SUPPORTING RESEARCH

EXTENSIONS IN MATHEMATICS™ Series

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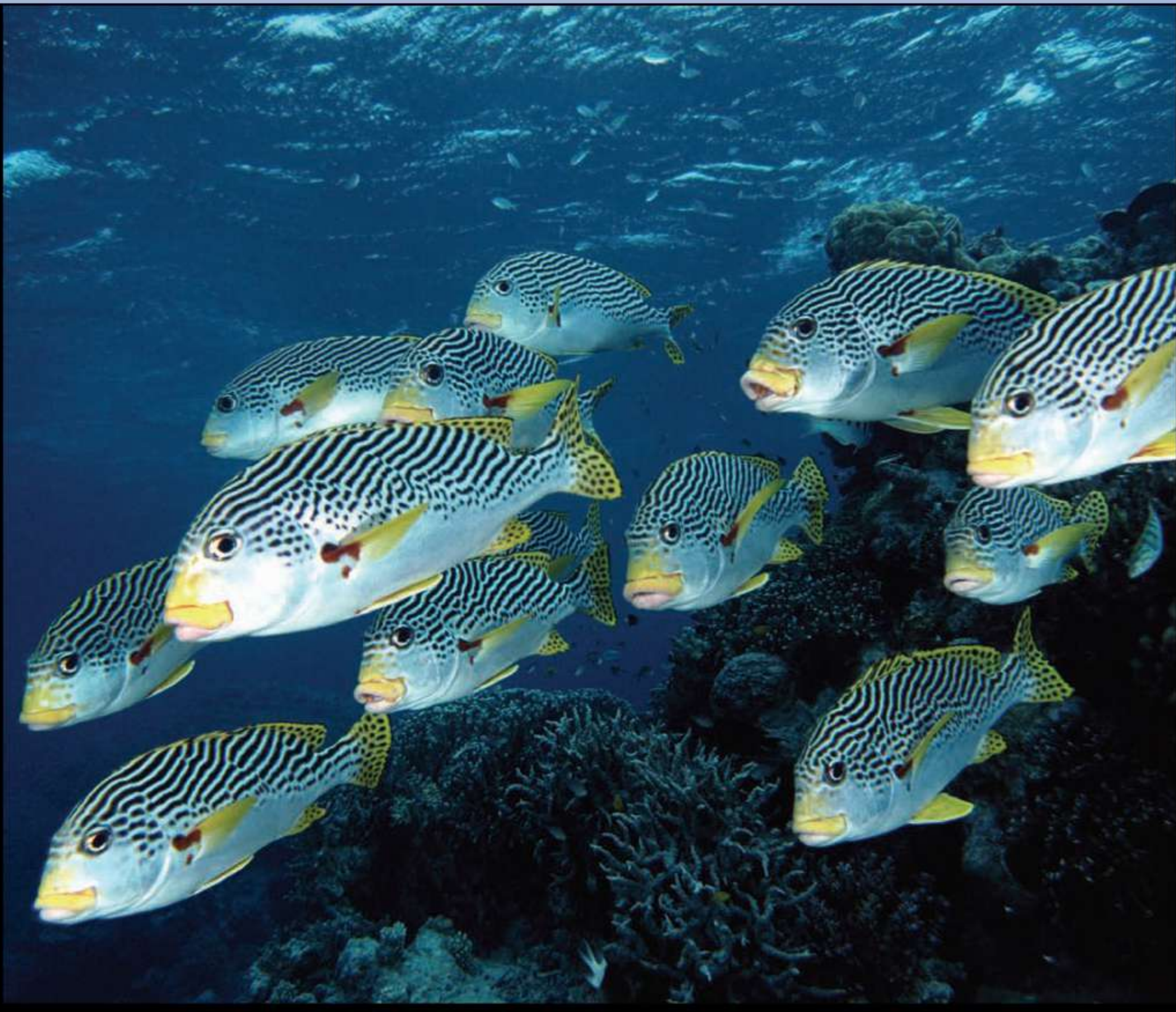
EXTENSIONS IN MATHEMATICS

- PROVIDES CHALLENGING INSTRUCTIONAL ACTIVITIES FOR 12 MATHEMATICS STRATEGIES
- STRENGTHENS PROBLEM-SOLVING SKILLS AND IMPROVES MATHS-RELATED WRITING SKILLS
- FEATURES ASSESSMENT IN MATHEMATICS, INCLUDING SELECTED-RESPONSE AND CONSTRUCTED-RESPONSE PROBLEMS



EXTENSIONS IN MATHEMATICS

A Research-Based Mathematics Strategy Series

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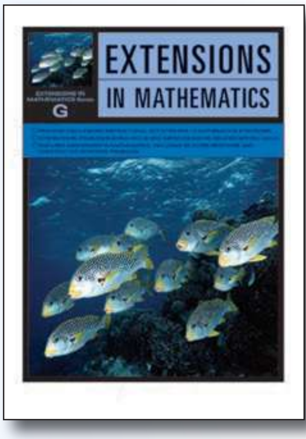

Mathematics Strategies

- Building Number Sense
- Using Estimation
- Applying Addition
- Applying Subtraction
- Applying Multiplication
- Applying Division
- Converting Time and Money
- Converting Customary and Metric Measures
- Using Algebra
- Using Geometry
- Determining Probability and Averages
- Interpreting Graphs and Charts

INTRODUCTION TO THE SERIES

The ***Extensions in Mathematics™ Series* is the in-depth,** advanced component of a maths program that spans from diagnosis through assessment. The ***Extensions in Mathematics™ Series*** is a research-based mathematics series that is designed to strengthen and extend students' problem-solving skills using graphic organisers in years one to nine. Learning opportunities that call for the use of graphic organisers include mathematical problem-solving, maths-related writing, and numbers-in-context reading selections. Mathematical learning continues through extension activities that offer cross-curricular learning and practical application experiences.

Additionally, strategies are aligned with the U.S. National Council of Teachers of Mathematics (NCTM) content and process standards. Each lesson is followed by assessment questions of varying formats. Students practise answering selected-response questions, constructed-response questions, including short-answer responses, and extended-answer responses. The constructed-response and extended-response questions require students to explain their problem-solving process. The ***Extensions in Mathematics™ Series*** provides students with integrated opportunities to work with maths concepts spatially, numerically, symbolically and in qualitative and quantitative formats.

The ***Extensions in Mathematics™ Series*** is based on research from several areas. The program is supported by the findings of the Committee for Mathematics Learning and NCTM, as well as other current mathematics research in the areas of metacognitive strategies, graphic organisers, mathematical literacy and multiple representations.

HOW DOES THE EXTENSIONS IN MATHEMATICS™ SERIES HELP STUDENTS BECOME PROFICIENT IN MATHS?

There is a current drive in mathematics education to meet 21st-century skills so that today's students will be competitive in tomorrow's workforce. Australian education ministers continue to make improved numeracy and literacy standards a national priority.

The Committee on Mathematics Learning (CML) was established to synthesise diverse research on mathematical learning, to provide research-based recommendations for teaching, teacher education and curriculum for improving student learning, and to advise and guide other educational entities about mathematical learning.

IS THERE A NEED TO HELP STUDENTS BECOME PROFICIENT IN MATHEMATICS?

The 2005 NAEP Mathematics Assessment results show that students need more help in becoming proficient: only 36% of assessed students perform at the proficient level.

Working towards mathematical proficiency is the end goal of every mathematics classroom.

From the review of mathematical research, CML developed the term mathematical proficiency to describe the desired end goal of every student's mathematical education. The committee also developed five interweaving strands that describe a mathematically proficient student. The five strands that CML identified and defined are:

- Conceptual understanding: comprehension of maths concepts, operations and relations
- Procedural fluency: skill in carrying out procedures flexibly, accurately, efficiently and appropriately
- Strategic competence: ability to formulate, represent and solve mathematical problems
- Adaptive reasoning: capacity for logical thought, reflection, explanation and justification
- Productive disposition: habitual inclination to see mathematics as sensible, useful and worthwhile, coupled with a belief in diligence and one's own efficacy.

The ***Extensions in Mathematics™ Series*** is geared to strengthen and extend students' problem-solving strategies using graphic organisers, maths-related writing, and numbers-in-context applications. The program utilises instructional strategies that build mathematical proficiency in students of all abilities. The following section is drawn from major findings of the Committee on Mathematical Learning and the publication *Adding It Up: Helping Children Learn Mathematics (2001).*

Conceptual Understanding

"Conceptual Understanding refers to an integrated and functional grasp of mathematical ideas" (p. 118). To have a firm grasp on mathematical ideas, students must be able to link their prior mathematical knowledge to new ideas. This connection between old knowledge and new knowledge promotes retention of the facts and procedures. The numbers and procedures have meaning to students. Students are then able to translate this knowledge into different representations. "Knowledge that has been learned with understanding provides a basis for generating new knowledge and for solving new and unfamiliar problems" (p. 119).

In the ***Extensions in Mathematics™ Series,*** each lesson activates prior knowledge and then connects students' knowledge to two new representations. Students read, discuss, and review the lesson's strategy in the Learn About the Strategy section and the Learn More About the Strategy section. The new representations for the mathematical strategy take the forms of a numbers-in-context problem and a graphic organiser that is used to solve the problem. The Learn About the Strategy section and the Learn More About the Strategy section each present the completed number-in-context problem and the graphic organiser as a model.

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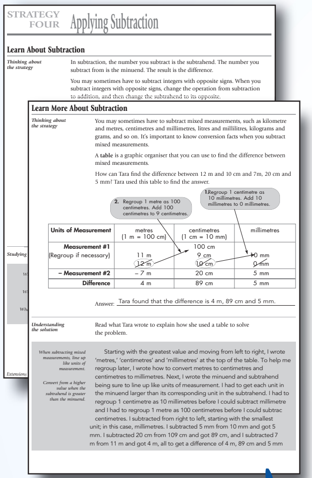

STRATEGY FOUR Applying Subtraction

Learn About Subtraction

Learn More About Subtraction

Conceptual Understanding takes place when students bridge understanding from old knowledge to newly gained knowledge. In mathematics, this bridge occurs through understanding new representations of concepts.

In this lesson, students learn strategies for solving problems that require adding positive and negative integers and adding mixed measures. Before beginning this lesson, you may wish to review with students some prerequisite skills, which include understanding positive and negative integers and knowing the basic equivalencies in customary measurement. Reproducible graphic organisers for this lesson are on pages 36 and 37.

Prerequisite skills are listed for each lesson, ensuring that students are prepared to start a new topic.

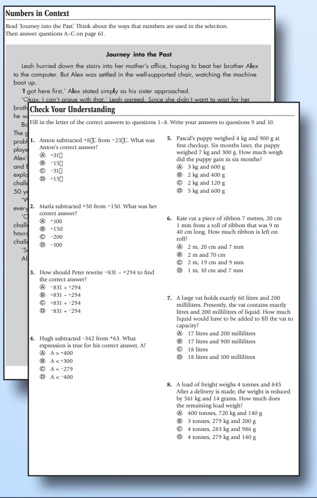
When students make connections, they gain confidence, which allows them to move to new levels of understanding. Additionally, teachers are provided, in the teacher's guide, prerequisite mathematics skills that students should have in place before the lesson begins. Teachers are directed to review key terms and concepts before each lesson begins.

Procedural Fluency

"Procedural fluency refers to the knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently" (p. 121). The goal is for students to have a firm understanding of mathematical concepts, which then allows students to use the appropriate computational strategies. Being procedurally fluent also entails that students can choose the appropriate computational procedure to solve mathematical problems. "Students need facility with a variety of computational tools, and they need to know how to select the appropriate tool for a given situation" (p. 121).

The ***Extensions in Mathematics™ Series*** provides students with consistent practice in the computational tools, graphic organisers, maths-related writing, and direct computation. Students also use these tools in a variety of mathematical settings as provided in the following sections of each lesson.

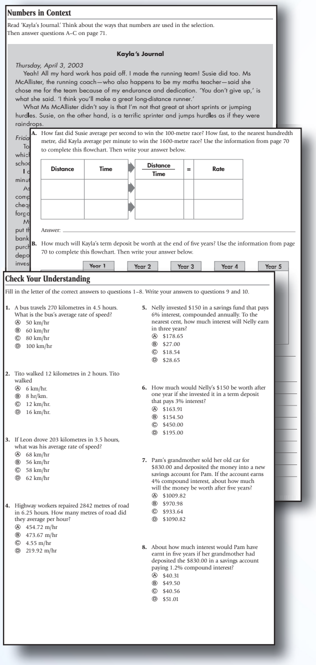
The Numbers in Context section provides students with practice in extracting relevant mathematical information from a reading selection and then transferring that information into graphic organisers to solve maths problems. The Solve a Problem section presents word problems that can be solved using both a graphic organiser and direct computation. Students then provide a written explanation of their problem-solving strategy. Students demonstrate their procedural fluency with the tools learned in the Check Your Understanding section of each lesson. Students complete the selected-response section by using either direct-computation skills or mental-maths skills.


Numbers in Context

Journey into the Past

Check Your Understanding

The Numbers in Context and Check Your Understanding activities are just two of several activities where students demonstrate procedural fluency.



Students apply multiple strategies as they work through the Numbers in Context activity, which includes graphic organisers and written explanations. The Check for Understanding activity engages additional strategies as students switch to computational problem solving.

Students demonstrate adaptive reasoning a minimum of 4 times in each lesson. With 12 lessons in each book, students have 48 opportunities to demonstrate mastery of adaptive reasoning.

Strategic Competence

Strategic Competence refers to the ability to formulate mathematical problems, represent them and solve them. Students need first to understand the situation, including key features, and then to generate a maths representation (numerically, symbolically, verbally, or graphically). They need to capture the core maths elements and ignore irrelevant features. Students need to develop and understand relationships in the problems and not just number crunch. Students also need to develop mental representations of problems, detect mathematical relationships, and devise novel solutions when needed.

Strategic competence doesn't apply to routine mathematical problems. A routine problem is one that the student knows how to solve almost immediately, such as a simple multiplication problem. Non-routine problems are those to which the student doesn't know the solution immediately. A non-routine problem is one that may be found in reading material or in real-life experiences. Flexibility of approach is key to solving non-routine problems. Developing strategic competence involves replacing cumbersome problem-solving procedures with ones that are more compact and efficient.

Students using the ***Extensions in Mathematics™ Series*** develop strategic competence as they progress through each lesson. Students are presented with two non-routine problems in the Numbers in Context activity. Students read a selection and then identify the key mathematical features. Students apply mental maths skills as they identify key features of the reading passage.

Students use graphic organisers to realise the mathematical relationships that are present in each problem. Students gain an additional mathematical experience with the non-routine problems throughout the Numbers in Context section. Students must analyse the reading selection and detect those numbers that are relevant. The section Explain Your Solution requires students to explain their problem-solving process to demonstrate their understanding of the mathematical relationships found in the problem.

Students develop strategic competence through the Check Your Understanding section of each lesson. In this section, students choose a mathematical representation to solve a problem. This section is designed to provide students with flexibility in choosing the most effective and efficient problem-solving solution. Additionally, students identify mathematical relationships through graphic organisers.

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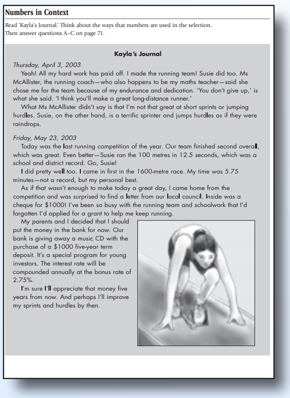
Extend Your Learning

Food Technology: Expanding a Recipe
In a cookbook or magazine, find three recipes that contain amounts that are mixed numbers, such as $2\frac{2}{3}$ cups flour and $1\frac{1}{4}$ cups chopped walnuts. Imagine that you need to increase the recipe to serve more people. How much more of each of these ingredients will you need to make $1\frac{1}{2}$ times the recipe? $2\frac{2}{3}$ times the recipe? $45\frac{1}{5}$ times the recipe? Make up other mixed numbers to multiply by to calculate the ingredients.

Real-world scenarios and applications challenge students' abilities to demonstrate productive disposition.

Numbers in Context


Numbers in Context

Read 'Kayla's Journal.' Think about the ways that numbers are used in the selection.
Then answer questions A–C on page 71.

Kayla's Journal

Thursday, April 3, 2003

Yeah! All my hard work has paid off. I made the running team! Susie did too. Ms McAllister, the running coach—who also happens to be my maths teacher—said she chose me for the team because of my endurance and dedication. 'You don't give up,' is what she said. 'I think you'll make a great long-distance runner.'

What Ms McAllister didn't say is that I'm not that great at short sprints or jumping hurdles. Susie, on the other hand, is a terrific sprinter and jumps hurdles as if they were raindrops.

Friday, May 23, 2003

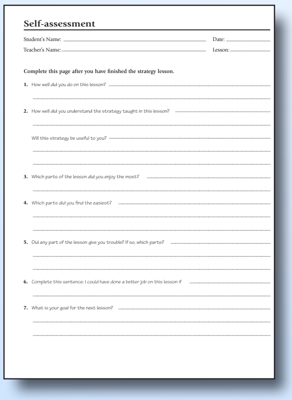
Today was the last running competition of the year. Our team finished second overall, which was great. Even better—Susie ran the 100 metres in 12.5 seconds, which was a school and district record. Go, Susie!

I did pretty well too. I came in first in the 1600-metre race. My time was 5.75 minutes—not a record, but my personal best.

As if that wasn't enough to make today a great day, I came home from the competition and was surprised to find a letter from our local council. Inside was a cheque for $1000! I've been so busy with the running team and schoolwork that I'd forgotten I'd applied for a grant to help me keep running.

My parents and I decided that I should put the money in the bank for now. Our bank is giving away a music CD with the purchase of a $1000 five-year term deposit. It's a special program for young investors. The interest rate will be compounded annually at the bonus rate of 2.75%.

I'm sure I'll appreciate that money five years from now. And perhaps I'll improve my sprints and hurdles by then.


Self-assessment

Student's Name: ____________ Date: ______
Teacher's Name: ____________ Lesson: ______

Complete this page after you have finished the strategy lesson.

1. How well did you do on this lesson?
2. How well did you understand the strategy taught in this lesson?
   Will this strategy be useful to you?
3. Which parts of the lesson did you enjoy the most?
4. Which parts did you find the easiest?
5. Did any part of the lesson give you trouble? If so, which parts?
6. Complete this sentence: I could have done a better job on this lesson if
7. What is your goal for the next lesson?

Students express their realisation of the value of mathematics through the Self-Assessment activity.

Adaptive Reasoning

Adaptive reasoning is defined as the capacity to think logically about the relationships among concepts and situations. Adaptive reasoning is the ability to justify a mathematical solution by demonstrating its logical development. Students use adaptive reasoning to prove the legitimacy of a chosen problem-solving strategy.

In the ***Extensions in Mathematics™ Series,*** students demonstrate their adaptive reasoning through the extended-response questions in the Explain the Solution section throughout each lesson. Students are exposed to adaptive reasoning in the modelled Learn About the Strategy section of each lesson. After reading the modelled response, students then have three more opportunities to demonstrate their adaptive reasoning of their problem-solving strategies. "One uses [adaptive reasoning] to navigate through the many facts, procedures, concepts, and solution methods and to see that they all fit together in some way, that they make sense" (p. 129).

Productive Disposition

Productive disposition is what occurs when the previous strands are realised. Productive disposition is students' realisation that maths makes sense and has value in their lives. In the ***Extensions in Mathematics™ Series,*** productive disposition takes place for students through three activities, Extend Your Learning, Numbers in Context, and the Self-assessment questionnaire. The Extend Your Learning section utilises cross-curricular activities to demonstrate to students how mathematics can play a part in other subjects, such as art, reading and real-life applications. Through hands-on activities, students see that maths adds value and dimension to other subject areas. The activity extends into real-world applications such as reading a newspaper, calculating the cost of a shopping list, or reading food labels for nutritional information. These real-world applications demonstrate to students the importance of learning maths because of the impact mathematics has on daily life.

The Numbers in Context activity is grounded in real-world application. Students read a selection and then extract relevant mathematical facts to answer mathematical questions based on the text.

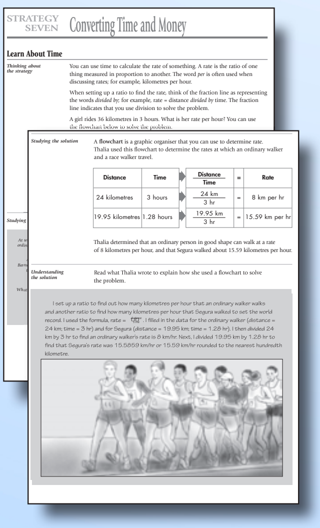
A final activity in the ***Extensions in Mathematics™ Series*** serves to monitor student's self-realisation of the value of mathematics, the Self-assessment questionnaire. Students complete a Self-assessment questionnaire after each lesson. This questionnaire requires students to consider in-depth the mathematical learning they have experienced in each lesson. Students completing this survey acknowledge their learning and how the lesson's mathematics makes sense to them.

Mathematical proficiency is attainable for students working in the ***Extensions in Mathematics™ Series***. By working through the many instructional strategies in the program, students develop the conceptual understanding, fluency, strategies, realisation and reasoning that mathematics is valuable and relevant to their lives in and outside the classroom environment.

HOW IS *EXTENSIONS IN MATHEMATICS™ SERIES* ORGANISED?

Lesson Parts	Instructional Strategies
Learn about the Strategy	Modelled/ Direct Instruction
Solve a Problem	Guided Instruction
Learn More About the Strategy	Strategy Extension
Solve a Problem	Guided Instruction
Numbers in Context	Number-Sense Development/ Real-World Connections
Check your Understanding	Independent Practice/ Extension Activities

Numbers in Context


STRATEGY SEVEN Converting Time and Money

Learn About Time

Each book in the ***Extensions in Mathematics™ Series*** has six parts to each strategy lesson. Scaffolded instruction is the organisational framework of the program. Scaffolded instruction benefits all types of students, including English-language learners (ELL). "Scaffolded instruction optimizes student learning by providing a supportive environment while facilitating student independence" (ERIC Document, 2002). The ***Extensions in Mathematics™ Series*** guides students through the learning process from modelled/direct instruction, through guided instruction, and finally to independent work.

Learn About the Strategy

Modelled/Direct Instruction

Students' exposure to the lesson's mathematics strategy begins with the section Learn About the Strategy. This section opens with an instructional page. Here problem-solving using the mathematics strategy is modelled and directly instructed. "Many students, particularly low-performing students, learn more quickly from a clear, concise explanation of what to do and how to do it" (Carnine, 1990). Teachers and students together read and discuss the mathematics strategy and the problem-solving process that is modelled. Margin notes guide students to think about the important information in the word problem.

Solve a Problem

Guided Instruction

In the Solve a Problem section, students apply the knowledge gained in the Learn About the Strategy section. Through the Solve a Problem section, students transfer their knowledge to a related problem. Students receive assistance through a partially completed graphic organiser. Students then make explicit their solution process by providing a written explanation of the solution method. Guided instruction is personalised instruction that helps students reflect upon and then articulate their problem-solving thought processes. "Teachers can help students change their original conceptions by helping students make their thinking visible so that misconceptions

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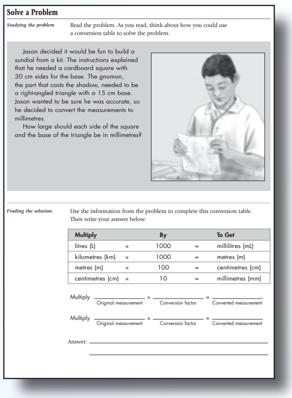


Solve a Problem

Studying the problem Read the problem. As you read, think about how you could use a conversion table to solve the problem.

Jason decided it would be fun to build a sundial from a kit. The instructions explained that he needed a cardboard square with 30 cm sides for the base. The gnomon, the part that casts the shadow, needed to be a right-angled triangle with a 15 cm base. Jason wanted to be sure he was accurate, so he decided to convert the measurements to millimetres.

How large should each side of the square and the base of the triangle be in millimetres?

Finding the solution Use the information from the problem to complete this conversion table. Then write your answer below.

Multiply		By		To Get
litres (L)	×	1000	=	millilitres (mL)
kilometres (km)	×	1000	=	metres (m)
metres (m)	×	100	=	centimetres (cm)
centimetres (cm)	×	10	=	millimetres (mm)

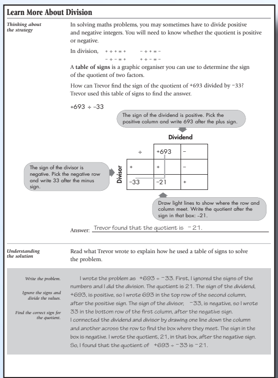
Multiply ____ Original measurement × ____ Conversion factor = ____ Converted measurement

Multiply ____ Original measurement × ____ Conversion factor = ____ Converted measurement

Answer: ____

"This feedback [from guided instruction] provides a formative assessment that the teacher-coach may use to suggest subsequent steps" (Jenkins & Keefe, 2001, p. 1).

Learn More About the Strategy



Learn More About Division

Thinking about the strategy In solving maths problems, you may sometimes have to divide positive and negative integers. You will need to know whether the quotient is positive or negative.

In division, + ÷ + = + − ÷ + = −
− ÷ − = + + ÷ − = −

A **table of signs** is a graphic organiser you can use to determine the sign of the quotient of two factors.

How can Trevor find the sign of the quotient of +693 divided by −33? Trevor used this table of signs to find the answer.

+693 ÷ −33

÷	+693	−
+	+	−
−33	−21	+

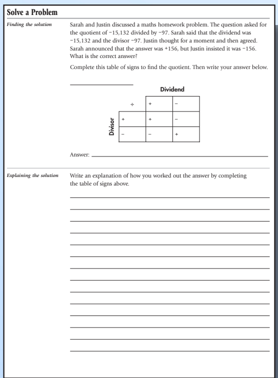
Answer: Trevor found that the quotient is −21.

Understanding the solution Read what Trevor wrote to explain how he used a table of signs to solve the problem.

Write the problem. *Ignore the signs and divide the values.* *Find the correct sign for the quotient.*

I wrote the problem as +693 ÷ −33. First, I ignored the signs of the numbers and I did the division. The quotient is 21. The sign of the dividend, +693, is positive, so I wrote 693 in the top row of the second column, after the positive sign. The sign of the divisor, −33, is negative, so I wrote 33 in the bottom row of the first column, after the negative sign. I connected the dividend and divisor by drawing one line down the column and another across the row to find the box where they meet. The sign in the box is negative. I wrote the quotient, 21, in that box, after the negative sign. So, I found that the quotient of +693 ÷ −33 is −21.

Solve a Problem



Solve a Problem

Finding the solution Sarah and Justin discussed a maths homework problem. The question asked for the quotient of −15,132 divided by −97. Sarah said that the dividend was −15,132 and the divisor −97. Justin thought for a moment and then agreed. Sarah announced that the answer was +156, but Justin insisted it was −156. What is the correct answer?

Complete this table of signs to find the quotient. Then write your answer below.

÷	+	−
+	+	−
−	−	+

Answer: ____

Explaining the solution Write an explanation of how you worked out the answer by completing the table of signs above.

can be corrected and so that students can be encouraged to think beyond the specific problem or to think about variations of the problem" (National Research Council, 1999, p. 78).

Learn More About the Strategy

Strategy Extension

This section provides students with an opportunity to elaborate upon their initial understanding of the lesson's strategy. Students make an additional connection to the primary mathematics strategy taught in the lesson. Students take on more responsibility for their learning as they move from direct instruction to application by solving a related strategy problem. Students continue the lesson by applying the strategy to a new graphic organiser and then writing an explanation for their solution method.

Solve a Problem

Guided Instruction

This section duplicates the guided-instruction strategy used in the initial Solve a Problem section. Students apply what they have learned from the Learn More About the Strategy section. Once more, students receive assistance through a partially completed graphic organiser. Students then make explicit their solution process by providing a written explanation of the solution method. Guided instruction helps students reflect upon and then articulate their thought processes in solving a mathematical problem.

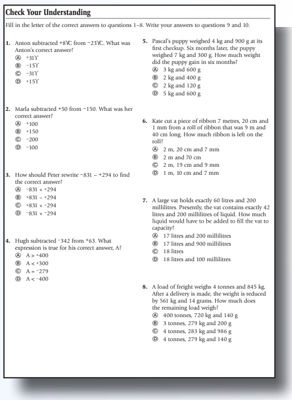
Numbers in Context

Number-Sense Development/Real-world Connections

The ***Extensions in Mathematics™ Series*** uses Numbers in Context as another problem-solving strategy to extend students' mathematical learning. Numbers in Context activities stress number sense so that students may make reasonable judgments about whether their mathematical solutions make sense in terms of quantities and relationships between quantities. "Without number sense, students make errors because they have a hard time judging whether their answers are reasonable. Emphasizing number sense involves dealing with numbers in context, visualizing quantities, and recognizing the relationships between quantities—in other words, concepts common to standardized tests" (Duke & Ritchhart, 2005).

Students read a passage that is centred on real-life scenarios. After students read the passage, they must answer three types of questions about the passage. The questions require students to extract the relevant numbers in context to answer the questions. Graphic organisers and qualitative responses are required to show mathematical comprehension.

Check Your Understanding



Check Your Understanding

Fill in the letter of the correct answers to questions 1–8. Write your answers to questions 9 and 10.

1. Anton subtracted +8°C from −23°C. What was Anton's correct answer?
Ⓐ +31°
Ⓑ −15°
Ⓒ −31°
Ⓓ +15°

2. Marla subtracted +50 from −150. What was her correct answer?
Ⓐ +100
Ⓑ +150
Ⓒ −200
Ⓓ −100

3. How should Peter rewrite −831 – +294 to find the correct answer?
Ⓐ −831 + +294
Ⓑ +831 – +294
Ⓒ +831 + −294
Ⓓ −831 + −294

4. Hugh subtracted −342 from +63. What expression is true for his correct answer, A?
Ⓐ A > +400
Ⓑ A < +300
Ⓒ A = −279
Ⓓ A < −400

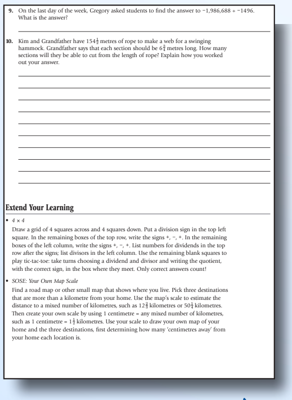
5. Pascal's puppy weighed 4 kg and 900 g at its first checkup. Six months later, the puppy weighed 7 kg and 300 g. How much weight did the puppy gain in six months?
Ⓐ 3 kg and 600 g
Ⓑ 2 kg and 400 g
Ⓒ 2 kg and 120 g
Ⓓ 5 kg and 600 g

6. Kate cut a piece of ribbon 7 metres, 20 cm and 1 mm from a roll of ribbon that was 9 m and 40 cm long. How much ribbon is left on the roll?
Ⓐ 2 m, 20 cm and 7 mm
Ⓑ 2 m and 70 cm
Ⓒ 2 m, 19 cm and 9 mm
Ⓓ 1 m, 10 cm and 7 mm

7. A large vat holds exactly 60 litres and 200 millilitres. Presently, the vat contains exactly 42 litres and 200 millilitres of liquid. How much liquid would have to be added to fill the vat to capacity?
Ⓐ 17 litres and 200 millilitres
Ⓑ 17 litres and 900 millilitres
Ⓒ 18 litres
Ⓓ 18 litres and 100 millilitres

8. A load of freight weighs 4 tonnes and 845 kg. After a delivery is made, the weight is reduced by 561 kg and 14 grams. How much does the remaining load weigh?
Ⓐ 400 tonnes, 720 kg and 140 g
Ⓑ 3 tonnes, 279 kg and 200 g
Ⓒ 4 tonnes, 283 kg and 986 g
Ⓓ 4 tonnes, 279 kg and 140 g

"Adequate test preparation significantly improves student attitudes toward test-taking and, hence, actual performance on high-stakes tests" (Chittooran & Miles, 2001, p. 42).



9. On the last day of the week, Gregory asked students to find the answer to −1,986,688 ÷ −1496. What is the answer?

10. Kim and Grandfather have $154\frac{1}{2}$ metres of rope to make a web for a swinging hammock. Grandfather says that each section should be $6\frac{3}{4}$ metres long. How many sections will they be able to cut from the length of rope? Explain how you worked out your answer.

Extend Your Learning

- *4 × 4*

  Draw a grid of 4 squares across and 4 squares down. Put a division sign in the top left square. In the remaining boxes of the top row, write the signs +, −, +. In the remaining boxes of the left column, write the signs +, −, +. List numbers for dividends in the top row after the signs; list divisors in the left column. Use the remaining blank squares to play tic-tac-toe: take turns choosing a dividend and divisor and writing the quotient, with the correct sign, in the box where they meet. Only correct answers count!
- *SOSE: Your Own Map Scale*

  Find a road map or other small map that shows where you live. Pick three destinations that are more than a kilometre from your home. Use the map's scale to estimate the distance to a mixed number of kilometres, such as $12\frac{2}{3}$ kilometres or $50\frac{1}{4}$ kilometres. Then create your own scale by using 1 centimetre = any mixed number of kilometres, such as 1 centimetre = $1\frac{1}{2}$ kilometres. Use your scale to draw your own map of your home and the three destinations, first determining how many 'centimetres away' from your home each location is.

Students engage in maths-related writing, a writing-to-learn activity.

"Students who use writing to learn have a considerable advantage over students who do not engage in writing for this purpose" (Vacca & Vacca, 2000, p. 220).

Check Your Understanding

Independent Practice

A true measure of success is when a student becomes an independent learner. The instructional goal of developing a class of independent learners is valued because "Reported patterns include that high-achieving students prefer independent study and are significantly more self-motivated, persistent, responsible, teacher and adult motivated, and prefer tactile rather than auditory instruction. They also strongly prefer self-direction, flexibility, and options as well as a minimum of structure and lecture" (Collinson, 2000).

When students reach the Check for Understanding section, they work independently much like they do in a testing situation. Gulek (2003) discusses the several benefits researchers have found about test preparation. Adequate and appropriate test preparation plays an important role in helping students demonstrate their knowledge and skills in high-stakes testing situations. Norton and Park (1996) found a significant relationship between test preparation and academic performance. Chittooran and Miles (2001) also concluded that "adequate test preparation significantly improves student attitudes toward test taking and, hence, actual performance on high-stakes tests" (p. 42). The ***Extensions in Mathematics™ Series*** offers additional practice with test preparation in each review lesson and in the Final Review portion of the text.

Students also receive assessment experience through short-response and extended-response questions. Throughout the ***Extensions in Mathematics™ Series***, students use this metacognitive strategy to reveal their problem-solving thought processes in both the Learn About the Strategy section and the Learn More About the Strategy section in each lesson. In the Check Your Understanding section, students graduate from a short-response question to an extended-response question. Students demonstrate that their understanding has transferred from the fundamental strategy to a more complex concept of a mathematical strategy.

Maths-related writing involves analytical writing by students to explain their thought processes when solving mathematical problems. This type of writing-to-learn is an effective metacognitive strategy for enhancing students' problem-solving skills. Vacca and Vacca (2000) emphasise that analytical writing reveals what learners think and understand in relation to the course material. Students are metacognitively aware of what they know and do not know about the material.

As a concluding activity of each lesson, students engage in extension activities, which span cross-curriculum topics. Students can apply mathematical concepts in other areas of life. Integrating other curriculum areas into the subject of mathematics is another instructional strength that the ***Extensions in Mathematics™ Series*** offers teachers and students.

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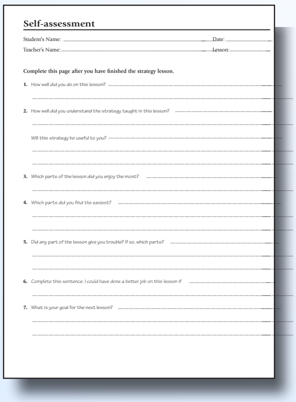
Summary of the Five Parts

The ***Extensions in Mathematics™ Series*** delivers a comprehensive and effective learning experience that provides comprehensive content coverage coupled with test-preparation practice. Many other researchers have documented the above teaching strategies as effective. Smith and Geller (2004) list these principles from research: cueing prior knowledge, scaffolded instruction, modelled and guided practice, and immediate feedback. The organisational framework of the Extensions in Mathematics™ Series is grounded in mathematics research, making the program an effective instructional tool for students who desire a deeper study of number relationships. Other research-based instructional features further support the ***Extensions in Mathematics™ Series.***

HOW DOES RESEARCH SUPPORT THE FEATURES IN THE EXTENSIONS IN MATHEMATICS™ SERIES?


Self-assessment

Student's Name: ____________ Date: ________

Teacher's Name: ____________ Lesson: ________

Complete this page after you have finished the strategy lesson.

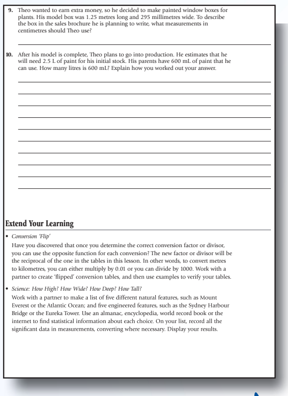
1. How well did you do on this lesson? ____________
2. How well did you understand the strategy taught in this lesson? ____________

   Will this strategy be useful to you? ____________
3. Which parts of the lesson did you enjoy the most? ____________
4. Which parts did you find the easiest? ____________
5. Did any part of the lesson give you trouble? If so, which parts? ____________
6. Complete this sentence: I could have done a better job on this lesson if ____________
7. What is your goal for the next lesson? ____________

The ***Extensions in Mathematics™ Series*** is a multi-tiered program that affords students a deeper exploration of a mathematics topic or concept. Along with scaffolded instruction, the ***Extensions in Mathematics™ Series*** is infused with additional research-based instructional features: metacognitive strategies, graphic organisers, multiple representations and development of mathematical literacy.

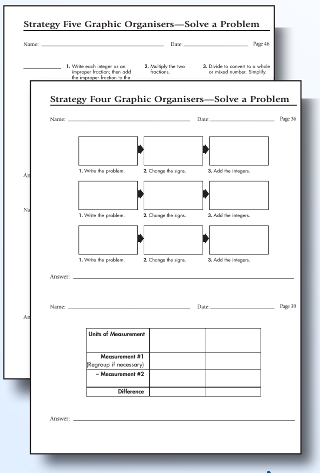
Metacognitive Strategies

Many of the instructional strategies used throughout this program are based on metacognitive research. Metacognition is "thinking about thinking, knowing 'what we know' and 'what we don't know'" (Blakey & Spence, 1990). Further support for metacognitive learning instruction is offered by the National Research Council (1999), "The model of the child as an empty vessel to be filled with knowledge provided by the teacher must be replaced. Instead, the teacher must actively inquire into students' thinking, creating classroom tasks and conditions under which student thinking can be revealed. Students' initial conceptions then provide the foundation on which the more formal understanding of the subject matter is built" (p. 15). The National Research Council (1999) defines metacognitive strategies as the ability of students to be able "to predict outcomes, explain to oneself in order to improve understanding, note failures to comprehend, activate background knowledge, plan ahead, and apportion time and memory" (p. 18). The National Research Council's recommendation stems from the Positive findings of integrating metacognitive learning strategies into classroom curriculum. Students develop the skills and intrinsic thought processes that lead them to become independent, successful learners.



Extend Your Learning

Students make their thought process explicit through the Explain Your Solution activities.



Strategy Five Graphic Organisers—Solve a Problem

Strategy Four Graphic Organisers—Solve a Problem

Students learn to work fluently through graphic organisers, such as flow charts and graphs, to demonstrate their understanding of a mathematics strategy.

Students using the Extensions in Mathematics™ Series demonstrate their mastery of a mathematics strategy by transferring and applying their knowledge into different representations of the mathematics concept.

In the ***Extensions in Mathematics™ Series,*** students employ metacognition as they progress through each lesson. In the Explain Your Solution portion of each lesson, students provide qualitative explanations about how they reached a solution. By making their thought process explicit, students and teachers alike can judge the reasonableness of their solution methods. The written explanation brings to the forefront students' awareness of their problem-solving process. Students are also focused on self-awareness when they complete the Self-assessment questionnaire. The questionnaire is administered after each lesson so that students have a chance to reflect upon and examine their experience with each mathematics strategy.

Graphic Organisers

A key finding in the learning and transfer literature is that organising information into a conceptual framework allows for greater "transfer"; that is, it allows the student to apply what was learned in new situations and to learn related information more quickly (National Research Council, 1999, p. 13). "Today it is important that young people understand the mathematics they are learning. Whether using computer graphics on the job or spreadsheets at home, people need to move fluently back and forth between graphs, tables of data, and formulas" (National Research Council (2001), p. 16).

In ***Extensions in Mathematics™ Series,*** students learn through graphic organisers such as grids, flowcharts, tables, charts, pyramids, calendars, schedules, stem-and-leaf plots, number lines, fraction strips, triangles, protractors, plane figures, bar graphs and line plots. These are graphical tools that help students see connections, relationships and patterns between numbers in a word problem. "Recording their questions, comments, connections, pictures, or graphic organizers to interpret a mathematical concept sometimes allows students to communicate their understanding to teachers in ways that are not reflected in symbolic problem solving" (Popp, 1997, p. 129).

Multiple Representations

Providing students with numerous opportunities to work with numbers in multiple external representations is an especially strong feature of the ***Extensions in Mathematics™ Series.*** "Representations such as numerals, algebraic equations, graphs, tables, diagrams, and charts are external manifestations of mathematical concepts that 'act as stimuli on the senses' and help us understand these concepts (Janvier, Girardon, & Morand, 1993, p. 81). Finally, representation also refers to the act of externalizing an internal, mental abstraction" (Pape & Tchoshanov, 2001, p. 118).

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In addition, NCTM recommends that students learn to be fluent and flexible in the use of representations. Pape & Tchoshanov (2001, p. 119) summarise NCTM's viewpoint on representation. "Within the NCTM (2000) document, 'the term representation refers both to process and to product … to the act of capturing a mathematical concept or relationship in some form and to the form itself' (p. 67). The new process standard calls for all students to be able to:

1. Create and use representations to organise, record, and communicate mathematical ideas;
2. Select, apply, and translate among mathematical representations to solve problems; and
3. Use representation(s) to model and interpret physical, social, and mathematical phenomena (p. 67)."

In the ***Extensions in Mathematics™ Series,*** students work through mathematics strategies using solutions that involve external representations such as graphic organisers, numerals and symbols. These representations are tools that help students organise their thoughts and then justify their solutions. "Since an external representation embodies the important relationships presented in a data or a word problem, they lighten the cognitive load of the individual and serve to organize the individual's further work on a problem. Given the representation, the learner may work on alternative parts of the problem. Representations then may be used to facilitate an argument and to support conclusions" (Pape & Tchoshanov, p. 118). Students using the ***Extensions in Mathematics™ Series*** demonstrate their mastery of a mathematics strategy by transferring and applying their knowledge into different representations of the mathematics concept.

Mathematical Literacy

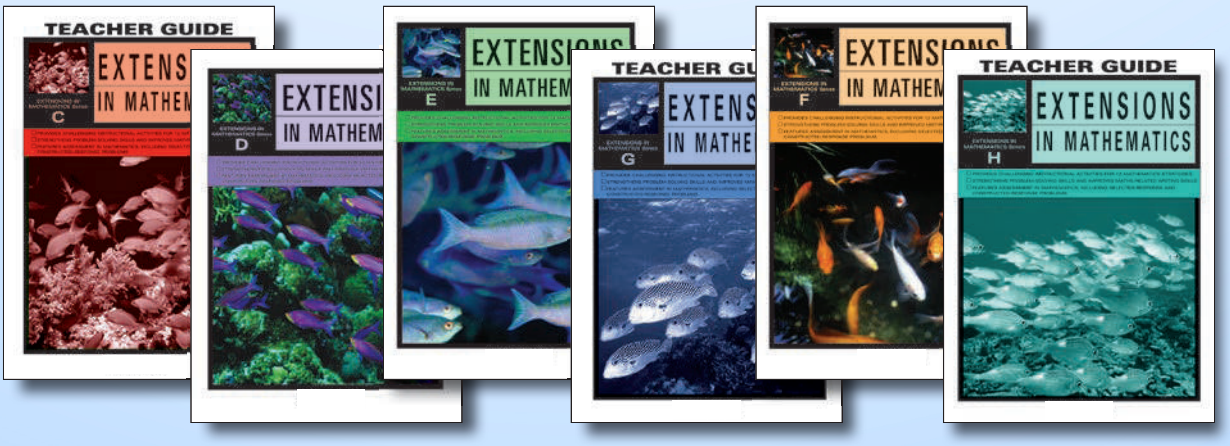
Students who are supported in their "speaking, writing, reading, and listening in mathematics classes reap dual benefits: they communicate to learn mathematics, and they learn to communicate mathematically" (NCTM, 2000, p. 60). Furthermore, "Researchers have emphasized the importance of mathematical communication to build students' capacity for mathematical thinking and reasoning" (Stein, Grover, & Henningsen, 1996). The ***Extensions in Mathematics™ Series*** develops students' mathematical literacy through the four areas of communication: speaking, writing, reading and listening. Peer-learning occurs throughout each lesson. Students are given the opportunity to speak about their mathematics reasoning. In turn, students listen to and analyse peers' reasoning to see if they are using sound mathematical reasoning. Students also write their own responses in the Solve a Problem section of each lesson.

Students work with peers as they progress through the literacy activities that shape each mathematical lesson.

Finally, each lesson, review lesson and final review lesson begin with a reading selection or graphic, which requires students to read. The students are then required to extract the relevant mathematics information to solve word problems about the reading selection or graphic. The ***Extensions in Mathematics™ Series*** strives to provide students with a balanced mathematical learning experience by providing ample opportunities to speak, write, listen and read content related to mathematics.

SUMMARY

The ***Extensions in Mathematics™ Series*** is an extension program that is built upon a research-based framework and is supported by research-based instructional strategies. Students will become mathematically proficient through its diverse types of activities. These activities require students to use multiple representations, such as graphic organisers, to demonstrate their mastery of a mathematics strategy. Scaffolded instruction provides the guideposts for students as they progress toward becoming independent learners. Metacognitive strategies give students insights into their mathematical thought processes strengthening their understanding and proficiency in mathematics. The ***Extensions in Mathematics™ Series*** is a program that applies research-based instructional experiences to improve and extend students' mathematical abilities.



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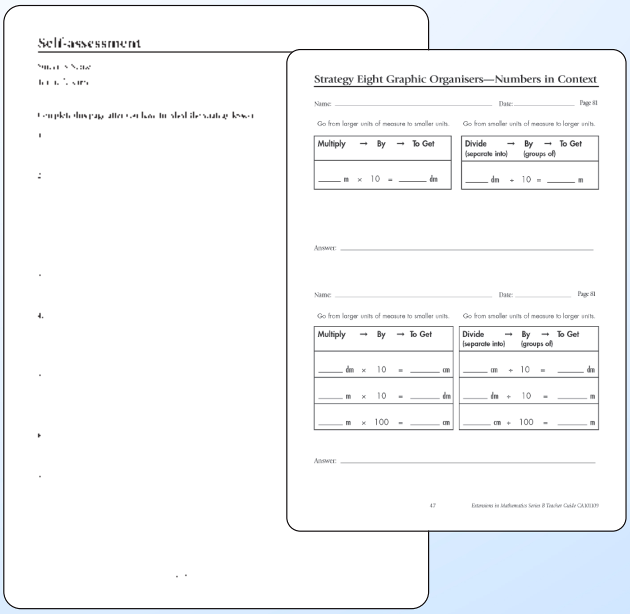
- Answers to the selected-response and short-answer questions in the practice lessons and the reviews.
- Reproducible graphic organisers and cloze-format explanations for Solve a Problem, Numbers in Context and Check Your Understanding activities.
- A reproducible Self-assessment sheet for students after they complete each lesson. Its purpose is to increase students' awareness of their own learning and help them set goals for improvement.



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