intercept. They learn to find the slope and the y-intercept from tables or graphs. The problems that students will use are real-world or mathematical problems, not straight calculation.

Why should the problems be based in a problem situation? The problem situation gives meaning to the slope and the y-intercept. The problem situation aids in understanding rather than being a word problem at the end of the section or chapter that gets skipped. Instruction should begin with the problem and understanding the problem. Then the meaning of the slope and y-intercept will be clear.

In 8.5F, students tell the differences and learn to identify a proportional or non-proportional situation from a table, graph, and equation. The discussion here will focus on tables and graphs. Equations are discussed with 8.5A and B.

An informal definition of slope is the rate of change from one point to another point on the line. It is a numerical representation of how much the line rises from one point to another and how far it moves across the grid from one point to another. Notice how much more meaning that has than the old $\frac{\text{rise}}{\text{run}}$, which has meaning for math teachers but not so much meaning for students.

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Example/Activity

The two problems below have essentially the same real-world situation. Both problems are linear. One problem has a y-intercept that is 0, whereas the y-intercept for the other is not 0.

Note: The problem situations below have slopes that are not 0 and are defined. Be sure that students also experience problem situations where the slope is 0 (horizontal lines) and situations where the slope is undefined (vertical lines).

Remember that the focus of this Student Expectation (SE) is to understand and be able to find slope and y-intercept from tables and graphs in problem situations. It isn't to bang out equations of lines.

Examples of Linear Problem Situations That Target Understanding and Finding Slope and y-Intercept

<table>
<tr><th></th><th>Problem Situation with 0 for y-intercept</th><th>Problem Situation with a number that is not 0 for the y-intercept</th></tr>
<tr><td>Problem Situation in Words</td>
<td><i>The dishwasher broke and the repair company was called. The repairman charges $60 per hour to fix the machine but does not have a service charge.</i>

The slope in this problem is a rate. It is a comparison between the number of hours and the cost per hour.

$\text{slope} = \frac{\$60}{1\text{ hour}}$ or $60 per hour or 60

The y-intercept is the point at which the work hasn't started yet but the repairman has walked in the door to do the repair.

y-intercept = $0 = 0</td>
<td><i>The washing machine broke and the repair company was called. The repairman charges $80 for a service fee plus $40 per hour.</i>

The slope in this problem is a rate. It is a comparison between the number of hours and the cost per hour.

$\text{slope} = \frac{\$40}{1\text{ hour}}$ or $40 per hour or 40

The y-intercept is the point at which the work hasn't started yet but the repairman has walked in the door to do the repair.

y-intercept = $80 = 80</td></tr>
<tr><td>Table</td>
<td>For tables, it is easiest to begin at 0. This will identify the y-intercept. If possible, include 1 in the table.
<table><tr><th>x</th><th>y</th></tr><tr><td>0</td><td>0</td></tr><tr><td>1</td><td>60</td></tr><tr><td>2</td><td>120</td></tr><tr><td>3</td><td>180</td></tr></table>
Find the slope in the table:
$\frac{y_2 - y_1}{x_2 - x_1} = \frac{60 - 0}{1 - 0} = \frac{60}{1} = 60$

The y-intercept is the point on the line where the line crosses the y-axis. All points on the y-axis have this form:
(0, _____)
Another way to say this is that all y-intercepts have a 0 for the x-coordinate. If 0 has been included in the table for x, identifying the y-intercept is as simple as identifying it in the table (0, 0).</td>
<td>For tables, it is easiest to begin at 0. This will identify the y-intercept. If possible, include 1 in the table.
<table><tr><th>x</th><th>y</th></tr><tr><td>0</td><td>80</td></tr><tr><td>1</td><td>120</td></tr><tr><td>2</td><td>160</td></tr><tr><td>3</td><td>200</td></tr></table>
Find the slope in the table:
$\frac{y_2 - y_1}{x_2 - x_1} = \frac{120 - 80}{1 - 0} = \frac{40}{1} = 40$

The y-intercept is the point on the line where the line crosses the y-axis. All points on the y-axis have this form:
(0, _____)
Another way to say this is that all y-intercepts have a 0 for the x coordinate. If 0 has been included in the table for x, identifying the y-intercept is as simple as identifying it in the table (0, 80).</td></tr>
</table>

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Table	Note: The relationship for this problem is proportional. The reason we know that it is proportional is that the ratios between the x- and y-coordinates will simplify to the same number. $\frac{60}{1}=60$ $\frac{120}{2}=60$ $\frac{180}{3}=60$ Note: If the table will be used for graphing, be sure to show more than two points in the table as a built-in arithmetic check. If two points are graphed, they will definitely make a line. If three points are graphed and they don't make a straight line, then the arithmetic for one of the points is incorrect.	Note: The relationship for this problem is *non-proportional*. The reason we know that it is non-proportional is that the ratios between the x- and y-coordinates yield different numbers. $\frac{120}{1}=120$ $\frac{160}{2}=80$ $\frac{200}{3}=66.7$ Note: If the table will be used for graphing, be sure to show more than two points in the table as a built-in arithmetic check. If two points are graphed, they will definitely make a line. If three points are graphed and they don't make a straight line, then the arithmetic for one of the points is incorrect.

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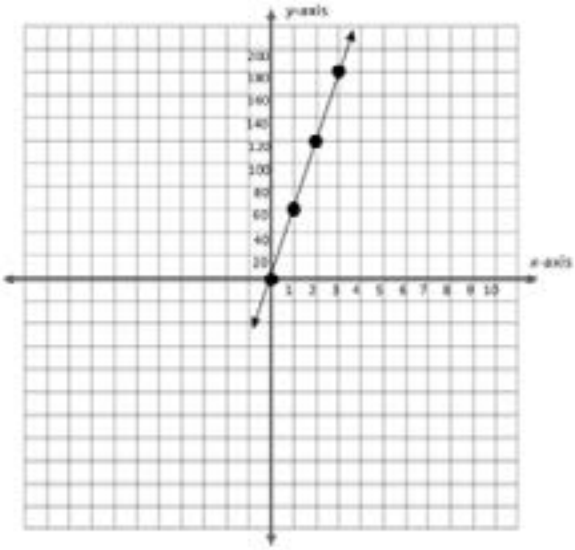
<table>
<tr>
<td>Graph</td>
<td>
Use the data in the table to make a graph of the problem situation.



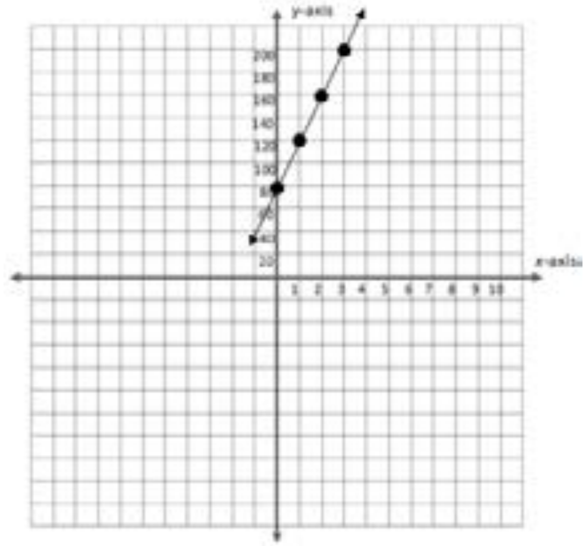
There are two ways to find the slope on the graph. First, you can choose two points and put the coordinates into the slope formula.

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{60 - 0}{1 - 0} = \frac{60}{1} = 60$$

This certainly works, providing no arithmetic errors are made.

Another way to find the slope is to use the points on the graph to count the vertical change and the horizontal change.

1. Choose any two points on the line.
2. Start with the vertical change. From one point to another, count up or down until you get to the second point. Remember the vertical scale on this graph is 20. If you go up to get to the second point (or toward the positives), the change is positive. If you go down, the change is negative.
3. Go back to the original point. Now find the horizontal change. If you go toward the positives, the horizontal change is positive. If you go toward the negatives, the horizontal change is negative.
</td>
<td>
Use the data in the table to make a graph of the problem situation.



There are two ways to find the slope on the graph. First, you can choose two points and put the coordinates into the slope formula.

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{120 - 80}{1 - 0} = \frac{40}{1} = 40$$

This certainly works, providing no arithmetic errors are made.

Another way to find the slope is to use the points on the graph to count the vertical change and the horizontal change.

1. Choose any two points on the line.
2. Start with the vertical change. From one point to another, count up or down until you get to the second point. Remember the vertical scale on this graph is 20. If you go up to get to the second point (or toward the positives), the change is positive. If you go down, the change is negative.
3. Go back to the original point. Now find the horizontal change. If you go toward the positives, the horizontal change is positive. If you go toward the negatives, the horizontal change is negative.
</td>
</tr>
</table>

is what leads to the equation. The only thing that changes as you move down the column is the value of x. The part that does not change is the equation.

$$y = x\,(60)$$

or, written more traditionally,

$$y = 60x$$

Note: Although this seems difficult for 8th graders and has been traditionally reserved for high school, the 7th grade Student Expectations (SEs) provide tremendous support for students to understand this. Keeping the numbers simple for students to build understanding will help students get the processes and concepts down. Once students are solid with that, move to more complicated numbers.

Finally, be sure to use your graphing technology with this SE. It's a perfect place, along with 8.5B, to see the difference between proportional and non-proportional situations, their tables, graphs, and equations.

8.5 Proportionality. The student applies mathematical process standards to use proportional and non-proportional relationships to develop foundational concepts of functions. The student is expected to:

8.5B represent linear non-proportional situations with tables, graphs and equations in the form of y = mx + b, where b ≠ 0. (RC2, Supporting Standard)

8.5F distinguish between proportional and non-proportional situations using tables, graphs, and equations in the form y = kx or y = mx + b, where b ≠ 0. (RC2, Supporting Standard)

In 8.4C, students found attributes of a line (the slope and the y-intercept) from a table and from a graph. All of the lines came from a mathematical or a real-world linear situation that was proportional. 8.5B now turns students' focus beyond the attributes to the line itself.

8.5B focuses on situations that are non-proportional. In other words, $\frac{y}{x}$ is not constant for the points on the line, but the rate of change is constant. The line may cross the y-axis at any point other than (0, 0). Therefore, the y-intercept has a value that is not 0.

In 8.5F, students find the differences and learn to identify a proportional or non-proportional situation from a table, graph, and equation. The discussion here will focus on equations that represent proportional relationships. Tables and graphs are discussed with **8.4C**. Proportional situations are discussed in **8.5A**.

Example/Activity

8.4C showed a non-proportional problem situation, its table, and its graph. The equation is shown here. For more information about the table and graph, see **8.4C**.

Equations that represent non-proportional situations are written in the form y = mx + b. m is the slope and b is the y-intercept.

Examples of problem situations that are non-proportional:

- It costs $10 to join a music club. Each song costs $0.25.
- Overdue library fines are $1 (for being tardy returning books) + 5 cents per day.
- Text messaging is $10 per month for the first 100 messages + 10 cents for each message after that (a totally impractical plan for any teenager).

For each of these situations, there is a flat fee plus a cost per item.

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