Let's Play MATH

How Families Can Learn Math Together—and Enjoy It

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FOREWORD BY KEITH DEVLIN

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If a child is to keep alive his inborn sense of wonder, he needs the companionship of at least one adult who can share it, rediscovering with him the joy, excitement, and mystery of the world we live in.
—Rachel Carson

The "Aha!" Factor

I sat on the bed, surrounded by stacks of notes, bills, and other papers. Our six-year-old bounced onto the other side.

"Mom, can we do some math?" she asked.

We delay academics, so I had not planned to do schoolwork with her at all. I suppose she was jealous of what she saw as her older siblings' Mommy-time.

I started to say I was busy, but stopped myself in mid-grumble. *You do that too often,* I scolded myself. I forced a smile.

"OK," I said. "Let's see what we can find."

I put the most important papers to the side. We counted the items that remained, then took some away and counted again.

Then I leaned over and whispered in her ear. "Guess what? We made cookies, just for you and me. And we're not going to share them with the big kids."

Her eyes grew wide. "Really?"

I told her we had six (imaginary) cookies. I piled up two notebooks, a used envelope, a sheet of scratch paper, a computer printout, and my pen. She divided the "cookies" between us. We giggled as we pretended to eat. Then she picked out seven new cookies for me to divide. I made exaggerated motions of cutting the electric bill in half. We counted things one by one and in pairs, paying attention to which numbers came out even and which numbers made us cut up the last cookie.

After ten minutes she went away happy, and I returned to my work. Subtraction, division, even and odd numbers, fractions, … in that short time, we had touched on more math than we might have found in a week's worth of workbook pages.

For young children, mathematical concepts are part of life's daily adventure. Their minds grapple with understanding abstract ideas such as *threeness:* the intangible yet real link between three blocks and three fingers and three raisins on a plate. But after a few sessions of "3 + 1 = 4, 3 + 2 = 5, 3 + 3 …" they begin to whine. Older children recoil from long division. High school students face torture like this: "The product of an integer and the next greater integer is 20 less than the square of the greater integer." Math becomes a tedious chore to put off as long as possible or to finish with slapdash speed.

Mathematics ought to be a game of discovery. It should give children the same *Eureka!* thrill that sent Archimedes running through town in his birthday suit. I call this the "Aha!" factor, the delight of solving a challenging puzzle. I aim for this "Aha!" factor when I bring home a brainteaser book from the library or a new game from the store.

The Problem with Traditional School Math

Why, as they grow up, do so many children learn to hate math? Why does the idea of math homework make so many parents feel like crying?

American mathematician Hassler Whitney once said that it is "no wonder you hate math. You never had a chance to see or do real math, which is easy and fun."

Easy? Yes, that's what he said. Of course, some parts of math can be difficult to master. Some math problems are extremely challenging. But compared to traditional school math, which requires us to memorize and recall arbitrary rules for the manipulation of abstract quantities, real mathematics is more like common sense—which makes it feel more natural.

As British mathematician, educator, and author W. W. Sawyer explained, "A widespread fallacy about teaching is the idea that remembering is easy and understanding difficult. John is a bright boy, we will teach him what the subject really means; Henry is dull, he will just have to learn things by heart. Now exactly the opposite is true: to remember things which you do not understand is extremely difficult."

Real mathematics is intriguing and full of wonder—an exploration of patterns and mysterious connections. It rewards us with the joy of the "Aha!" feeling. These characteristics make it easy to stick with real math, even when a particular concept or problem presents a difficult challenge. Workbook math, on the other hand, is several pages of long division by hand followed by a rousing chorus of the fraction song: "Ours is not to reason why, just invert and multiply."

Real math is the surprising fact that the odd numbers add up to perfect squares (1, 1 + 3, 1 + 3 + 5, etc.) and the satisfaction of seeing why it must be so. Did your algebra teacher ever explain to you that a *square number* is literally a number that can be arranged to make a square? Try it for yourself:

- Gather a bunch of pennies, or any small items that will not roll away when you set them out in rows. Place one of them in front of you on the table. Imagine drawing a frame around it: one penny makes a (very small) square. One row, with one item in each row.
- Now, put out three more pennies. How will you add them to the first one to form a new, bigger square? Arrange them in a small L-shape around the original penny to make two rows with two pennies in each row.
- Set out five more pennies. Without moving the current four, how can you place these five to form the next square? Three rows of three.
- Then how many will you have to add to make four rows of four?

Each new set of pennies must add an extra row and column to the current square, plus a corner penny where the new row and column meet. The row and column match exactly, making an even number, and then the extra penny at the corner makes it odd. Can you see that the "next odd number" pattern will continue as long as there are pennies to add? And that it could keep going forever in your imagination?

Twenty-five is a square number, because we can arrange twenty-five items to make a square: five rows with five items in each row.

The point of the penny square is not to memorize the square numbers or to get any particular "right answer," but to see numbers in a new way. To understand that numbers are related to each other. To realize we can show such relationships with diagrams or physical models. The more relationships like this our children explore, the more they see numbers as familiar friends.

A focus on answer-getting and test performance can ruin mathematics, distorting a discipline that is half art and half sport. Imagine a piano teacher who insisted her students spend six years on scales and exercises of gradually increasing difficulty before she would let them attempt a piece of actual music. Or a football coach who made his team run laps and do sit-ups every day, but let them play only two or three games a year, and scrimmage games at that. How many people would become bored with music or learn to hate football under such instruction?

As every coach knows, skill grows through practice. But practice has no meaning unless the team has a real game to play. And the best type of practice takes advantage of the benefits of cross-training by emphasizing variety rather than repetitive drills. Mathematical cross-training will include games, puzzles, stories, patterns, physical models, and the challenge of thinking things through.

Our children do need to learn how to perform routine calculations, as piano players must practice scales and football players lift weights. More important, however, our children need to learn why those operations work. And they must never be led to think that calculations are the essence of mathematics.

"A teacher of mathematics has a great opportunity," wrote Hungarian math professor George Polya. "If he fills his allotted time with drilling his students in routine operations he kills their interest, hampers their intellectual development, and misuses his opportunity.

"But if he challenges the curiosity of his students by setting them problems proportionate to their knowledge, and helps them to solve their problems with stimulating questions, he may give them a taste for, and some means of, independent thinking."

Playing with Numbers

Writing for *Family Life* magazine, mathematician and music critic Edward Rothstein described a game he invented for his daughter:

> *"What number am I? If you add me to myself, you get four."*

I gave that question to my six-year-old daughter during a family car trip. Then her sister, age nine, wanted to play. I tried a question with bigger numbers, but she rolled her eyes. "That's too easy, Mom."

So I asked her:

> *"What number am I? If you take away one-fourth of me and then add two, you get seventeen." [answer1]*

The older our children get, the harder their parents have to work. For my twelve-year-old son, I asked:

"What number am I? If you multiply me by myself and add one, you get half as many as the number of pennies in a dollar." [answer2]

That kept him busy for a few minutes. After he figured it out, he came back with:

"What number am I? If you divide me by two and take away four, then add five, then multiply by three and divide by two and add seven, you get me again."

"What?" I asked. He repeated the question.

"This is actually a number?" I asked. "You figured out an answer to this?"

He nodded, with the smug grin of a preteen who knows he has Mom skewered.

I pulled out a notebook and pen. He repeated his series of calculations, and this time I wrote it down. I figured the answer had to be zero or one, those magic numbers that make multiplying easy, but neither worked.

I tried one hundred. No luck.

I heard a chuckle from the back seat.

"Wait," I said. "Give me a chance."

My husband was driving, but he glanced over at the notebook. "You know," he offered, "you could set that up as an equation."

No way. The boy had not needed algebra to figure it out, so neither

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Answers

You can find "Answers to Sample Problems" in the appendixes.

Problem 1 has two possible solutions, depending on how you understand the words in the question. My daughter did not see it the same way I did. Her answer caught me by surprise — it was three times the number I expected — and yet after she explained her reasoning, I had to admit that her solution, too, was correct.

Let this be a warning: if your child's answer is not the same as yours, don't assume she is wrong. Ask her to explain how she figured it out. Then listen with care. Children almost always have a logical reason for their answers. Language is a complicated thing, and even a math problem may be open to different interpretations.

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did I. Still guessing, I tried ten, then fifty, then twenty. OK, that narrowed it down. Now I knew the answer had to be between twenty and fifty, but I had run out of easy numbers.

I nibbled on the end of my pen.

My son hummed to himself.

"I've got it." I spun around as far as the seat belt allowed. "The answer is ___" [answer3]

"Nope."

"WHAT?"

I looked at my scratch paper. I worked the numbers again, coming up with the same answer. I read the steps of my calculation out loud.

He agreed that my number would work, but it was not the one he had in mind. I would have to guess again.

Hubby protested that there couldn't be another answer. If the equation doesn't have an x^2 or something similar, there can't be more than one solution.

The kid stood his ground, smirking.

I conceded. "What's your number?"

"Infinity. It doesn't matter what you multiply or take away, it's always infinity."

Aha! He was right. Well, sort of right: infinity isn't a real number, so you can't calculate with it that way. But it's good enough for middle school.

Even better, he had managed to stump the adults.

Playing with Shapes and Patterns

Even the simplest objects can provide an opportunity for mathematical play. The following story comes from math teacher Christopher Danielson, the primary organizer behind the hands-on "Math On-A-Stick" exhibit at the 2015 Minnesota State Fair.

Danielson writes …

I have spent the last four days playing and talking math with kids of all ages for eleven hours a day, paying close attention to how children behave in this space we've built. My number one message coming out of this work is *Let the children play.*

When children come to the egg table at Math On-A-Stick, they know right away what to do. There are brightly colored plastic eggs, and there are large, flat thirty-egg cartons. The eggs go in the cartons.

No one needs to give them instructions.

A typical three- or four-year old will fill the cartons haphazardly. She won't be concerned with the order she fills it, nor with the colors she uses, nor anything else. She'll just put eggs into the carton one at a time in a seemingly random order.

But when that kid plays a second or third time, emptying and filling her egg carton—without being told to do so—she usually begins to see new possibilities. After five or ten minutes of playing eggs, this child is filling the carton in rows or columns. Or she's making patterns such as pink, yellow, pink, yellow, and so on. Or she's counting the eggs as she puts them in the carton. Or she's orienting all of the eggs so they are pointy-side up.

The longer the child plays, the richer the mathematical activity she engages in. This is because the materials themselves have math built into them. The rows and columns of the egg crate; the colors and shape of the eggs; the fact that the eggs can separate into halves—all of these are mathematical features that kids notice and begin to play with as they spend time at the table.

We have seen four-year-olds spend an hour playing with the eggs.

I have observed that the children who receive the least instruction from parents, volunteers, or me are the most likely to persist. These are the children who will spend twenty minutes or more exploring the possibilities in the eggs.

The children who receive instructions from adults are least likely to persist. When a parent or volunteer says, "Make a pattern," kids are likely to do one of two things:

- Make a pattern, quit, and move to something else.
- Stop playing without making a pattern.

We adults have a responsibility to let the children play. We can be there to listen to their ideas as they do. We can play in parallel by getting our own egg cartons out and filling these cartons with our own ideas.

But when we tell kids to "make a pattern" or "use the colors," we are asking the children to fill their carton with our ideas, rather than allowing them to explore their own.

—Christopher Danielson

What Is Our Goal?

As children approach school age, today's parents face a bewildering array of choices. Many find it useful to write out (or at least to talk through) their educational ideals. Is our mission like filling the empty bucket of a child's mind, or is it more like lighting a fire that will grow and spread on its own? Or is our role not to "teach" at all, but rather to walk alongside and assist our children as they explore the world? How we define our goals will make an enormous difference in how we approach the day by day adventure of learning.

In the same way, before we can figure out how to help our children with math, we need to think about our goals. What does it take to understand mathematics? Is it truly necessary for our children? After all, computers and calculators crunch most of the numbers in our modern world. Some children must grow up to program those computers, but what if my kids have other plans?

Do I want my children to learn math only because the state requires it? The state requires our children to learn math so they will be functionally literate. That may not sound like a lofty goal, but think about what "functionally literate in math" means:

- Filling out an IRS Form 1040 with its Schedule A, Schedule B, Schedule SE, and all the rest.
- Reading a mortgage and understanding how a fixed- or variable-rate loan will affect family finances.
- Following newspaper articles about the governor's budget proposal or discerning the relevance of political polls.
- Knowing that a 40% chance of rain on Saturday and a 60% chance of rain on Sunday doesn't mean there is a 100% chance of rain this weekend.

Mathematical literacy is a worthy challenge. But most of us want more than literacy for our children. We want them to be educated. An educated person is interested in more than merely what is useful. He or she loves to learn, studies for the sake of gaining knowledge, and grows in wisdom.

Few people read Shakespeare because his plays are useful. But an educated person will enjoy Shakespeare because his stories are interesting and his dialogue insightful. Likewise, much of math does not seem to be useful, but it can be fascinating.

If you want to experience the joy and artistry of math, consider *fractals,* which are intricate patterns of regular irregularity that capture something of the complex beauty in nature. The men who first discovered fractals in the late nineteenth century called them "monster curves." They could not imagine any use for such absurdities. Yet today, these monsters are used to compress images and other data files and have become a staple tool in film makers' special-effects kit.

Children who are educated in math will gain practical skills. But what is more important, those who enjoy learning for its own sake will find plenty to fascinate them.

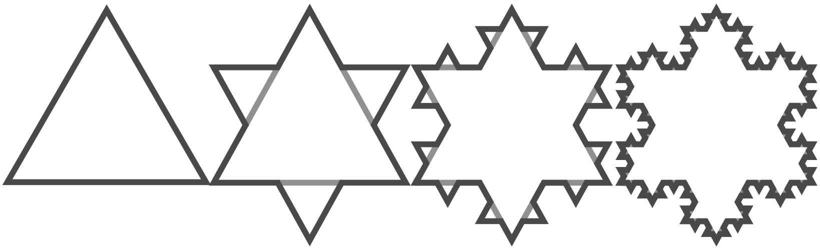
Math the Mathematician's Way

> *Real mathematics is not just formulaic tutoring. My hope is that children learn to think about mathematics as a kind of mental play.*
>
> —Edward Rothstein

Mathematics is mental play, the essence of creative problem solving. This is the truth we need to impart to our children, more important than fractions or decimals or even the times tables. Math is a game, playing with ideas.

Traditional school math is a lock-step sequence of topics, but math the mathematician's way is a social adventure of exploring and sharing new ideas. Math the mathematician's way is fun. It can even be beautiful. Listen to how real mathematicians, both professionals and amateurs who enjoy working with math, describe their subject:

> *I love mathematics principally because it is beautiful, because man has breathed his spirit of play into it, and because it has given him his greatest game—the encompassing of the infinite.*
>
> —Rózsa Péter



Step by step, the boundary of the Koch Island fractal — sometimes called the Koch Snowflake — approaches infinite raggedness, much as an actual coastline appears more and more jagged under increasing magnification.

By allowing random variation as they design a fractal, artists can create natural-looking forms such as this mountainous coastline in winter.

There is no ulterior practical purpose here. I'm just playing. That's what math is: wondering, playing, amusing yourself with your imagination.

—Paul Lockhart

Puzzles in one sense, better than any other single branch of mathematics, reflect its always youthful, unspoiled, and inquiring spirit. When a man stops wondering and asking and playing, he is through.

—Edward Kasner

If mathematics education communicated this playful aspect of the subject, I don't think innumeracy would be as widespread as it is.

—John Allen Paulos

W. W. Sawyer wrote a book called *Mathematician's Delight,* in which he described mathematical thinking this way: "Everyone knows that it is easy to do a puzzle if someone has told you the answer. That is simply a test of memory. You can claim to be a mathematician only if you can solve puzzles that you have never studied before. That is the test of reasoning."

It is also the test of life. Every day we face puzzles we have not studied before. Math taught the mathematician's way prepares us to approach problems with confidence. It teaches us to see our mistakes as stepping stones to learning. It reminds us that there may be more than one right answer, as my children and I discovered when we played "What Number Am I?"

Math taught the mathematician's way gives our children practice struggling with challenging problems. It lets them enjoy that "Aha!" thrill when they find a solution. Math the mathematician's way prepares them for careers or college. It gives them tools they can use throughout their lives. It gives them confidence by letting them succeed at something difficult. When kids solve a puzzle that stumped their parents, they know they can handle anything.

EDUCATION / MATHEMATICS

Learning Math Is a Game, Playing with Ideas

"Want to help your kids with math? Don't help with the homework. Get them to engage with math by doing things together—many of which don't even look like math. *Let's Play Math* is charming, intelligent, and practical; full of family fun and sound advice."

—Ian Stewart, author of *Professor Stewart's Casebook of Mathematical Mysteries*

"It revolutionized our homeschool this year."

—Caitlin Fitzpatrick Curley, author of *My Little Poppies* blog

"A crash course in how to enjoy math with your children! Denise Gaskins uses her years of experience to show parents how to teach math with games, stories, puzzles, manipulatives, and living books.

"Full of useful advice and pedagogical insight, this book is a treasure trove for parents who want to help their children appreciate the beauty, history, and fun of math but don't know where to start."

—Kate Snow, author of *Preschool Math at Home*

"This is the math helper I wish I'd had years ago."

—Anne White, author of *Minds More Awake*

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