11 Let's Think about How to Find Area

Area of Quadrilaterals and Triangles

Textbook pp. 150 to 170 | Suggested number of lessons: 13

1 Goal of the Unit

- Students will understand how to find the areas of parallelograms, triangles, trapezoids, and rhombuses. Students will also derive formulas and use them to calculate areas.

Interest, Motivation, and Disposition

- Students think about how to find the areas of parallelograms, triangles, trapezoids, and rhombuses using previously studied methods of finding areas. They then attempt to find the areas through calculation.

Mathematical Reasoning

- Using previously studied methods of finding area as a basis, students devise ways to find the areas of parallelograms, triangles, trapezoids, and rhombuses, and they can derive formulas for finding the area.

Skills and Procedures

- Students can use the formulas for areas of parallelograms, triangles, trapezoids, and rhombuses to find the areas of the respective shapes.

Knowledge and Understanding

- Students understand how to find the areas of parallelograms, triangles, trapezoids, and rhombuses through calculation.

2 Unit Outline

Suggested number of lessons: 13

Sub-Units	Lesson	Textbook Pages	Primary Learning Content
1. How to Find the Area of Parallelograms	1	150-152	• Find the area of a parallelogram.
	2	153-154	• Identify the base and the height of a parallelogram. • Derive and apply the area formula for parallelograms.
	3	154-156	• Understand that the area formula for a parallelogram applies even when the height is outside of a parallelogram.
2. How to Find the Area of Triangles	4	157-158	• Find the area of a triangle.
	5	159-160	• Identify the base and height of a triangle. • Derive and apply the area formula for triangles.
	6	160-162	• Understand that the area formula for triangles applies even when the height is outside of a triangle.
3. How to Find the Area of Various Quadrilaterals	7	163-164	• Find the area of a trapezoid.
	8	164-165	• Derive and apply the area formula for trapezoids and its application.
	9	166-167	• Find the area of a rhombus. • Derive and apply the area formula for rhombuses.
	10	167	• Find the areas of irregular shapes using grids. (Let's Try)
4. The Relationship between Height and Area	11	168	• Determine the relationship between the area and height of a parallelogram when its base length is constant.
Summary	12	169	• Deepen understanding of math content. (Power Builder)
	13	170	• Check understanding of math content. (Mastery Problems)
Wonderful Problems	(14)	258-259	• Development Problems (Let's Try Wonderful Problems!)
Review		171	• Check students' understanding of previously learned content. (No allocation for instructional time)

3 Explanation of the Mathematics

1 Goals:

The main goal of this unit is for students to be able to calculate the areas of basic geometric figures (polygons) by measuring the necessary lengths and applying previously studied methods of finding the areas of rectangles and squares, and by coming up with new formulas and using them.

Specifically, the aim is to foster students' ability to think in logical steps using methods of finding areas of previously studied figures to think about and explain how to find the areas of parallelograms, triangles, trapezoids, and rhombuses, as well as deriving new formulas.

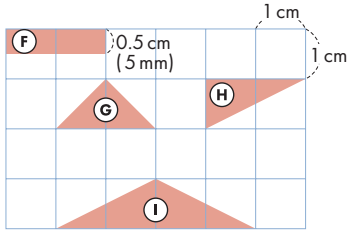
2 What students have learned previously:

In Grade 4, Unit 4, "Perpendicular/Parallel Lines and Quadrilaterals," students studied the concept and properties of trapezoids, parallelograms, and rhombuses, as well as how to draw such figures. Additionally, they studied the concept of diagonals as well as the manipulative activities of decomposing and composing quadrilaterals.

Further, in Grade 4, Unit 11, "How to Measure and Express Area," when studying the areas of rectangles and squares, students learned to quantify and express area by finding how many 1-cm^2 or other unit of area there are.

In addition to the area formulas for rectangles and squares, students have explored preservation of quantity and equivalent area transformations by dividing 1 cm^2 into various shapes and by finding the areas of composite figures.

Grade 4, Unit 10, p. 151



3 Ideas to be emphasized:

♦ Area formulas

This unit organizes how to find the areas of parallelograms, triangles, trapezoids, and rhombuses as formulas. Focus on student understanding of the formulas while relating them to the process of creating each formula as well as enabling them to use the formulas.

♦ The proportional relationship between height and area

Additionally, the examination of the proportional relationship between the heights and areas of parallelograms and triangles is important as well. These are opportunities to examine the relationships of quantities in terms of proportion, and they serve as a bridge to the study of proportions in later grades.

4 Instructional points to consider:

In this unit, students study geometric figures in the order of parallelograms, triangles, trapezoids, and then rhombuses. No matter which stage they are in, students must apply previous studies and think about how to find the given area. The understanding and application of the formulas of area are important, but instead of inculcating the area formulas, carefully teach students to think about how to find the areas. It is important to provide many opportunities for students to explain how to find areas and learn from one another—in other words, opportunities for mathematical communication—through activities in which they use diagrams to think about how to find areas. The exchanges are important as a process for increasing expressive ability.

Support

Accommodations for students who are struggling:

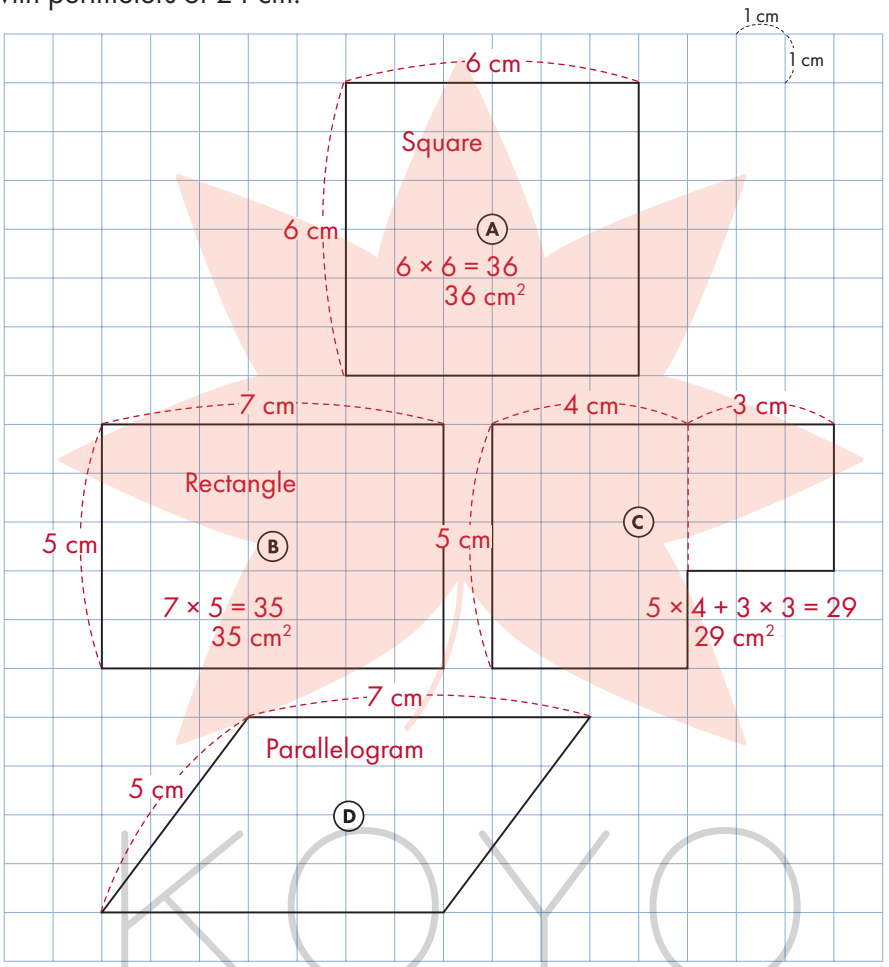
It can be difficult for students who have trouble interpreting diagrams to grasp geometric figures drawn on grids, such as those in the textbook. In these cases, have students trace outlines of the figures.

Specifically pointing out "bases" and "heights" helps students who have difficulty with language comprehension understand the problems.

Allow students who have difficulty with calculations to use a multiplication table or calculator.

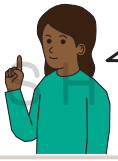
Even though the perimeters are the same ...

1 As shown below, draw and compare the areas of many different shapes with perimeters of 24 cm.



Even though the perimeters are the same, their areas look different.

Jude



Can you put Figures (A), (B), (C), and (D) in order from the largest to the smallest area?

Yes, once the area of the parallelogram is found.

Lesson 1 of 13

Goal

- Students think about how to find the area of a parallelogram, and can explain how to do so.

Materials

T An enlarged copy of the diagram on p. 150, a diagram of the parallelogram on a grid, a blackboard with a grid

S Copies of the parallelogram on p. 151, scissors, ruler, graph paper

1 Introduction

- Students draw various geometric figures with a perimeter of 24 cm.

Hatsumon Let's draw various shapes with a perimeter of 24 cm.

- First, have students draw the figures freely on graph paper.
- Then, present the four figures in the textbook (Ⓐ, Ⓑ, Ⓒ, and Ⓓ) on the blackboard.
- Have students find the areas and present their findings.
- Students arrange figures Ⓐ, Ⓑ, Ⓒ, and Ⓓ in descending order of area.

Hatsumon Let's list the four figures Ⓐ, Ⓑ, Ⓒ, and Ⓓ in descending order of area.

[Anticipated responses]

a. Square Ⓐ's area is $6 \times 6 = 36$ cm^2.

b. The area of rectangle Ⓑ is $7 \times 5 = 35$ cm^2.

c. If we separate figure Ⓒ into shapes we know how to find the area of, the area is 29 cm^2.

d. Ⓓ is a parallelogram, so it's difficult to find the area.

- Increase students' curiosity and interest in geometric figures through discussion.

- **"Even though the perimeters are the same ..."**

Present the various plane figures with the same perimeter on p. 150. Increase students' curiosity and interest in the areas of figures through discussion of area in general as well as the fact that the area of a figure does not necessarily depend on its perimeter.

In Grade 2, students learned the definitions and properties of rectangles, squares, and right triangles. In Grade 4, they learned how to find the areas of rectangles and squares, as well as the definitions and properties of parallelograms, trapezoids, and rhombuses.

In this introduction, students review content they have studied up to this point through by drawing figures with a perimeter of 24 cm and finding their areas. In addition to helping students recall how to find the areas of previously studied figures such as rectangles, squares, and composite figures, the introduction confirms students have not yet learned to find the areas of parallelograms, thus increasing their curiosity and interest in how to find the areas of figures.

The suggested length of time for this introduction is 10 minutes.

11

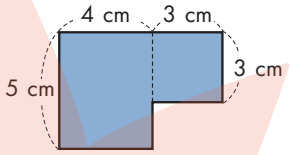
Area of Quadrilaterals and Triangles

Let's Think about How to Find Area

2

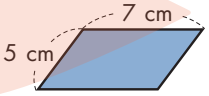
Grant: Since (A) is a square and (B) is a rectangle, we can find their areas using a formula, can't we?

Lili: We can find the area of (C) by splitting it into a rectangle and a square.



Walter: (D) is a parallelogram. Since there are slanted sides, counting the number of 1-cm^2 squares is ...

Annabelle: I wonder if we can find the area of a parallelogram by multiplying the lengths of two adjacent sides.

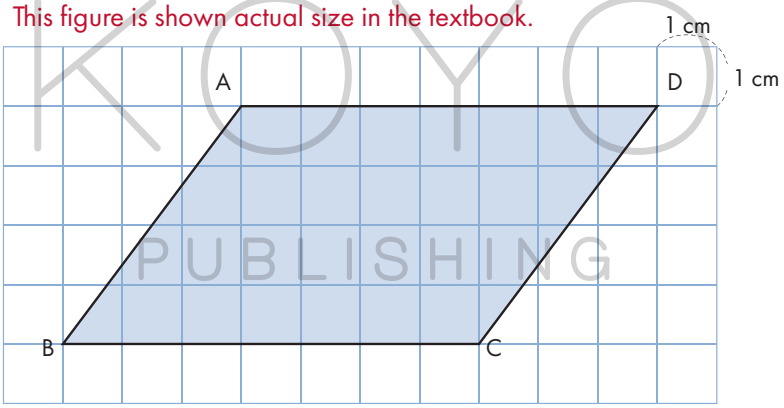


1 How to Find the Area of Parallelograms

1 Think about how to find the area of parallelogram ABCD below.

3

This figure is shown actual size in the textbook.



2 Grasping the problem

● Students look at the top of p. 151 and understand the task of this lesson is to find the area of a parallelogram.

Hatsumon	Can we find the area of a parallelogram by multiplying the lengths of two adjacent sides?

[Anticipated responses]

a. $7 \times 5 = 35$, so I wonder if 35 cm^2 is the answer.
b. The area of rectangle (B) is 35 cm^2, but the area of the parallelogram doesn't look the same.
c. I think multiplying 5 cm, a slanted side, by 7 cm is wrong.

● Students read and understand Problem **1**.

Problem	How can we find the area of parallelogram ABCD?

3 Independent problem solving

● Students think about how to find the area of a parallelogram.

[Anticipated responses]

a. If we count the squares in the grid, it looks to be about 28 cm^2.
b. It looks like we can change the shape into a rectangle.

Hatsumon	Can we change the shape of the parallelogram so we can find the area?

[Anticipated responses and support]

a. Students change the shape into that of a rectangle with a length of 7 cm and a width of 4 cm.

➡ Confirm that the area could be found if the shape were a rectangle, and have students think about what part of the parallelogram must be cut and moved to form a rectangle.

IMD Students realize that they can transform the parallelogram into a rectangle, and attempt to think about how to find the area of the parallelogram. (Observation, Statement)

▪ About showing the actual sizes of geometric figures

On textbook pages 150, 151, 157, 160, 163, 164, and 166 there are figures in the textbook shown in their actual sizes. The goal of this is to fully foster students' sense of quantity and to allow them to think about how to find the area in a way that does not rely solely on calculation.

When providing instruction, touch upon the fact that the figures are shown in their actual sizes when appropriate to enable students to feel a sense of reality toward area when they are finding areas.

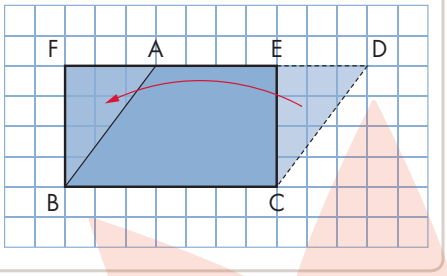
▪ Introduction from parallelograms

Finding the area of plane figures can be introduced from parallelograms or from triangles, but in general, the former is more common. This is because when finding the area of a figure, students' natural inclination is to think, "can we change it into a rectangle or square?" which they have studied previously. It is easier to change a parallelogram into a rectangle through equivalent area transformation than to transform a triangle.

4 **1** Explain the two students' ideas below.

Support Show the transitions with actual figures to help students understand the diagrams.

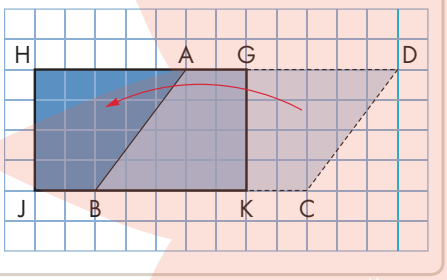
Yoko



Victor: Yoko moved triangle ECD to change parallelogram ABCD into rectangle FBCE to find the area.

Why did Yoko change the shape into a rectangle?

Govind



Eliza: Govind moved quadrilateral GKCD to change parallelogram ABCD into ...

rectangle HJKG to find the area.

5 **2** The area of this parallelogram is equal to the area of a rectangle with what dimensions? A rectangle with a length of 7 cm and a width of 4 cm.

3 What is the area of this parallelogram?

$7 \times 4 = 28$ Answer: 28 cm^2

SUMMARY

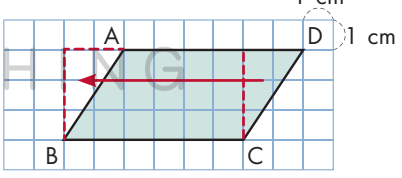
The area of a parallelogram can be found by changing it into a rectangle.

Even though the shapes are changed, the area stays the same, doesn't it?

Using the length and width of a rectangle obtained through equivalent area transformation will lead to deriving the formula for the area of a parallelogram.

The preservation of area is an assumption used in the summary.

6 **1** Using the two students' ideas above, find the area of the parallelogram on the right.



Do an equivalent area transformation to change the figure into a rectangle with a length of 6 cm and a width of 3 cm.

$6 \times 8 = 18$ Answer: 18 cm^2

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4 Discussion

- 1 Students present their ideas.

Hatsumon Let's explain our ideas using the diagram.

[Anticipated responses]

a. If we move triangle ECD, we can change parallelogram ABCD into rectangle FBCE to find the area of the parallelogram. (Yoko's idea)

b. If we draw a perpendicular line from the top to the bottom, we can slide the shape GKCD to make rectangle HJKG. (Govind's idea)

- Instead of having each student present his or her whole idea, have others take over the explanations using the diagram and other clues.

MR Students think about how to find the area of a parallelogram by transforming the parallelogram into a rectangle without changing the area, and they explain their ideas in logical steps. (Presentation, Notebook)

- Students think about why they changed the shape into a rectangle.

Hatsumon Why did we change the shape into a rectangle?

[Anticipated responses and support]

a. Because we couldn't find the area when it was a parallelogram.

b. Because changing the shape into a rectangle allows us to use a formula to find the area.

➡ Place an emphasis on the fact students are applying previous studies and carefully review the equivalent area transformation process using a diagram.

5 Summary

- 2 3 Students solve the problems.
- Confirm that the parallelogram becomes a rectangle with a length of 7 cm and a width of 4 cm, and that the area is $7 \times 4 = 28$ cm^2.
- Conclude that the area of a parallelogram can be found by changing it into a rectangle through equivalent area transformation.
- Have students summarize what they learned in their notebooks.

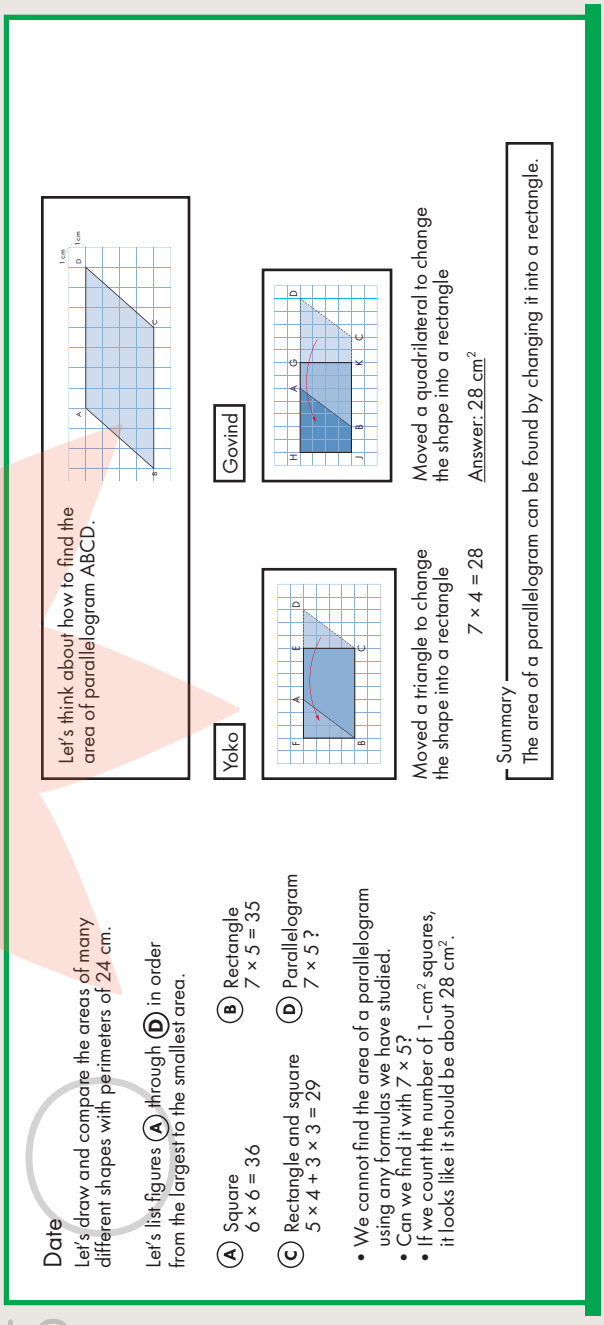
6 Follow-up problem

- Students solve Problem 1.

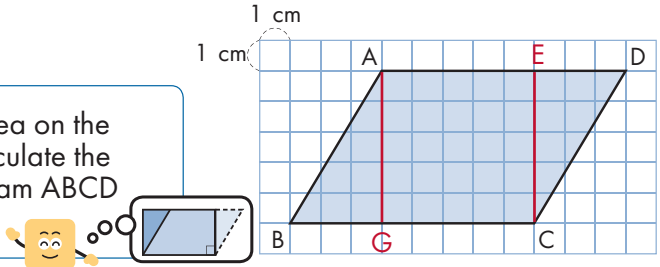
[Anticipated response]

a. It becomes a rectangle with a length of 6 cm and a width of 3 cm, so the area is 6×3, which is 18 cm^2.

▪ Example of board organization (Lesson 1)



2 Based on Yoko's idea on the previous page, calculate the area of parallelogram ABCD on the right.

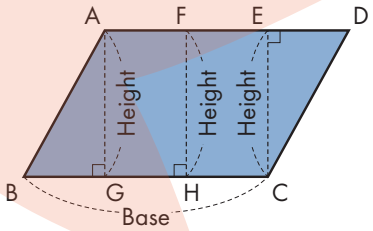


? Let's come up with a formula to find the area of a parallelogram.

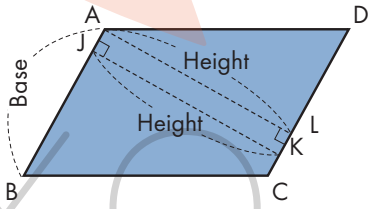
1 The area of the parallelogram above is equal to the area of a rectangle with what dimensions? A rectangle with a length of 8 cm and a width of 5 cm
Which parts of the parallelogram are equal to the length and the width of this rectangle? See the diagram above.
Length: Side AD, Side BC
Width: Line AG, Line EC, etc.

Draw the part that is the same length as the length of the rectangle in the figure above.

3 In the parallelogram to the right, if we make side BC as the **base**, the length of the line segments that are perpendicular to the base such as line segment EC is called the **height** of the parallelogram.



If we consider side AB as the base, the height will be the length of the line segments such as those shown on the right.



If it is difficult for students to understand a base that is at an angle, have them rotate their textbooks so the base is horizontal to them.

2 Calculate the area of the parallelogram above by considering side BC as the base. 8 × 5 = 40 Answer: 40 cm^2

4 SUMMARY

The area of a parallelogram can be calculated using the following formula.

Area of parallelogram = Base × Height

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Support
Have students draw a diagram to understand this.

Lesson 2 of 13

Goal

- Students derive the formula for finding the area of a parallelogram, and use it to find the areas of parallelograms.

Materials

T A diagram of the parallelogram in a grid

S Copies of the parallelogram on p. 151, ruler

1 Grasping the problem

- Students read and understand Problem 2.

Problem Based on Yoko's idea on the previous page, calculate the area of parallelogram ABCD.

- ? Students read and understand the task of this lesson.

2 Independent problem solving

- 1 Students relate the lengths of the sides of parallelograms to those of rectangles.

Hatsumon How many centimeters are the length and width of the rectangle that has the same area as parallelogram ABCD? What parts of parallelogram ABCD are they the same length as?

- Have students draw the parts that have the same length in the diagram from the textbook.

3 Discussion

- Students learn the definitions of "base" and "height" as they pertain to parallelograms.

Hatsumon If we consider side BC as the base, which part of the figure is the height? If, instead, we consider side AB as the base, which part of the figure is the height?

[Anticipated responses and support]

a. The height when we make side BC the base is AG, FH, or EC.

b. The height when we make side AB the base is AL or JK.

➡ Have students explain their answers while tracing the diagram.

- Confirm that each height is the same length.

- 2 Students find the area of parallelogram ABCD using side BC as the base.

- Help students understand through discussion that the area can be found using base × height.

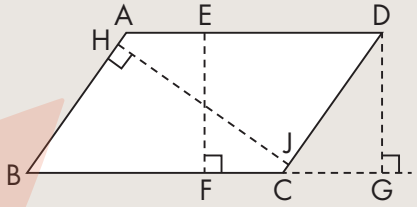
MR Students focus on the length and width of the transformed rectangle to think about and explain the area formula for a parallelogram. (Presentation, Notebook)

4 Summary

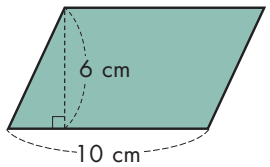
- Students read the boxed section and summarize what they learned in their notebooks.

▪ About the base and height of a parallelogram

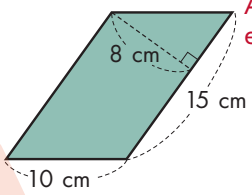
For parallelograms, if the base is determined, the height can be determined as well. The base must be determined before finding a line that is perpendicular to the base and labeling it as the height. In the diagram below, this line corresponds to lines EF and DG when the base is side BC. When the base is set as side AB, line HJ, which is perpendicular to side AB, is the height.



5 **2** Calculate the area of the parallelograms below.

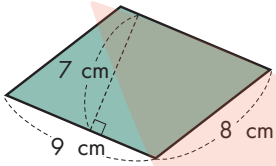
① 

10 × 6 = 60 Answer: 60 cm²

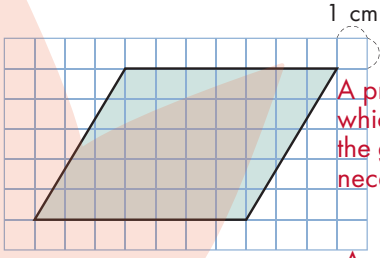
② 

A problem that contains excess information

15 × 8 = 120
Answer: 120 cm²

③ 

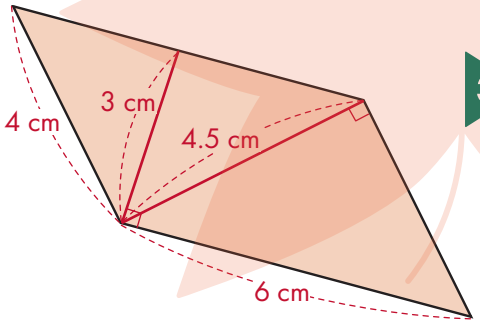
9 × 7 = 63 Answer: 63 cm²

④ 

A problem in which students use the grid to find the necessary lengths

7 × 5 = 35
Answer: 35 cm²

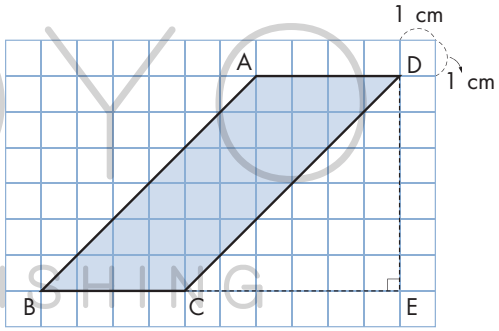
A O Additional problems → Page 244

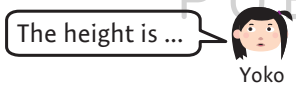


3 Measure the necessary lengths and calculate the area of the parallelogram on the left.

6 × 3 = 18 Answer: 18 cm²
(Alternative solution)
4 × 4.5 = 18 Answer: 18 cm²

1 **3** Think about how to find the area of parallelogram ABCD on the right when side BC is considered as the base.





A problem in which height is outside the parallelogram

Lesson 3