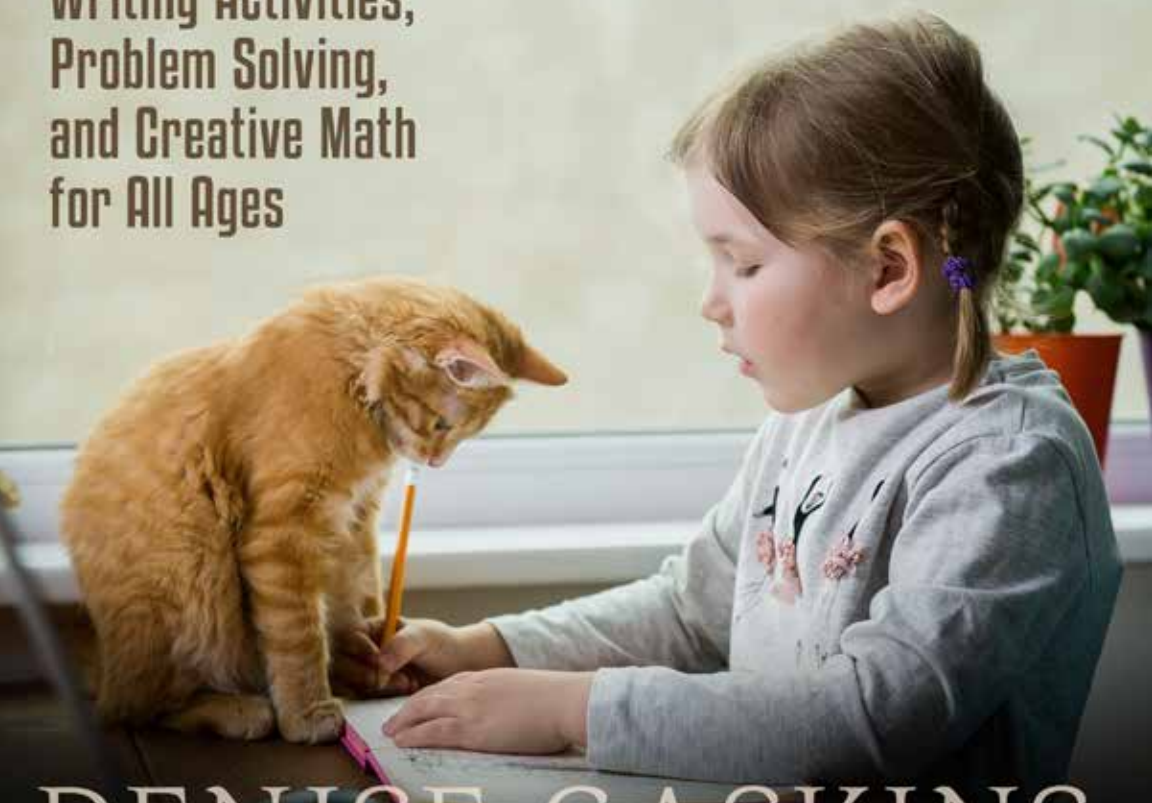
PLAYFUL MATH SINGLES

312 Things To Do with a MATH JOURNAL

Games, Number Play, Writing Activities, Problem Solving, and Creative Math for All Ages



DENISE GASKINS

Author of Let's Play Math: How Families Can Learn Math Together—and Enjoy It

From My Journaling Beta-Testers

"We really enjoyed these!"

"I remember doing pages and pages of dull equations with no creativity or puzzle-thinking, but now as a homeschool mom, I'm actually enjoying math for the first time! My daughter's math skills have skyrocketed and she always asks to start homeschool with math."

"Thank you for a great intro to Playful Math!"

"All of the kids were excited about their journals. My oldest kept going without prompting and did several more pages on his own."

"We had a lot of fun doing your math prompts. We had never done any math journaling before, but we will certainly integrate this into our weekly routine from now on."

A PLAYFUL MATH SINGLE

312 Things To Do WITH A Math Journal

GAMES, NUMBER PLAY, WRITING ACTIVITIES, PROBLEM SOLVING, AND CREATIVE MATH FOR ALL AGES

Denise Gaskins

Tabletop Academy Press

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This is the wonderful thing about just thinking and playing with half-formed thoughts: often exciting ideas will flash into your brain when you least expect them.

— James Tanton

Preface: Cats and Math

In all the books I write, my goal is to encourage families to explore the world of math in a new way. To enjoy thinking and playing with ideas. To delight in the beauty of numbers, shapes, and patterns. And to fight for true understanding, doing whatever it takes to help math make sense for our children.

With that "fight for understanding" on my mind, back in January 2021 while the Covid-19 pandemic raged on, I launched my first math journaling Kickstarter project, called "Make 100 Math Rebels."

To my surprise, my daughter's cats Cimorene and Puck signed on to lead the Kickstarter promotions. Because cats know the Internet, and they know how to make people do whatever they want.

Or at least, that's what they told me.

Cimorene thought everyone should order one of the big sets of three paperback or hardcover journals. Books come in boxes, after all, and boxes are important to cats.

Puck agreed that boxes are a good thing. But he thought people should get into journaling in any format they liked. Puck values curiosity and creative thinking, and math journaling is all about teaching students to explore ideas and think creatively about math.

The Kickstarter succeeded beyond expectations. More than a hundred parents and teachers signed on to help me create three beautiful

journals for adventurous students, with full-color, parchment-style pages that make writing fun. Along with the journals, I offered supporters a checklist of one hundred math journaling prompts to help draw out their children's mathematical thinking.

Months later, I've tripled that original list into a full book with more than three hundred ideas to spark creative, liberal-arts mathematics. As I'm wrapping up work on this book, the cats are still plotting ways to spread the news about writing to learn math.

Cimorene worries that many children (and their parents) struggle with a fear of math. She thinks that's because school math can seem stiff and rigid. To children, it can feel like "Do what I say, whether it makes sense or not."

That's a horrid feeling. It reminds Cimorene of being trapped in the carrier bag for a trip to the vet. She wants everyone to know that math journaling is not like that. In fact, journaling makes number play fun like catnip.

Nobody wants a trip to the vet. Cimorene hopes you'll take her advice and try a math journaling prompt instead.

But Puck thinks most people are confused by the idea of math journals.

Cats know how important it can be for students to explore math and try new things. Playing with ideas is how kittens (and humans) learn. Many people understand that children need to do hands-on experiments in science. But Puck believes that most adults don't know how to do a math experiment.

The Cat Escape Puzzle

To show how your children can experience the joy of creative reasoning, Puck decided to create a puzzle about saving cats from their mortal enemy.

> *Imagine the dog ran into the kitchen, so the cats need to get off the floor. There are three chairs around the table. There are two cats, and*

they don't like to share a seat. How many different ways can the cats jump onto the chairs?

When Puck was a little barn kitten, his mama taught him that the best way to learn is to figure things out for yourself. So he won't give you the answer to his puzzle.

Your children may draw pictures, write explanations, or use equations. They can work alone or with a friend. When someone finds an answer that makes sense to them, and their friend can't find anything they missed, that's good enough.

And then the fun begins. The real point of a math experiment is to change something in the problem and see how that changes the answer.

What new Cat Escape Puzzle will your children create? What if there are four chairs, or three cats, or only one cat? What if there are more chairs? What if there's only one chair? (A math horror story, from Puck's point of view!) How might the puzzle change if the cats were willing to share a seat?

What questions will you ask?

The Princess Puzzle

Cimorene refuses to let Puck have all the math journaling fun. She wants children to understand that there are many approaches to solving any math problem, so she created a new cat puzzle of her own.

The Princess of Cats has a luxuriously soft tail about 12 inches (30 centimeters) long. Her tail is three times the length of her noble head. Her beautiful, furry body is as long as head and tail together. How long is the Princess from her delicate nose to the tip of her majestic tail?

What can children do with a problem like this? They may want to make a list of the things they know from the story. Perhaps they will draw a picture of the cat and label the proportions. Each will take their

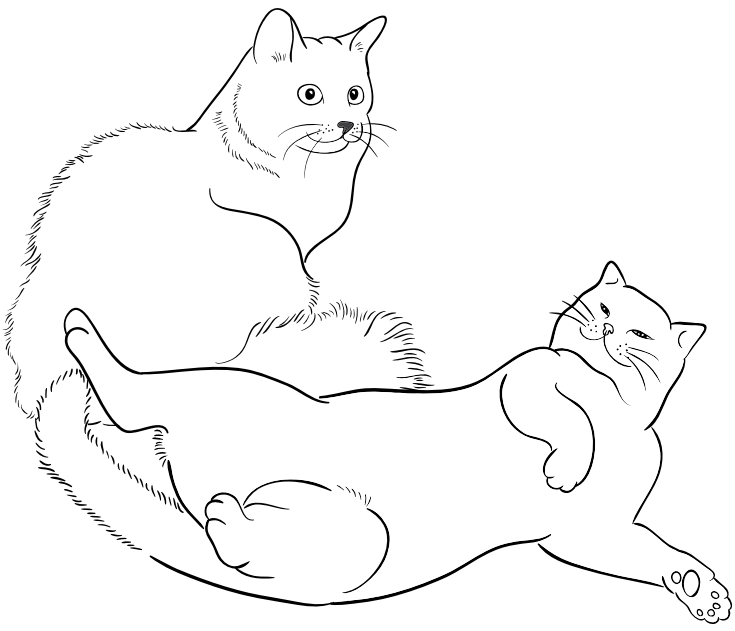
own approach to figure it out.

And then the best part of any math journal prompt is when kids make their own math. Will they write a puzzle about their own pet? Or about their favorite animal? Encourage your children to be math makers, sharing their creations with their friends and family.

As every cat knows, learning is a lifelong adventure that everyone can enjoy.

Listen to Cimorene and Puck, and help your children explore their own ideas about numbers, shapes, and patterns through journaling. I hope your family has as much fun playing with these prompts as the cats and I had writing them.

—Denise Gaskins, with Cimorene and Puck
Rural Illinois, September 2021



Section I

Writing to Learn Math

Mathematics is not about following rules. It's about playing and exploring and fighting and looking for clues and sometimes breaking things.

Einstein called play the highest form of research. And a math teacher who lets their students play with math gives them the gift of ownership.

Playing with math can feel like running through the woods when you were a kid. And even if you were on a path, it felt like it all belonged to you.

Parents, if you want to know how to nurture the mathematical instincts of your children, play is the answer.

What books are to reading, play is to mathematics.

—*Dan Finkel*

Mathematics as a liberal art: In this 16th-century engraving by Gregor Reisch, Lady Arithmetica generously shares her wisdom.

When I was in front of the class demonstrating and explaining, I was learning a great deal, but many of my students were not. My definition of a good teacher has since changed from "one who explains things so well that students understand" to "one who gets students to explain things so well that they can be understood."

— Steve Reinhart

Chapter 1: What Is a Math Journal?

Once upon a time, mathematics was considered a liberal art—an important part of any well-rounded education. Artists painted images of the angelic ladies Arithmetica and Geometria sharing their wisdom with human scholars.

Somehow, over the centuries, math lost its connection both to wisdom and to art.

Now, too often, the school math curriculum forces students on a relentless treadmill from kindergarten to calculus. Our test-driven culture rewards a fast memory and leads children to believe that "math" means cramming facts and procedures into their heads so they can perform on demand.

It's no wonder many kids grow up thinking they're no good at math.

And far too many parents feel unable to help their children learn. They worry about their kids falling behind, which raises the stress level to the point of tears. Mom and Dad are frustrated. The child is discouraged. Doing math homework feels like stumbling through an emotional minefield.

How can we help our children step off this treadmill and rediscover the liberal art of mathematics?

The problem is, we're all a product of our own schooling. Just as we are hoping to shape our children and their future through training

them, we were shaped by our own childhoods. And for most of us, our schooling gave us a totally wrong idea of what math is all about.

School and society teach us to view mathematics as a race. You run as fast as you can from one topic to the next. You must get the answer quickly. You need to follow instructions and score high on tests, and then you win. Or if you don't, you're a loser.

But let me give you a new vision of mathematics. I want you to think of math as a nature walk. There's a whole wide, wild world of interesting things—more ideas, more patterns, more concepts than you and your children would ever have time to study. And everywhere you look, there's something cool to discover.

In his book *Measurement,* math teacher Paul Lockhart compares doing math to a jungle safari:

> *"Mathematical reality is an infinite jungle full of enchanting mysteries, but the jungle does not give up its secrets easily. Be prepared to struggle, both intellectually and creatively.*
>
> *"The important thing is not to be afraid. So you try some crazy idea, and it doesn't work. That puts you in some pretty good company! Archimedes, Gauss, you and I—we're all groping our way through mathematical reality, trying to understand what is going on, making guesses, trying out ideas, mostly failing.*
>
> *"And then every once in a while, you succeed … And that feeling of unlocking an eternal mystery is what keeps you going back to the jungle to get scratched up all over again."*

If you explore this mathematical world with your children, you're not behind. Wherever you are, you're not behind, because *there is no behind.* There's only "We're going this direction," or "Let's move that way," or "Hey, look what I found over here." If your children are thinking and wondering and making sense of the math they find, they're going to learn. They're going to grow.

The key to helping our children have success with math is to focus on teaching the real thing. Real math is about making sense of ideas.

Real math is about creative reasoning.

School math rewards children who follow directions, even though it's tedious to memorize stuff that you really don't understand. And to always follow someone else's rules, that's boring. But to figure out things for yourself can be exciting.

When you embrace this adventure of learning math through playful exploration, you'll be surprised how much fun thinking hard can be. It doesn't matter whether your students are homeschooled or in a classroom, distance learning or in person. Everyone can enjoy the experience of playing around with math.

In this book, I'll teach you one of the best ways I know to put real math into practice and help children experience math as a nature walk: math journaling.

Recording Their Mathematical Journey

In a math journal, children explore their own concepts about numbers, shapes, and patterns through drawing or writing in response to a question. Journaling teaches them to see with mathematical eyes—not just to remember what we adults tell them, but to create their own math.

All they need is a piece of paper, a pencil, and a good prompt to launch their mathematical journey. The prompts in this book include number play, math art, story problems, mini-essays, geometry investigations, brain-teasers, number patterns, research projects, and much more.

My journaling prompts invite students to take any rabbit trail that interests them and discover whatever they will, without worrying about grades, testing, or state standards. Everyone can enjoy journaling because creativity is fun. And when children get a chance to be creative in an area they normally think of as drudgery, it feels like a refreshing treat.

Through journaling, children come to realize that learning is more than memorizing facts and procedures, and they develop a richer

mathematical mindset. As they explore their own thoughts, they begin to see connections and make sense of math topics. They grow confident in their ability to think through new problems.

When students write about what they're learning, they build deeper layers of understanding. The process of wrestling ideas into words forces them to pin down nebulous thoughts and decide what they really believe. Journaling gets children actively involved in their own learning. They are more likely to remember what they learn when they write it down.

For children who struggle with numbers and abstraction, writing offers a more familiar way to grapple with concepts. It helps them see themselves as mathematical thinkers.

For students who find math easy, writing reminds them that there's more to being good at math than just getting the right answers. And for those who struggle with words and language, writing about math can feel more natural than many language-based writing assignments.

Math journaling can help you as a parent or teacher, too. If you want to know what your students understand about math, their writing gives you a glimpse into how they are thinking. Some teachers use journal writing as an "exit slip," asking students to jot down a sentence or two about each lesson before leaving class.

Five Types of Journaling Prompts

In Section II of this book, I've organized the math journaling prompts into twelve categories, which may seem overwhelming at first glance. Here's a simpler way to classify the prompts by the type of reasoning involved.

1: Game Prompts

Game prompts break through the idea that math is dull and boring. They help students develop a positive attitude toward math while practicing their number skills or strategic thinking.

For example:

Basic Nim (two players): Draw 10–15 circles (called "stones"). On your turn, mark out one or two of the stones, removing them from play. Whoever marks the last stone wins the game.

Game prompts can also serve as fodder for the other types of prompt questions. We might ask students to analyze the mathematics of the game, to determine whether either player has an advantage, or to explain how they make strategic decisions during game play.

2: Content Prompts

Content prompt questions deal with the concepts of math and the topics studied. They can range from a short summary of a recent lesson to an in-depth research report on math history. Or they may pose a number-play puzzle or a word problem for students to investigate.

For example:

Choose any base number and investigate its powers. For example: If you choose a base of three, the powers are $3^1 = 3$, $3^2 = 3 \times 3 = 9$, $3^3 = 3 \times 3 \times 3 = 27$, $3^4 = 3 \times 3 \times 3 \times 3 = 81$, etc. Extend the list as far as you can. What patterns do you see in the powers of your base number? What other questions can you ask?

Content prompts help students see the bigger picture of a topic. Too often we teach by breaking a math topic into small, bite-size chunks. But writing helps students to step back and put all those little pieces in perspective.

3: Artistic Prompts

Artistic prompts encourage children to express their creativity in playing with mathematical designs. The prompt may propose a geometric or numerical constraint for the artwork. Or it may be open-ended, allowing the students to choose their own responses.

For example:

Use dotty graph paper. Connect dots to create an eight-sided shape. Are all the sides of your octagon the same length? How can you tell? What kind of design can you make with octagons?

Artistic prompts inspire children to make mental connections in a way that abstract number problems can never do. Students feel the relationships of angles and lines as they draw a shape. And these prompts may lead to informal geometry proofs, like determining whether the sides really are the same length.

4: Process Prompts

Process prompts explore and explain the way a student solves problems. They ask learners to organize their ideas and reflect on their problem-solving strategies. Process prompts involve *metacognition,* which means "thinking about your own thinking."

For example:

Describe a mistake you made in math, or a problem you missed on a quiz or test. What went wrong? How will you avoid this error the next time? Do you understand the problem now, or is there something more you need to learn about it?

Process prompts help students recognize their own understanding. Too often, math class is about learning to follow other people's thoughts, not about thinking for ourselves. But students already have many ideas about math, and the best way of teaching is to draw out and strengthen those ideas.

5: Affective Prompts

Affective prompts ask about the student's feelings and attitude toward mathematics. This includes self-assessment: How is your math comprehension growing? What is easy for you, and what is most difficult?

For example:

Have you heard that your brain keeps growing the more you use it? And that mistakes help you learn even more than when you get things right? How do these scientific discoveries affect your attitude toward math?

Affective prompts support students in relating math to their own personal experience. They make math seem more "real" to students, more relevant to things they care about, more meaningful. Writing helps students take ownership of their math experience.

And One More Type: Quotation Prompts

When you're looking for ways to prompt student writing, short quotations can be a great resource. I love quotations: Everything I might possibly want to say, someone else has already said it better than I ever could.

You can share one of your own favorite quotes or search for a new quip online. You may want to sample the tidbits on my blog's "Math and Education Quotations" resource page.†

Short-Response Prompts

Let students choose how they want to react to the quotation. Or offer one of the following questions:

- What did the author mean? Put the thought in your own words.
- Do you agree or disagree? Why?
- Is it a general principle, or only for specific situations? Describe a time when the quote might apply, or when it might not.

† *denisegaskins.com/quotations*

- Tell a time in your life when you lived up to the quotation—or when you wish you had.
- How does the quote relate to math, science, history, or another subject?

Research Prompts

Short exercises are great writing practice. But occasionally you may want to assign deeper essay topics, such as:

- Look up the author's name online. Who are/were they, and why do people care what they said?
- What have others said about the same topic? Search out a variety of quotes related to this one. How are they similar? How are they different?
- Does thinking about the quotation make you want to change anything, in yourself or in the world? How could you put that idea into action?

This Is Not a Lesson

Math journaling is different from the normal process of learning math. A typical school math book asks questions where the teacher always knows the answer. This turns math into a performance subject in which our children are constantly being judged. Some students enjoy the chance to show off their knowledge, while others feel like failures.

But journaling prompts ask questions for which we adults do not know the answer because the topic gets filtered through each child's own mind. Students come to a task at their own level and explore their own ideas. Everyone may learn something different, but they all grow as mathematicians.

The journal prompt is not a lesson to be learned. Even with the research prompts that require a student to seek out new information, there is no specific thing we want them to see. It's more like a directed

nature exploration: "What can we find hiding under this log?" We're building awareness, helping them see that there's more to mathematics than they realized.

The journal prompt is not a quiz to be graded or an essay to be judged. Even when a prompt has one specific right answer (which is rare), its primary purpose is to draw out each student's own creative reasoning. How they approach the problem is much more important than whether they figure out an answer.

Our role as parents and teachers is to listen to the children. We want to hear their ideas and understand what's going on inside their minds. When we ask about their own thoughts, our children are the experts. And that's an enormously powerful feeling.

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