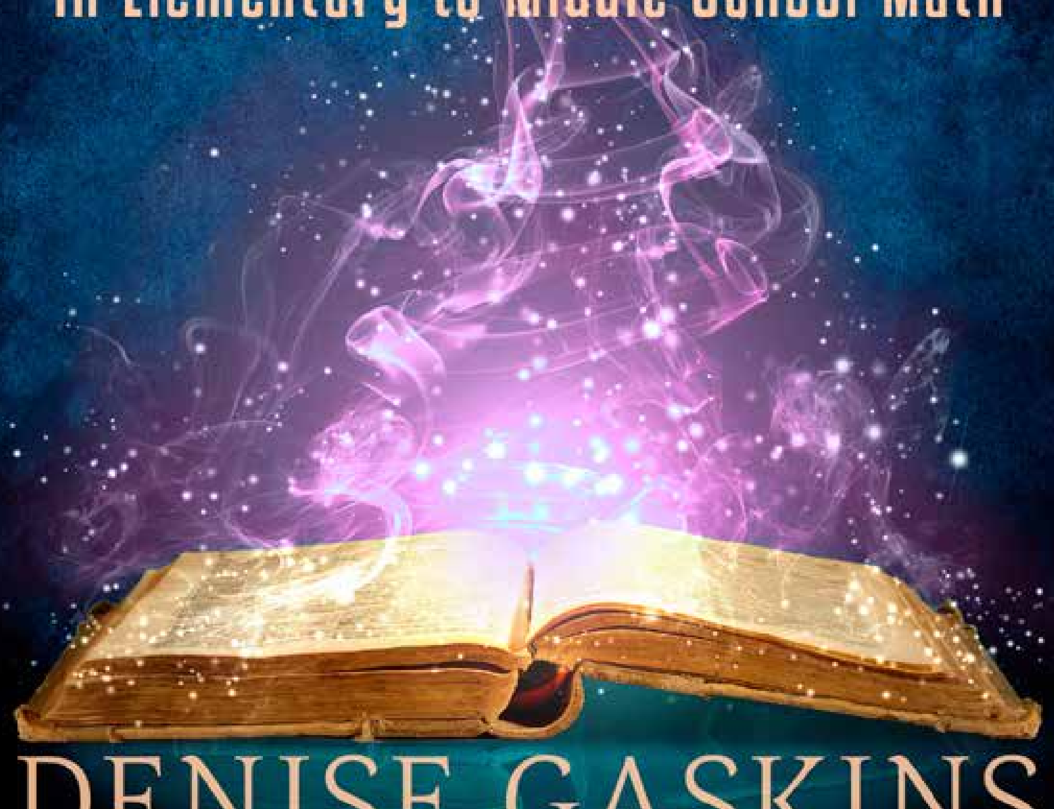
PLAYFUL MATH SINGLES

WORD PROBLEMS from Literature

Help Students Master Problem Solving in Elementary to Middle School Math



DENISE GASKINS

Author of Let's Play Math: How Families Can Learn Math Together—and Enjoy It

Books by Denise Gaskins

Let's Play Math: How Families Can
Learn Math Together—and Enjoy It

Math You Can Play Series
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A PLAYFUL MATH SINGLE

WORD PROBLEMS from Literature

Help Students Master Problem Solving in Elementary to Middle School Math

SECOND EDITION

Denise Gaskins

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Contents

Preface to the Second Edition

As a math coach, I love showing adults and children how to look at math with fresh eyes, to explore the adventure of learning math as mental play, which is the essence of creative problem-solving. Mathematics is not just rules and rote memory. Math itself is a game, playing with ideas.

I've written several books to help families play math together. But at heart, I've always been a fiction fan—especially fantasy fiction. And this book, *Word Problems from Literature,* lets me bring that love of story to the surface.

This is one of my all-time favorite books, and I've had so much fun with this new edition: adding stories, writing make-your-own-problem prompts, sneaking extra teaching tips into the worked-out solutions, creating an almost-magical guide to helping kids reason their way through math problems.

To provide support when your children get stuck, I added my favorite problem-solving tip, the Four Questions that push students to apply their own common sense, emphasizing the importance of reasoning about math. And to give you a boost when *you* get stuck, I beefed up the explanations of the advanced puzzles, adding several tips on understanding and teaching fraction division and other monster calculations.

There's a new section in the student workbook on "What If I'm Not Good at Math?" to help children develop a problem-solving mindset. Plus, I wrote more than 30 new word problem prompts to get your students writing their own creative math stories.

Most of all, I've tried as much as possible to encourage both adults and students to work at making sense of the math, seeing how the numbers relate, avoiding the crutch of standard textbook rules so you can experience the joy of figuring things out for yourself.

Those of us who made it through school math by memorizing and following rules eventually paid the price. We came to the point where our minds could hold no more, where the rules we had learned all jumbled together, where we felt lost in the dark as the rock began to crumble beneath our metaphorical feet while the wizard rammed his staff into the ground and cried, "You shall not pass!"

Some people reach the point of mental exhaustion with upper-elementary fractions or middle school ratios and percents, while others make it into algebra or geometry before they crash. A rare few have a good enough mental filing system that they can proceed as far as calculus before it all falls apart.

Estimates vary, but anywhere from half to three-fourths of adults suffer from some level of math anxiety due to their school experience.

Can we spare our children this fate?

We must strive to teach math in a way that makes sense, where children don't just memorize the rules we tell them but see clearly how math concepts connect, drawing their own conclusions, building their understanding into a solid foundation for future learning.

And we must give our students the tools to build on this foundation, problem-solving tools that will help them face and conquer any new math monster that comes their way.

This book will help you do that.

Have fun playing math with your kids!

—Denise Gaskins,
rural Illinois, July 2022

Storying—
encountering the world
and understanding it contextually
by shaping ideas,
facts,
experience itself
into stories—
is one of the most fundamental means
of making meaning.
As such, it is an activity
that pervades all learning.

—Gordon Wells

Word Problems as Mental Manipulatives

It is the duty
of all teachers,
and of teachers of mathematics
in particular,
to expose their students
to problems
much more than to facts.

—Paul Halmos

1

Puzzles Build Mathematical Thinking

Archimedes tried to find the distance around a circle and almost discovered calculus. Pierre de Fermat predicted the result of a gambling game and laid the foundations of probability. Leonhard Euler went for an afternoon walk over the bridges of Königsberg and invented topology. Georg Cantor created a way to count infinity and opened a whole new world of modern math.

Through the centuries, mathematics has grown as mathematicians struggled with and solved challenging puzzles.

Problems are the raw material of math, the ore we dig, grind up, and melt, refining it to produce ideas. Our understanding of math grows as we play with problems, puzzle them out, and look for connections to other situations. The threads that connect these problems become the web of ideas we call mathematics. Each puzzle we solve adds a new thread to the web, or strengthens one that already exists, or both.

If we want our children to learn real math, we need to offer them plenty of problems to solve. A child may work through several pages of number calculations by rote, following memorized steps, but a good problem demands more thought.

The story in a word problem puts flesh on the abstract bones of arithmetic, encouraging children to ponder what it means for one thing to be bigger than another, or smaller, or faster, or slower, or made up of several parts.

Math professor Herb Gross says: "We teachers so often hear students summarize a course by saying, 'I could do everything except the word problems.' Sadly, in the textbook of life, there are only word problems."

Our children will meet numbers in many guises throughout their lives. Few of these will be as straight-forward as a textbook word problem, but real-life problems and school math stories will always have certain things in common. Quantities will be related to each other in a given proportion. Situations will be complex, and solutions may require many steps.

Story problems give students a chance to grapple with these complexities in a controlled environment, where we can increase the difficulty in stages over several years.

The Purpose of Word Problems

Word problems feed a student's imagination. Like other puzzles, word problems are often artificial, but that needn't diminish our pleasure in solving them.

In working a puzzle, we expect to find difficulties and setbacks. That's part of the game. Similarly, when students approach word problems as puzzles, they become less concerned with rushing to an answer and more interested in figuring out the relationships within the story.

Playing with puzzles strengthens mathematical understanding, according to retired professor Andre Toom. Originally from Russia, Toom taught math in several colleges and universities around the world.

Here's how he explains the purpose of story problems:

> "Word problems are very valuable—not only to master mathematics, but also for general development.

"Especially valuable are word problems solved with minimal scholarship, without algebra, even sometimes without arithmetic, just by plain common sense. The more naive and ingenuous is the solution, the more it provides the child contact with abstract reality and independence from authority, the more independent and creative thinker the child becomes.

"When we teach children to solve problems in school, we do not expect them to meet exactly and literally the same problems in later life. Mathematical education would be next to useless if its only use were literal. We want much more, we want to teach children to solve problems in general.

"In this respect traditional word problems are especially valuable, because to solve a word problem, you have to understand what is said there.

"This function of word problems is very poorly understood. The main educative value of word problems is that they serve as mental manipulatives, paving children's road to abstract thinking.

"For example, coins, nuts, and buttons are clearly distinct and countable, and for this reason are convenient to represent relations between whole numbers. The youngest children need some real, tangible tokens, while older ones can imagine them, which is a further step of intellectual development. That is why coin problems are so appropriate in elementary school.

"Pumps and other mechanical appliances are easy to imagine working at a constant rate. Problems involving rate and speed should be common already in middle school. Trains, cars, and ships are so widely used in textbooks not because all students are expected to go into the transportation business, but for another, much more sound reason: These objects are easy to imagine moving at constant speeds.

"There is an important similarity between children's play and mathematics: In both cases, creative imagination is essential."

—*Andre Toom*

The Trouble with Word Problems

How can we teach our students to reason their way through math problems? We must help them develop the ability to translate real-world situations into mathematical language.

Most young children solve story problems by the flash-of-insight method, hearing the problem and knowing by instinct how to solve it. This is fine for simple problems like "Four kittens played with a yarn ball. Two more kittens came to join the fun. Then how many kittens were playing with the yarn ball?"

When problems grow more difficult, however, that flash of insight becomes less reliable. We find our children fidgeting with their paper, staring out the window, complaining, "I don't know what to do. It's too hard."

They need a tool that will work when insight fails.

In solving a textbook word problem, students blaze a trail through the unknown. They must:

- Read the problem and understand what it's asking.
- Translate the problem into a mathematical calculation or algebraic equation.
- Do the calculation or solve the equation.
- Make sense of the answer, explaining how it relates to the original problem.

The first two steps give students the most trouble. They could calculate just fine, if they could decide which calculation to do.

But they don't know how to translate the problem from English into Mathish.

One common classroom approach emphasizes *key* or *signal* words. For instance, we can tell our children that a problem asking "How many more?" will probably require subtraction. The question asks for the difference between two quantities, and *difference* is the answer when you subtract. But this technique only works for the simplest word problems.

Unfortunately, key words can mislead a careless reader.

For example, consider this question: "What must we add to 2 to get 7 as the sum?" The words *add* and *sum* are designed to lead an unwary child into the trap of answering 2 + 7 = 9.

Or this one: "There are 21 girls in a class. There are 3 times as many girls as boys. How many boys are in the class?" Would the word *times* trick your child into multiplying 21 × 3 = 63?

I do teach a few key words to my students. My favorite is the translation "*of* = multiply" when dealing with fraction and percent problems. But I want my kids to read a math problem and to analyze what is happening, no matter what words are used to describe the situation. For this, they need a more powerful tool.

Four Common-Sense Questions To Solve Any Problem

Because all people begin their school career by solving problems intuitively, children expect to look at a math puzzle and instantly see the answer. As students progress through the years, their math problems develop layers of complexity. Solution by intuitive leap becomes impossible. Too often a frustrated child concludes, "I'm just not good at math."

But the truth is that *nobody* is good at math, if you define "good at math" to mean they can see the answer instantly. Here's a more useful definition: You're good at math if you have problem-solving tools and know how to use them.

And *that* is something everyone can learn.

When children are faced with a math problem, they need to combine the given facts in some way to reach the required answer. But rarely can they do it in a single leap. So encourage them to take one little step at a time. Even if they can't solve the problem, can they think of a way to get closer to the goal?

Teach your child to ask these four questions…

(1) What Do I Know?

Read the problem carefully. Reread it until you can describe the situation in your own words.

List the facts or information given in the problem. Notice math vocabulary words like *factor, multiple, area,* or *perimeter.* What do you remember about those topics?

Sometimes a problem tries to trick you. Watch out for mixed units: If one length is given in inches and another is given in yards, make them consistent.

Try to express the facts in math symbols or using the visual algebra of a bar model diagram.

(2) What Do I Want?

Describe the goal, what the problem is asking you to find. What will your answer look like?

Notice important words like *product, sum, next,* or *not.* Small words like "not" are especially easy to miss.

Try to express the goal in math symbols or using the visual algebra of a bar model diagram.

(3) What Can I Do?

Imagine yourself in the story situation, applying your hard-earned common sense. If this actually happened to you, what would you do?

Mix things around in your mind. Combine the given facts.

Have you worked a problem like this before? How did you solve that one? Will that method, or something like it, work here?

If you're using a bar model, think about ways you might move or cut the blocks to discover new relationships.

Try a tool from your problem-solving toolbox:

- Draw and label a diagram or sketch.
- Act the problem out, step by step.
- Make a systematic list, chart, table, or graph.
- Look for a pattern.
- Simplify the problem. Try it with smaller numbers.
- Change your focus. Restate the problem in another way.
- Look for a related problem. How is it the same? How is it different?
- Think about "before" and "after" situations.
- Work backwards. Start at the end of the problem and find a path back to the beginning.
- Guess and check. Try something to see if it works, and then make sure you know why.

If you are completely stumped, explain the problem to another person. Talking it over might help unclog your brain, opening your eyes to clues you had missed until you put them into words. Or take a break to let the problem simmer in your unconscious mind while you do other things.

(4) Does It Make Sense?

Don't neglect this last step! When you think you have found the answer, always look back at the original problem one more time.

Does your answer make sense? Did you leave anything out?

Lay the Foundation: One-Step Problems

2

Word Problems Inspired by *Mr. Popper's Penguins*

A family of four adopts several penguins and teaches them to perform tricks.

In this chapter, I'll demonstrate the bar model problem-solving tool in action with a series of early-elementary word problems. For your reading pleasure, I've written stories that might happen in a situation like the family-favorite read-aloud book, *Mr. Popper's Penguins* by Richard and Florence Atwater.

Try your hand at the problems before reading my solutions. Can you draw a bar model diagram to represent each story situation?

[1]

During the winter, Papa read 34 books about Antarctica. Then he read 5 books about penguins. How many books did Papa read in all?

[2]

When Papa opened the windows and let snow come into the living room, his children made snowballs. The girl made 18 snowballs. Her brother made 14 more than she did. How many snowballs did the boy make? How many snowballs did the children make altogether?

[3]

Papa had 78 fish. The penguins ate 40 of them. How many fish did Papa have left?

[4]

The family dressed in their best clothes for their meeting with the show manager. Mama had a ribbon 90 centimeters long. She had 35 cm left after making a bow for her daughter's hair. How much ribbon did Mama use to make the bow?

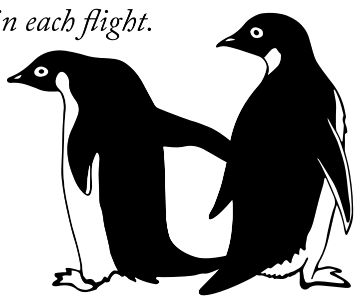
[5]

The penguins did theater shows for 2 weeks. They performed 4 shows every week. How many shows did the penguins perform?

[6]

While they were staying at the hotel, Papa put a leash on one of the penguins and took him for a walk. They climbed up 3 flights of stairs. There were 10 steps in each flight.

Then the penguin flopped onto his stomach and slid down all the stairs. He pulled Papa with him all the way. How many steps did Papa fall down?



Bar Models and Answers

[1]

During the winter, Papa read 34 books about Antarctica. Then he read 5 books about penguins. How many books did Papa read in all?

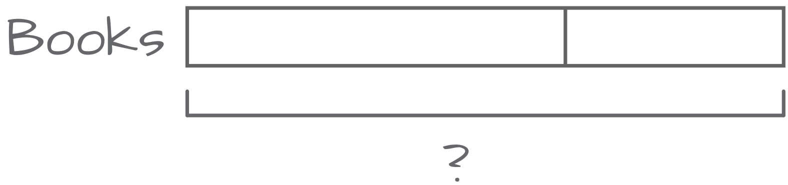
One of the best ways to approach a word problem is to ignore the numbers. Encourage your child to think about what is happening and describe the story in general words, without using numbers: "Papa read some books. He read a lot of books about Antarctica. And he read some books about penguins, too."

Whenever you or your child feel confused about a word problem, come back to this basic starting point. Tell the story without numbers, so you can focus on the relationships. After you make sense of what is happening, then you can put the numbers back in.

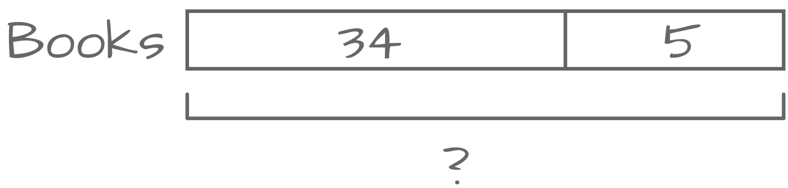
We can imagine the books standing together on a library shelf, sorted by topic. We draw one long bar to represent all the books. Then we split it into two parts: books about Antarctica, and books about penguins.

Notice that the diagram is not to scale. That's the algebraic power of the bar model: The bars are variables, and we needn't worry about getting a perfect proportion. As long as we fit our model to the story relationships, the numbers will work themselves out.

The story asks us to find the total number of books, so we indicate this with a bracket either above or below the bar. We put a question mark for our goal.



Now we've drawn out the basic relationship in our word problem, so we're ready to add in the numbers. Each number goes inside the bar that represents that amount.



The whole is the sum of its parts, so we need to add to find our answer.

$$34 + 5 = 39$$

Papa read thirty-nine books.

[2]

When Papa opened the windows and let snow come into the living room, his children made snowballs. The girl made 18 snowballs. Her brother made 14 more than she did. How many snowballs did the boy make? How many snowballs did the children make altogether?

In early arithmetic, most addition and subtraction problems involve active situations: joining, separating, growing, shrinking, giving, and taking away. The movement in these stories offers children a strong hint about how to do the math.

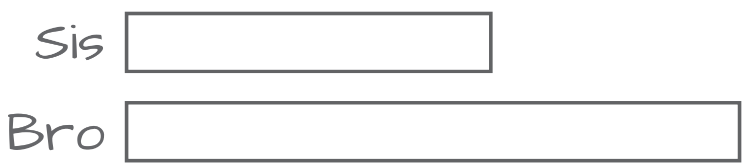
But when problems are static, without movement, they pose a greater challenge. Stories about collections with different categories of items, groups with different types of people or animals, or comparisons between quantities of different sizes—none of these situations give students an action clue that shows what to do.

With young children first learning to deal with static situations, I like to offer them an action they can visualize. I might say something like:

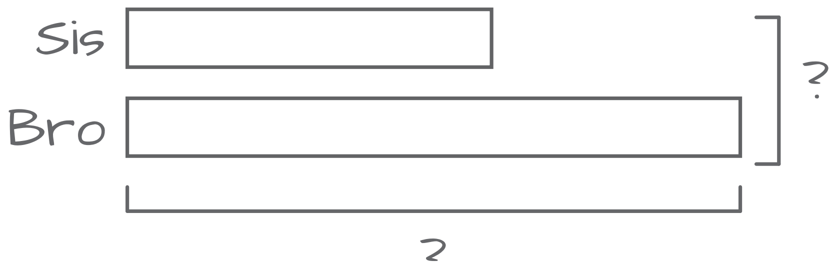
"Pretend that we lined up all the snowballs side-by-side, with the boy's snowballs in one row and his sister's snowballs in another. Which line would be longer? How many of the snowballs would sit side-by-side? How many would be left without a partner? How many more snowballs would the girl have to make if she wanted the rows to match?"

To model a comparison, we draw a bar for each thing we're comparing. The left edges of the bars line up, making it easy to see which is longer or shorter.

Here our bars will represent the number of snowballs each child made. The boy made more snowballs than his sister, so his bar needs to be longer.

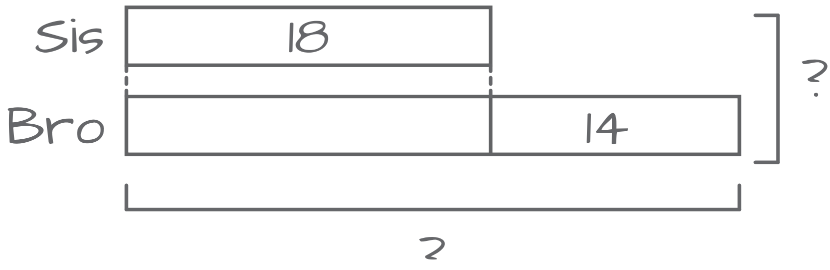


The question has two parts. We need to figure out how many snowballs the boy made, and the total of all the snowballs together. Add brackets and question marks to show the quantities we need to find.



That gives us the basic situation of our word problem. Now we are ready to fill in the number clues. It's easy to write 18 in the bar for the girl's snowballs, but how can we show that her brother made fourteen more than she did?

When two blocks are the same length, we often draw dotted lines to connect their endpoints. We mark off a section of the brother's bar to make a block equal to his sister's, then write 14 into the extra chunk.



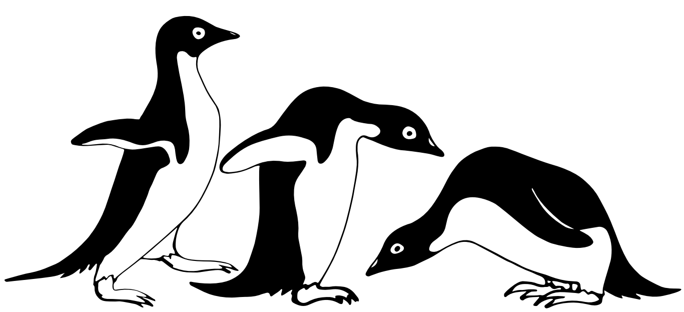
The whole is the sum of its parts, so we need to add the two chunks to find out the boy's number.

$$18 + 14 = 32$$

The boy made thirty-two snowballs. And now we can add both children's amounts together to find the total.

$$18 + 32 = 50$$

The children made fifty snowballs altogether.



Make Your Own Math

One of my favorite math journaling activities is to have children write and share their own problems. We'll talk about this more in Chapter 7, but for now you can try the following story problem prompts.

What kind of things do the characters in your story world count? Do they measure or cut things, or do crafts? What do they like to collect or share, or what are their favorite foods? What kind of math stories will you create?

[7]

Write a changing-amount problem set in the world of a book or movie you enjoy. Your story will include:

- ♦ A beginning amount.
- ♦ Some type of change—joining or separation, giving or taking, growth or shrinking.
- ♦ And the final amount.

Tell any two of these numbers. Then ask your reader to figure out the third.

[8]

Write a collection problem set in the world of a book or movie you enjoy. Your story will include:

- ♦ A whole amount (the collection).
- ♦ Two types or groups of people, animals, or things that are parts of the collection.

Leave either the whole or one of the parts a secret. Then ask your reader to find it out.

[9]

Write a comparison problem set in the world of a book or movie you enjoy. Your story will include:

- A smaller amount.
- A larger amount.
- The difference between them.

Leave one of these numbers a secret. Then ask your reader to figure it out.

[10]

Write a problem that contains some sort of this-per-that unit.

For example, where in your story world would people think about the number of legs per animal, cookies per child, dollars per item they buy, or something like that?

Your problem has three numbers:

- How many same-size units.
- The size of a single unit.
- The total amount.

Tell any two of these numbers. Then ask your reader to figure out the third.

[11]

Write any kind of problem you like.

A Note about Copyright and Trademarks

Most of the books and movies mentioned in this book are the protected intellectual property of their authors or estates, or of the company which bought those rights.

I have created math problems inspired by these stories, but I've peopled my work with generic characters (a father, a faun, a rogue starship captain) to avoid infringing on the original author's creation.

When you and your students write problems for your own private use, feel free to use your favorite characters from any story. This is a form of fan fiction. But if you decide to share your creation beyond your own home or classroom, then be sure to "genericize" it first. Change or remove the proper names, using general descriptions instead.

For example, if your children love the Star Wars movies, they might want to use a Jedi in their story problem. Instead, encourage them to write about "an interstellar justice warrior with an energy sword." Or "an alien master of martial arts training a cocky but inexperienced young apprentice."

We'd love to add your child's story to the Student Math Makers Gallery.†

† *tabletopacademy.net/math-makers*

EDUCATION / MATHEMATICS

Help Your Child Succeed in Math

You can help prevent math anxiety by giving your children the mental tools they need to conquer math problems.

Young children expect to look at a word problem and instantly see the answer. But as they get older, their textbook math problems also grow in difficulty, so this solution-by-intuitive-leap becomes impossible. Too often the frustrated child concludes, "I'm just not good at math."

But with practice, any student can learn to master word problems.

Word Problems from Literature features math puzzles for elementary and middle school students inspired by classic books such as *Mr. Popper's Penguins* and *The Hobbit*. Denise Gaskins demonstrates step by step how to solve these problems—and how to build a strong foundation of problem-solving skills that can handle any situation.

And when you finish the puzzles in this book, Denise shows you how to create your own word problems from literature, using your child's favorite story worlds.

You'll love this book, because it prepares your children for mathematical success. Order your copy of *Word Problems from Literature* today.

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If you're using these word problems with children, check out the companion *Word Problems Student Workbook*.

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