# 3 Let's Think about Multiplication of Decimal Numbers

*Multiplication of Decimal Numbers*

Textbook pp. 36 - 48 | Suggested number of lessons: 11

## 1 Goal of the Unit

- By the end of this unit, students will understand the meaning of multiplication calculations in which the multiplier is a decimal number and how to carry out the calculation. Students will understand that the properties of calculations and operations previously used for whole number calculations work with calculations with decimal numbers.

**Interest, Motivation, and Disposition**

- Students think about and explain the meaning and method of multiplication calculations in which the multiplier is a decimal number, using the properties of multiplication and number lines.

**Mathematical Reasoning**

- Using the properties of multiplication and number lines, students think about and explain how to do multiplication calculations when the multiplier is a decimal number.

**Skills and Procedures**

- Students can do multiplication calculations in which the multiplier is a decimal number.

**Knowledge and Understanding**

- Students understand how to do multiplication calculations in which the multiplier is a decimal number.

## 2 Unit Outline

Suggested number of lessons: 11

| Sub-Units | Lesson | Textbook Pages | Primary Learning Content |
|---|---|---|---|
| 1. Multiplication of Decimal Numbers | 1 | 36-38 | • Understand the meaning of multiplying by a decimal number. |
| | 2 | 38-39 | • Calculate whole numbers × decimal numbers. |
| | 3 | 40-41 | • Calculate decimal numbers × decimal numbers and understand the algorithm. |
| | 4 | 41 | • Calculate decimal numbers × decimal numbers (including cases in which 0s at the end of the product are removed). |
| | 5 | 42 | • Understand that the product is less than the multiplicand when multiplying by a proper decimal number. |
| | 6 | 43 | • Apply the area and volume formulas for rectangles and cuboids when the side lengths are decimal numbers. |
| | 7 | 44 | • Apply the commutative, associative, and distributive properties to decimal numbers. |
| 2. Times As Much with Decimal Numbers | 8 | 45 | • Use decimal numbers, including proper decimal numbers, to express times as much. |
| | 9 | 46 | • Use multiplication to find the quantity being compared in cases of times as much involving decimal numbers. |
| Summary | 10 | 47 | • Deepen understanding of math content. (Power Builder) |
| | 11 | 48 | • Check understanding of math content. (Mastery Problems) |
| Review | | 49 | • Check students' understanding of previously learned content. (No allocation for instructional time) |

## 3 Explanation of the Mathematics

### 1 Goals:

In this unit, the meaning of multiplication is expanded to cases in which the multiplier is a decimal number. Students will understand how to do those calculations and apply their knowledge. They will be able to apply the properties of calculations and operations, previously only used for whole numbers, to decimal numbers.

### 2 What students have learned previously:

Students studied the structure of decimal numbers as well as how to add and subtract decimal numbers in Grade 3, Unit 12, "Decimal Numbers," within the range of the $\frac{1}{10}$s place and in Grade 4, Unit 6, "Structure of Decimal Numbers" within the range of the $\frac{1}{1000}$s place. They learned that decimal numbers are base-10 numbers like whole numbers. Students learned the meaning of decimal numbers × whole numbers and how to do such calculations in Grade 4, Unit 14, "Multiplication and Division of Decimal Numbers." Have students recall these previous studies as needed while progressing through this unit.

### 3 Ideas to be emphasized:

♦ Expanding the meaning of multiplication

Until now, even when the multiplicand was a decimal number, every multiplication problem students have studied was multiplying by a whole number. For example, in the case of 2 × 3, the calculation can be thought of as 2 + 2 + 2, or "adding 2 three times." However, applying this idea to 2 × 2.3 leads to "adding 2 2.3 times," which does not make obvious sense and cannot easily be explained. Therefore, in this unit, students relearn the meaning of multiplication. Specifically, the meaning of multiplication is expanded to mean calculations for finding the number that is however much the size of the multiplier. This allows the application of multiplication when the multiplier is a decimal number in the same manner as when the multiplier was a whole number, using number lines and word math sentences as a basis.

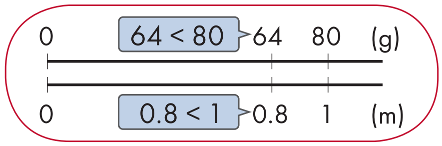
♦ Focusing on the properties of multiplication calculations and changing decimal numbers into whole numbers

Students use their knowledge of the properties of multiplication of whole numbers when multiplying decimal numbers. They first multiply the multiplicand and/or multiplier by a multiple of 10, and divide the product by the same multiple of 10. The algorithm calculation is done in the same way as with whole numbers, and students must determine the position of the decimal point as well. By relating calculations involving decimal numbers to calculations they have studied previously, students will infer how to do multiplication calculations involving decimal numbers on their own.

♦ The fact that when the multiplier is less than 1, the product is less than the multiplicand

Because they have only studied cases in which the multiplier is greater than 1 in multiplication involving whole numbers, there are students who believe that multiplication always leads to a product greater than them multiplicand. In this unit, employ methods such as using a number line to help students understand the relationship between the multiplicand and the product as well as doing calculations and comparing the numerical values to correct students' oversimplified reasoning.

Grade 5, Unit 3, p. 42




♦ Times as much and multiplication

Teach students that even when the number that expresses times as much is a decimal number, the number that shows a certain number of times as much can be found using base quantity × times as much = quantity being compared. An example that employs this is thinking of 2 as 1 when solving 2 × 3.2 and finding the number that corresponds to 3.2. Understanding this line of thinking serves as an introduction to the study of rates in the future. When providing instruction, it is also important to use number lines to help students visually grasp the relationships between quantities.

*Support*

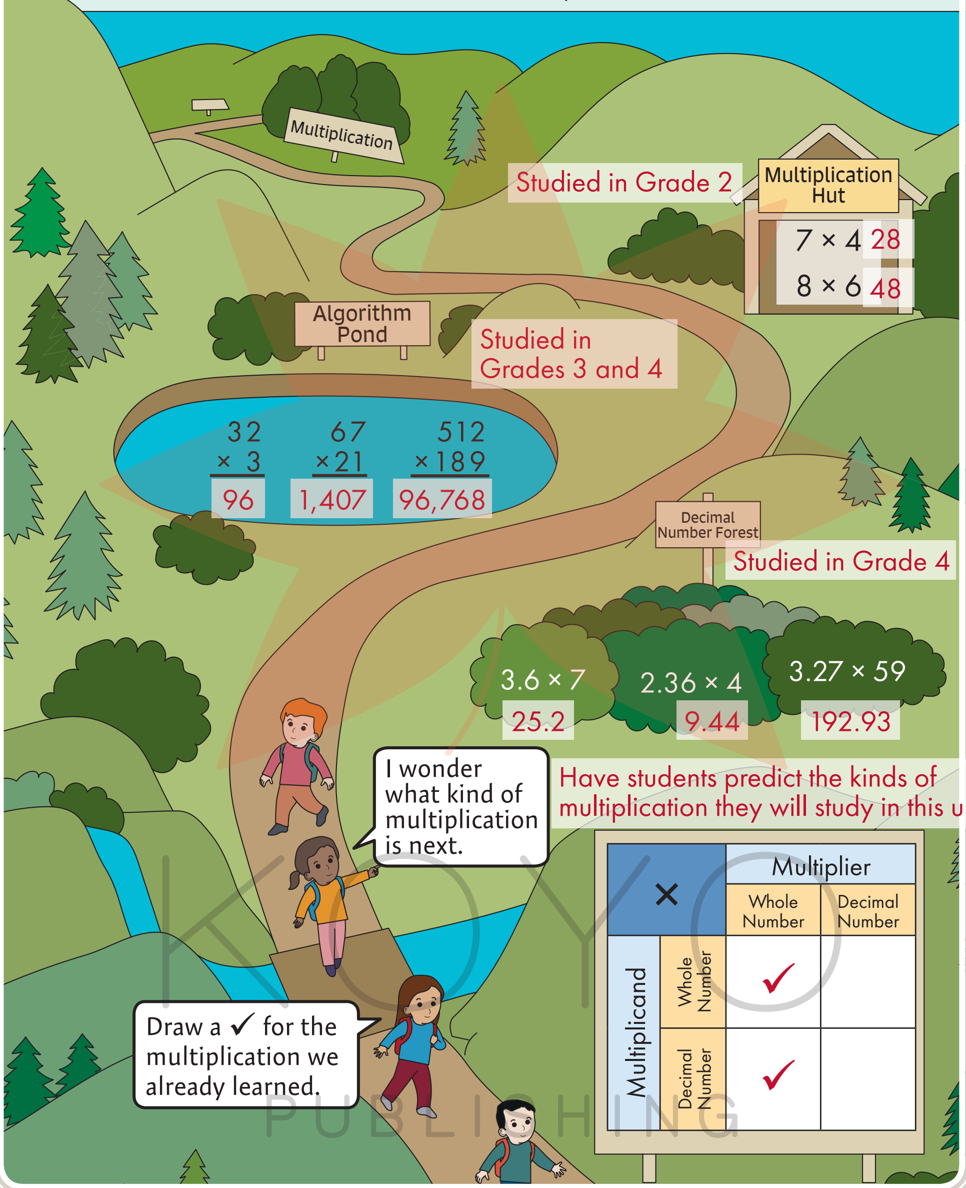
**Accommodations for students who are struggling:**

Some factors that can cause students to be poor at calculations are a poor grasp of the concept of numbers (a weak sense of quantity of numbers), weak short-term memory (an inability to remember where in the calculation one is when regrouping), and issues with visual/perceptual cognition (an inability to align digits). It can be difficult for some students to visualize the numbers involved when multiplying decimal numbers, and they can struggle with multiplication involving decimal numbers. Provide students with support that responds to their needs, such as helping them visualize quantities through the use of tangible objects, diagrams, and number lines, using grids for algorithm calculations, and verbalizing the method and steps of calculation.

# What kinds of multiplication have we studied?

Lesson 1

1 Let's review what we have studied about multiplication.



**Goal**

- Students understand the meaning of multiplying by a decimal number.

**Materials**

T Enlarged copy of number line on p. 37

## 1 Introduction

- Students look at the illustration of children hiking and review the kinds of multiplication calculations they have studied previously.

*Hatsumon* We have previously done calculations in which a decimal number is multiplied by a whole number. Can you do the following calculations?
(1) 3.6 × 7
(2) 2.36 × 4
(3) 3.27 × 59

[Anticipated response and support]

a. Students do the calculations.

➡ Check not only students' answers, but also whether they did the calculations correctly.

*Hatsumon* Are there types of multiplication involving decimal numbers that we have not yet studied? Let's use the table at the bottom to find out.

[Anticipated response and support]

a. We have not yet studied calculations of whole numbers × decimal numbers or decimal numbers × decimal numbers.

➡ Confirm that students have not yet studied multiplication calculations in which the multiplier is a decimal number.

*Support* Have students recall the multiplication table as well as the algorithm calculation process of whole numbers × whole numbers and decimal numbers × whole numbers.

*Support* This unit makes frequent use of number lines. Provide students who seem to be having difficulty interpreting number lines with additional review of number lines, such as the "Math Story" on textbook p. 35.

- **"What kinds of multiplication have we studied?"**

Have students look at the illustration on p. 36, discuss calculations students have studied previously, and increase their curiosity and interest in multiplication calculations where the multiplier is a decimal number, which they have yet to study.

Prepare students with what kinds of multiplication they will study by having them focus on multiplication calculations they have studied up to and through Grade 4.

The goals of this page are to organize the kinds of multiplication students have studied previously and to get students to think about calculations where the multiplier is a decimal number.

An additional goal is to help students recall how to do calculations by reviewing calculations of decimal numbers × whole numbers. If there are students who are struggling, find the reasons they are having trouble, provide individual instruction, and make sure that they are motivated to study decimal numbers × decimal numbers.

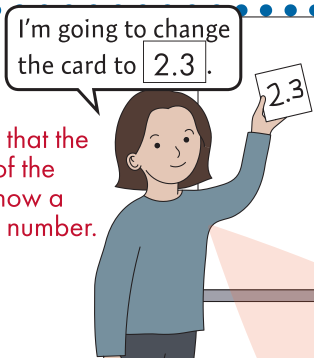
The suggested length of time for this introduction is 10 minutes.

3

Multiplication of Decimal Numbers

# Let's Think about Multiplication of Decimal Numbers

2

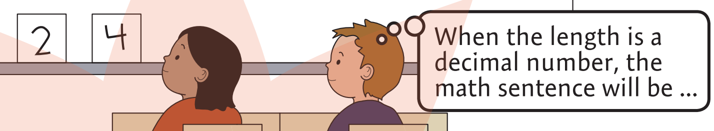



Confirm that the weight of the rope is now a decimal number.

1 meter of rope weighs 80 g. I bought 3 m of the rope.

How much does it weigh?

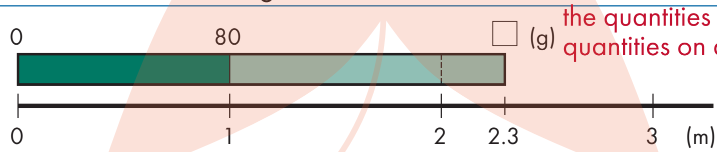
80 × 3 = 240 Answer: 240 g




## 1 Multiplication of Decimal Numbers

**1** 1 meter of rope weighs 80 g. I bought 2.3 m of the rope. How much does it weigh?

Confirm the relationship between the quantities by expressing the quantities on a number line.




3 ? Let's think about what math sentence we should write.

Confirm that using the meaning of multiplication used when the multiplier was a whole number makes it difficult to interpret the math sentence when the multiplier is a decimal number.

Math Sentence 80 × 2.3


Sam

If it were 3 m, we could think of it as three times the weight of 1 m of rope, but ...

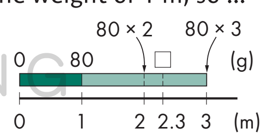

Govind

| Weight of 1 m | × | Length bought | = | Total weight |

Word math sentence


Jayla

If we buy 2 m or 3 m, the weight will be 2 and 3 times the weight of 1 m, so ...




The meanings of proportional relationships and times as much

4 ★1 Explain why the math sentence is written in this way.

37

## 2 Grasping the problem

- Students look at the illustration on p. 37, discuss what the characters are doing, and develop an awareness of the task.

(Anticipated responses)

a. Students think about the weight of the rope.
b. Students think about the weight of the rope when the length is a decimal number.

- Students read and understand Problem 1.

| Problem | 1 meter of rope weighs 80 g. I bought 2.3 m of the rope. How much does it weigh? |
|---|---|

Support For students who are having trouble visualizing the length of 2.3 m, use some rope or ribbon to help them grasp the actual length.

## 3 Independent problem solving

- ? Students construct a math sentence according to the meaning of the problem.

| Hatsumon | What math sentence should we write? |
|---|---|

[Anticipated responses and support]

a. Recalling cases involving whole numbers, students attempt to insert the values into the word math sentence to construct a math sentence.
b. Students attempt to interpret the number line to construct a math sentence.
c. Students are at a loss.

➡ Encourage students who are having trouble constructing a math sentence to use the number line and word math sentence as clues.

➡ Have students who are still having trouble read Govind's thought bubble on p. 37. Advise them to recall what they did for whole numbers to solve the problem.

IMD Students realize that "number of objects in each group × number of groups," which they have been using until this point, is difficult to visualize when the number of groups is a decimal number, so they attempt to expand the meaning of multiplication. (Observation)

## 4 Discussion

- ★1 Students explain the reasoning behind the constructed math sentence.

| Hatsumon | Let's explain why 80 × 2.3 is the math sentence we should use. |
|---|---|

(Continued next page)

### Understanding the meaning of multiplication through the number line

The idea of repeated addition used when the multiplier was a whole number does not lend itself well to situations in which the multiplier is a decimal number, since the construction of the math sentence and how to find the answer cannot be explained. Accordingly, use a math sentence to help students relearn the meaning of multiplication.

This unit takes the following steps with number lines:

① Express one quantity using a number line and the other using a tape diagram. (p. 37)

② Express two quantities using two number lines. (p. 40)

In the number line on p. 37, the tick marks along the tape diagram above the number line show the weight of the rope, whereas the tick marks along the number line below show the length of the rope. Additionally, the number line shows that for every length, there is a corresponding weight. Help students realize that there is a weight that corresponds to 2.3 m and that if the cost of 1 m is made 2.3 times as much in the same way as though the multiplier were a whole number—that is, using 80 × 2.3—the weight can be found.

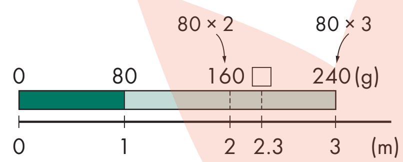
The word math sentence "number of objects in each group (unit quantity) × number of groups = total quantity" can be used to expand the meaning of multiplication. The textbook expresses the relationship using specific words such as "Weight of 1 m" and "Length bought."

Govind is replacing the words in the word math sentence with numbers.

I thought we could think of the lengths as if they were whole numbers.

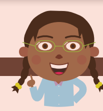

Weight of 1 m (g) × Length bought (m) = Total weight (g)

| | | | | |
|---|---|---|---|---|
| 2 m ..... | 80 | × | 2 | = 160 |
| 3 m ..... | 80 | × | 3 | = 240 |
| 2.3 m ..... | 80 | × | 2.3 | = ☐ |




$80 \times 2.3 = \square$

Jayla is constructing the math sentence using the fact that the weight of the rope is proportional to its length.

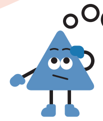



The weight of 1 m of rope is 80 g. I thought the weight of 2.3 m of rope should be 2.3 times 80 g, and that's why I thought we could use multiplication.

5 **SUMMARY**

Even when the length of rope is a decimal number, we can use a multiplication sentence to find the total weight, just like we did when the lengths were whole numbers.

1 $80 \times 2.3$

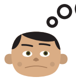


About how much will it weigh?
It will be greater than 80 × 2, but 80 × 3 ...

Have students use the number line to make estimates visually.

? Let's think about how to calculate.

2 Acknowledge the idea of using the commutative property and place importance on the idea of using the same method used for whole numbers.




We can find the weight of 0.1 m of rope, then find 23 times as much ...

He is thinking using 0.1 as a unit.


We can find the weight of 23 m of the rope first, then find $\frac{1}{10}$ of that weight ...

She is using the length that is 10 times as great as a unit.

[Anticipated responses and support]

a. If we put the numbers into the math sentence we used for whole numbers of "Weight of 1 m" × "Length bought" = "Total weight," we get 80 × 2.3. (Govind's idea)

➡ Have students verify whether this can be explained using the number line as well.

b. The number line is similar for both 3 m and 2.3 m. When buying 3 m, the weight was 3 times as much as 80, and when buying 2.3 m, the weight is 2.3 times as much as 80, so we should use multiplication. (Jayla's idea)

➡ Compare the cases of 2 m and 3 m with 2.3 m. Then, confirm that the relationship is the same as that of multiplying by a whole number.

MR Students think about the meaning of multiplying by a decimal number using number lines and calculations they have studied previously, and explain it in logical steps. (Presentation, Notebook)

### 5 Summary

- Conclude that the math sentence should be 80 × 2.3.

# Lesson 2 of 11

**Goal**

- Students understand how to do calculations of whole numbers × decimal numbers, and can carry out such calculations.

**Materials**

T Enlarged copy of number lines on p. 39

### 1 Grasping the problem

- Students estimate the answer.
- Estimation is important for preventing mistakes when finding the product.

[Anticipated responses]

a. Since 80 × 3 is 240, the product of 80 × 2.3 should be less than 240.

b. Since 80 × 2 is 160, the product of 80 × 2.3 should be greater than 160.

c. The product of 80 × 2.3 should be between 160 and 240.

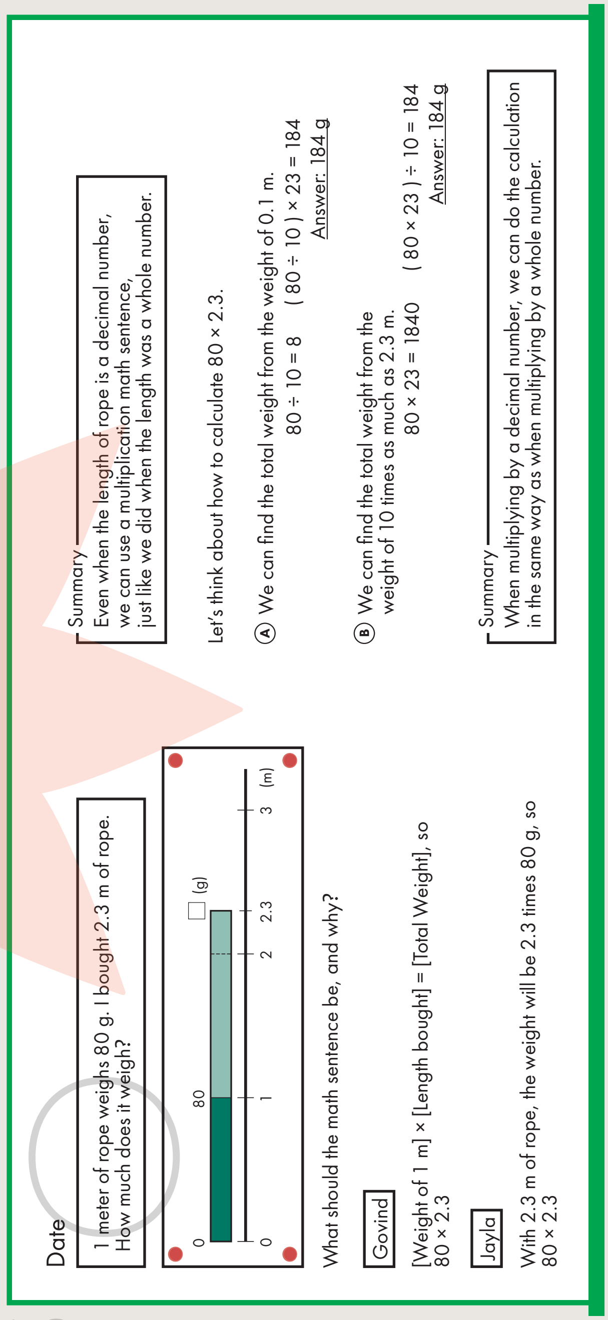
- ? Students read and understand the task of this lesson.

Hatsumon | Let's think about how to calculate 80 × 2.3.

### 2 Independent problem solving

- Students think about how to calculate 80 × 2.3.

**Example of board organization (Lessons 1-2)**




**Support** **Interpreting number lines**

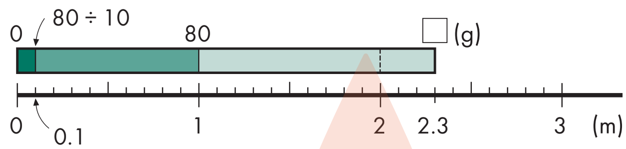
Students who have weak visual cognition may become confused due to not knowing what to focus their attention on. Make sure they understand that the key is to look at how many times as much as 1 the length is, and aim for students to understand the number line through visual and gradual explanation.

**3**


Sam

2.3 m is made up of 23 0.1-m pieces. So, we can find the weight of 0.1 m of rope, and then find what 23 times that weight is.

| 8 g | ÷ 10 | 80 g |
|---|---|---|
| 0.1 m | ÷ 10 | 1 m |

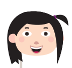



- Weight of 0.1 m ... 80 ÷ 10
- Total weight of 2.3 m ... (80 ÷ 10) × 23 The calculation is using whole numbers.

80 × 2.3 = 80 ÷ 10 × 23

= 184

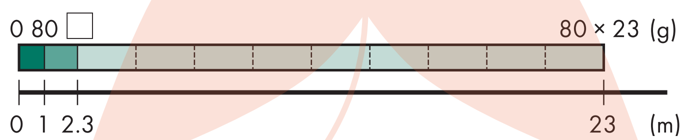
Answer: 184 g


Yoko

If the length of the rope is 10 times as long, the weight is also 10 times as much.

80 × 2.3 = 184
↓ × 10 ↓ × 10 ÷ 10
80 × 23 = 1840

Students may be given a card with the property involved in the calculation in the box to the left for regular use.




- Weight of 23 m ... 80 × 23
- Total weight of 2.3 m ... 80 × 23 ÷ 10 Confirm that the calculation is using whole numbers.

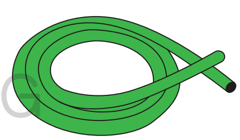
80 × 2.3 = 80 × 23 ÷ 10

= 184

Answer: 184 g

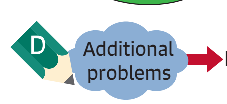
**4** You can multiply by a decimal number using whole number calculations.

**5** **1** 1 meter of hose weighs 180 g. How much will 1.6 m of this hose weigh?



180 × 1.6 = 288

Answer: 288 g



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- The key to doing the calculation is to think of the problem in the same way as with a multiplier that is a whole number. Have students think about how many times as much the multiplier, 2.3, must be made to convert it to a whole number.

[Anticipated responses and support]

a. Students use the weight of 0.1 m as a unit.
b. Students use the weight of 23 m as a unit.
c. Students find the weights of 2 m and 0.3 m.
➡ Have students think of a way to explain their methods in an easily understood manner using a number line or other tool.
➡ Have students who found the task simple find other ways of solving the problem.
d. Students are at a loss.
➡ Advise students to read Sam's and Yoko's ideas and find the numbers that go in the □.

## 3 Discussion

- Students present their ideas to one another and examine them.

[Anticipated responses and support]

a. 2.3 m is made of 23 0.1 m, so I first found the weight of 0.1 m, and then I found what 23 of those would weigh. (Sam's idea)
b. I made 2.3 10 times as much, found the weight of 23 m, and found $\frac{1}{10}$ of that weight. (Yoko's idea)
➡ Have students relate words, the math sentence, and the number line with one another when explaining their answers.
c. I separated 2.3 m into 2 m and 0.3 m, found the weight of each, and then found the total weight.

MR Students use the number line and calculations they have studied previously to think about and explain in logical steps how to do calculations of multiplying by a decimal number. (Presentation, Notebook)

- Clarify the focus of the discussion.

| *Hatsumon* | What do these ideas have in common? |
| --- | --- |

[Anticipated response]

a. They are doing the calculation by converting the decimal number into a whole number.

## 4 Summary

- Conclude that the answer can be found by converting the decimal number into a whole number.

## 5 Application problems

- Students solve Problem 1.
- Students write their study reflections.

### Study reflections and evaluations

Lessons 1 and 2 are important for examining whether students can apply their knowledge even when the multiplier is a decimal number, so have students write study reflections after class.

(1) I was surprised to learn that we can multiply numbers even when the multiplier is a decimal number. I would like to try multiplication with decimal numbers in a variety of situations.

The student attempts to proactively apply what (s)he learned to everyday life.

(2) I think Sam's idea is very good. With his method, I think I can do calculations with any decimal number.

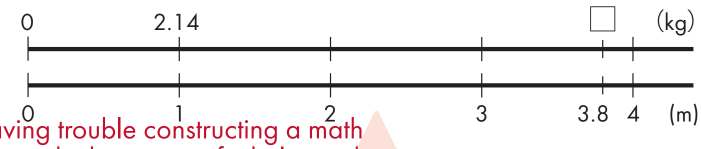
The student attempts to find the advantages of the method.

(3) I would like to see if this works even when multiplying by 0.8, which is less than 1.

The student creates a developmental task.

Lesson 3

**1** **2** 1 m of a pipe weighs 2.14 kg.
How much will 3.8 m of this pipe weigh?




If students are having trouble constructing a math sentence, have them think in terms of whole numbers.

Math Sentence 2.14 × 3.8

Have students estimate the product using the number line.

About how many kg will it be?

**2** Let's think about how to calculate.

Jayla: I wonder if we can make both 2.14 and 3.8 into whole numbers.

**3** ★1 Explain the calculation method on the right.

2.14 × 3.8 = 8.132
× 100, × 10, × 1000, ÷ 1,000
214 × 38 = 8,132

To calculate 2.14 × 3.8, first multiply 2.14 by 100 and 3.8 by 10 to make them 214 and 38. Then, calculate 214 × 38. Finally, divide this product by 1,000.

$2.14 \times 3.8 = 214 \times 38 \div 1{,}000$

$= 8.132$

Answer: 8.132 kg

Let's think about how to calculate 2.14 × 3.8 using the multiplication algorithm.

**Position of the decimal point**

```
    2.1 4   — 100 times →      2 1 4 ·····Move (2) places to the right.
×     3.8   — 10 times →  ×      3 8 ·····Move (1) place to the right.
  1 7 1 2                    1 7 1 2               (2 + 1)
  6 4 2                      6 4 2
  8.1 3 2   — 1000 times →   8 1 3 2 ·····Move (3) places to the left.
              1/1000
```

Have students explain this by relating it to what was summarized in ★1.

## Goal

- Students understand how to do calculations of decimal numbers × decimal numbers.

### Materials

T Enlarged copy of number line on p. 40

### 1 Grasping the problem

- Students read and understand Problem 2.

**Problem** How heavy is 3.8 m of a pipe when 1 m weighs 2.14 kg?

- Students construct a math sentence.

**Hatsumon** What should the math sentence be?

- Students estimate the answer.

**Hatsumon** About how much will the answer to 2.14 × 3.8 be?

[Anticipated responses]

a. Students estimate the product using the number line.
b. Students estimate the answer using a calculation involving whole numbers, such as 2 × 3 or 2 × 4.

### 2 Independent problem solving

- ? Students think about how to calculate 2.14 × 3.8.

**Hatsumon** Let's think about how to calculate 2.14 × 3.8.

- Have students who are having trouble recall Yoko's idea from the previous lesson and think about how many times as much the multiplicand and multiplier must be made respectively to convert them to whole numbers.

MR Students think about and explain how to do calculations of decimal numbers × decimal numbers by using the properties of multiplication and their knowledge of multiplying whole numbers. (Observation, Statement)

### 3 Discussion

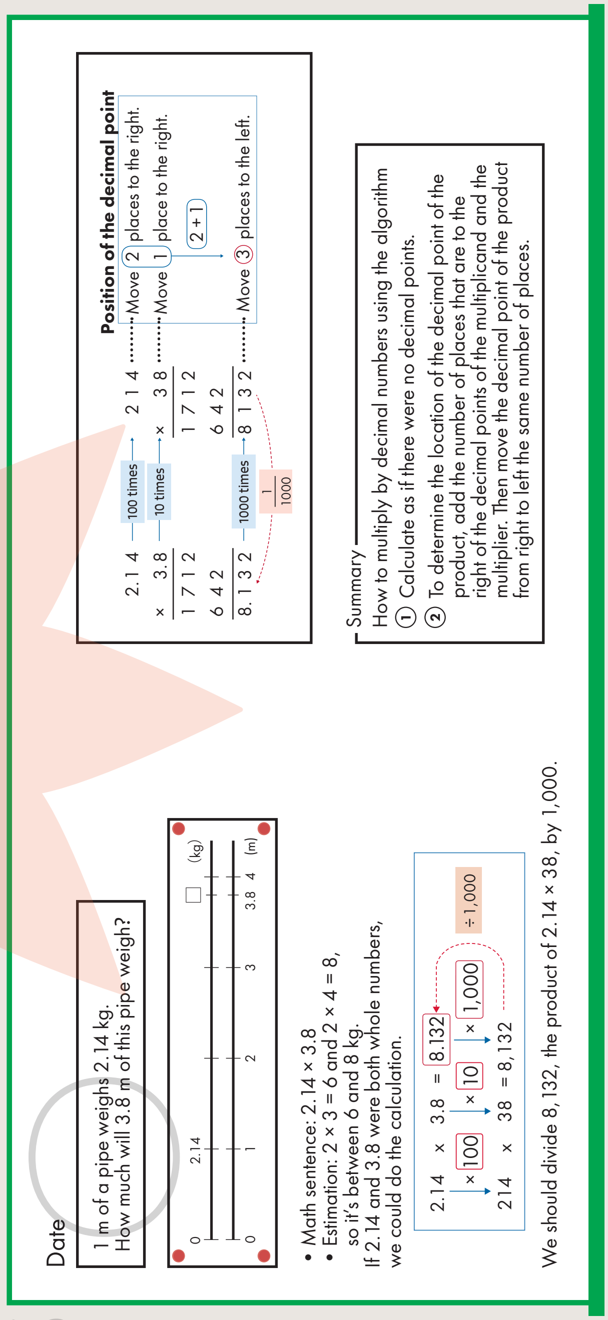
- ★1 Summarize how to calculate 2.14 × 3.8.
- ? Students think about how to calculate 2.14 × 3.8 using the algorithm.

**Hatsumon** Let's think about how to do the calculation using the multiplication algorithm, using the algorithm calculation of 214 × 38 as a basis.

- Have students think about how to do the algorithm calculation by comparing it to the algorithm calculation of whole numbers × whole numbers.

(Continued next page)

■ **Example of board organization (Lessons 3-4)**



Summarize this by relating it to what was summarized in 1.

4

How to multiply by decimal numbers using the algorithm

1 Calculate as if there were no decimal points.
2 To determine the location of the decimal point of the product, add the number of places that are to the right of the decimal points of the multiplicand and the multiplier. Then move the decimal point of the product from right to left the same number of places.

```
    2.1 4  → 2 places to the right
×     3.8  → 1 place to the right
  1 7 1 2        2 + 1
  6 4 2
  8.1 3 2  ← 3 places to the left
```

Even when the multiplier is a decimal number, we can calculate just like we did with whole numbers, can't we?

**2** Find the product of each of the following based on 176 × 54 = 9504.

1 17.6 × 54 — 950.4
2 176 × 5.4 — 950.4
3 1.76 × 5.4 — 9.504

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**3** Place the decimal points in the products.

```
1    1.7        2      7 6.5
   × 2.3            ×    8.3
     5 1              2 2 9 5
   3 4              6 1 2 0
   3.9 1            6 3 4.9 5
```

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A, B, etc. denote the type of each problem. Students may use a calculator to check their answers.

**4** Estimate the products first, and then calculate using the multiplication algorithm.

A Decimal number × Decimal number

1 4.37 × 5.6 — 24.472
2 3.81 × 7.4 — 28.194
3 3.9 × 2.1 — 8.19
4 19.6 × 3.02 — 59.192
5 54 × 6.8 — 367.2
6 816 × 2.3 — 1876.8

B Whole number × Decimal number

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**3** Think about how the multiplication algorithm was used in the calculation to the right.

```
1     4.9 2        2     0.1 8
   ×    7.5           ×    3.4
    2 4 6 0                7 2
  3 4 4 4                5 4
  3 6.9 0 0              0.6 1 2
```

C Remove the extra 0s in the product.

D Add a 0 in the ones place.

1 The unneeded 0s are removed.
2 The necessary 0 is added.

**5**

C 1 2.35 × 5.6 — 13.16
2 3.6 × 9.5 — 34.2
3 875 × 1.2 — 1,050
D 4 0.17 × 1.2 — 0.204
5 0.23 × 3.1 — 0.713
6 0.6 × 1.5 — 0.9

D' Remove the 0s in the product, add a 0.

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Lesson 4

[Anticipated responses and support]

a. We make 8,132 $\frac{1}{1000}$ times as much, so we should move the decimal point of 8,132 by 3 places.

➡ Have students think about where in the algorithm calculation they can look to find "3 places."

b. We moved the decimal point 2 places for 2.14 and 1 place for 3.8, so we should move the decimal point of 8, 132 by 3 places.

### 4 Summary

● Students summarize how to do multiplication calculations of decimal numbers using the algorithm.

| | |
|---|---|
| *Hatsumon* | Summarize in your own words how to multiply decimal numbers using the algorithm. |

- Make sure that students sufficiently understand the steps of calculation, ① and ②.
- Students can focus on the difference between how the decimal point is placed in the multiplication algorithm versus the addition and subtraction algorithms.

K&U Students understand how to do algorithm calculations of decimal numbers × decimal numbers. (Statement, Notebook)

# Lesson 4 of 11

## Goal

Students understand how to do algorithm calculations of decimal numbers × decimal numbers (including cases in which 0s at the end of the product are removed and in which a 0 must be added to the product), and can carry out such calculations.

**1** Students solve Problems ▶2, ▶3 and ▶4.

- Many students tend to erroneously place the decimal point in the same place as the multiplicand or multiplier, so if this is the case, review the principles of decimal point placement to help them recognize their mistake.

**2** Students read Problem 3 and explain how to do algorithm calculations ① and ②.

[Anticipated responses]

a. In ①, the decimal point is written in the product before erasing the 0s to the right of the decimal point.

b. In ②, a 0 is added for the ones place.

**3** Students solve Problem ▶5.

S&P Students can do algorithm calculations of decimal numbers × decimal numbers (including cases in which 0s at the end of the product are removed and in which a 0 must be added for the ones place of the product).

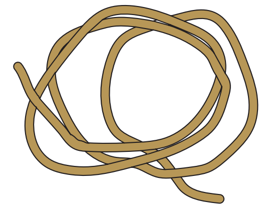
**Proficiency in calculations and the use of calculators**

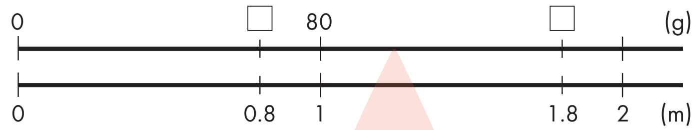
In the study of calculations, the effective application of calculators is important.

The goal here is for students to become proficient in doing calculations and sufficiently understand the principles of calculation. Therefore, instead of having students use calculators first, it is better to have them use calculators to check their answers only after doing calculations with the algorithm or only use calculators to do calculations that involve many digits.

A "proper decimal number" is a decimal number between 0 and 1 (which when expressed as a fraction is a proper fraction).

**4** The weight of 1 m of rope is 80 g.
How much will 1.8 m of this rope weigh?
How much will 0.8 m of this rope weigh?






**1** Write math sentences, then find the answers.

Weight of 1.8 m ... Math Sentence 80 × 1.8 = 144 Answer: 144 g

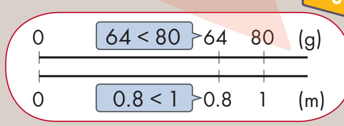
Weight of 0.8 m ... Math Sentence 80 × 0.8 = 64 Answer: 64 g

? Let's investigate the relationship between the multiplier and the size of the product.

**2** Which product is less than the multiplicand 80? 64, the weight of 0.8 m
Have students think about why the product is less than the multiplicand.

SUMMARY

When you multiply by a number less than 1, the product will be less than the multiplicand.




**6** Which of the following will have a product that is less than 6? Ⓐ and Ⓓ
Ⓐ 6 × 0.9 Ⓑ 6 × 1.4 Ⓒ 6 × 2.08 Ⓓ 6 × 0.85

Ⓔ The multiplier is a proper decimal number
Ⓕ Both numbers are proper decimal numbers; a 0 is added

**7**
① 8.3 × 0.7 = 5.81
② 29.3 × 0.4 = 11.72
③ 0.9 × 0.6 = 0.54
④ 0.2 × 0.03 = 0.006
⑤ 0.5 × 0.8 = 0.4
⑥ 1.25 × 0.4 = 0.5

Ⓖ A 0 is removed, a 0 is added
Ⓖ' Two 0s are removed, a 0 is added

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## Goal

- Students understand that when multiplying by a proper decimal number, the product is less than the multiplicand.

### Materials

**T** Enlarged copy of number line on p. 42

**1** Students read and understand Problem **4**.

| Hatsumon | What do we know from the problem, and what are we looking for? |
|---|---|

[Anticipated responses]

a. We know the weight of 1 m of rope.
b. We are looking for the weights of 1.8 m and 0.8 m of rope.

**2** ★1 Students think about the math sentences used to find the weights of 1.8 m and 0.8 m of rope, and then they find the products of the math sentences.

**3** ? Students examine the relationship between the multiplier and the product.

| Hatsumon | Let's look at the number line and the math sentences to examine the relationship between the multiplier and the product. |
|---|---|

[Anticipated responses and support]

a. I think when we do multiplication, the product is always greater than the multiplicand.
➡ Help students understand that there are cases in which the product is less than the multiplicand by using the number line and multiplying with several proper decimal numbers.
b. Looking at the diagram, it looks like the product is less than the multiplicand when we multiply by 0.8.

MR Students focus on the size of multipliers using 1 as a basis to think about and explain the size relationship between the multiplicand and the product. (Presentation, Notebook)

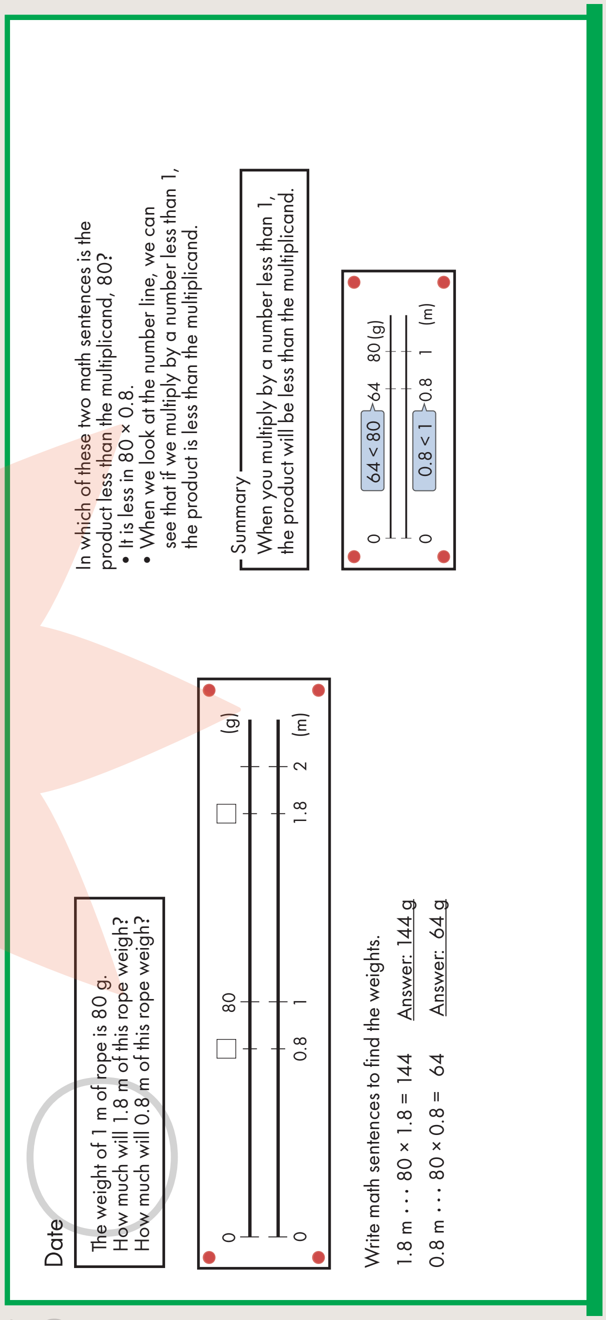
**4** ★2 Summarize the relationship between the size of the product and multiplier.

- Provide the summary while showing the relationship visually using the number line.

**5** Students solve Problems 6 and 7.

S&P Students can look at a multiplier and determine the size relationship between the product and the multiplicand. (Notebook, Presentation)

**Example of board organization (Lesson 5)**

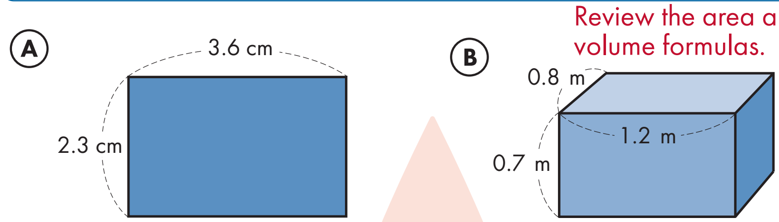



**Multiplying by a proper decimal number**

➔ See notes on p. 81 of this Teacher's Guide.

## 5 Find the area of rectangle (A) and the volume of cuboid (B).

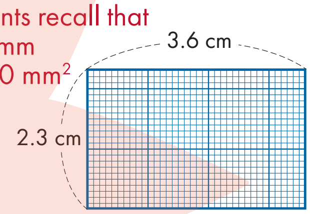
Review the area and volume formulas.




? Let's see if we can use the formulas when the lengths are decimal numbers.

Let's start with rectangle (A).

Have students recall that 1 cm = 10 mm
$1\ cm^2 = 100\ mm^2$




**1** How many squares with 1-mm sides are inside rectangle (A)?
$23 \times 36 = 828$ Answer: 828 squares

100 squares with 1-mm sides will equal 1 $cm^2$, won't they?

**2** How many $cm^2$ is the area of rectangle (A)?
$828 \div 100 = 8.28$ Answer: 8.28 $cm^2$

Confirm that the answer is being solved for using a smaller unit.

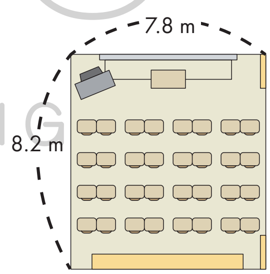
**3** Check to see if we can find the area of rectangle (A) with the math sentence $2.3 \times 3.6$.
$2.3 \times 3.6 = 8.28$ Answer: 8.28 $cm^2$

Have students recall that 1 m = 100 cm.
$1\ m^2 = 10{,}000\ cm^2$
$1\ m^3 = 1{,}000{,}000\ cm^3$

**4** Calculate volume of (B) using both centimeters and meters for the lengths of the edges, then compare the results.
$80 \times 120 \times 70 = 672{,}000$ $672{,}000\ cm^3 = 0.672\ m^3$
$0.8 \times 1.2 \times 0.7 = 0.672$ They are equal.

SUMMARY
We can use the formulas to calculate area and volume even when the lengths of the sides or edges are decimal numbers.

**8** How many $m^2$ is the area of a classroom that is 8.2 m long and 7.8 m wide?
$8.2 \times 7.8 = 63.96$ Answer: 63.96 $m^2$



## Goal

- Students can apply the formulas of area and volume even when the lengths of the sides of rectangles and cuboids are decimal numbers.

### Materials

T Blackboard with a grid

**1** Students read and understand Problem 5.

*Hatsumon* Let's find out whether we can use the formulas of area and volume even when the side lengths are decimal numbers.

**2** ? Students think about how to find the areas of rectangles when the side lengths are decimal numbers.

*Hatsumon* How should we find the area of a rectangle when the side lengths are decimal numbers?

- Tell students that by converting the units to millimeters, the areas can be found by multiplying whole numbers. Confirm that when finding areas in this manner, 100 squares with 1-mm sides make 1 $cm^2$.
- Students should estimate the area first for later verification of where they place the decimal point.

**3** 1, 2 Students convert the unit to millimeters to find the area of the rectangle.

**4** 3 Students compare the result from their calculation using the method in 1 and 2 with the result from calculating 2.3 × 3.6.

**5** 4 Students find the volume of Ⓑ using both centimeters and meters and compare the result when converting the unit to centimeters with the result when calculating using meters, 0.8 × 1.2 × 0.7.

**6** Conclude that even when the edge lengths are decimal numbers, the formulas of area and volume still apply.

**7** Students solve Problem 8.

S&P Students can apply the formulas of area and volume even when the lengths of the sides of rectangles and cuboids are decimal numbers. (Notebook, Presentation)

### Multiplying by a proper decimal number

Many students believe that the product is always greater than the multiplicand. This is due to every multiplication calculation until this point fitting this description, with the exception of when the multiplier was 0. However, students should realize that when the multiplier is a decimal number, the product can be less than the multiplicand.

This is visually evident when looking at the number line (student textbook p. 42). When multiplying by a number less than 1, the position of the □ shows that even with numbers other than 0.8, the product would be less than the multiplicand, 80.

Students tend to be surprised when the product is less than the multiplicand. They begin to think, "This is strange" and "Why is this the case?" This is a chance to increase students' curiosity and interest in mathematics, so handle this section carefully.

### Support Support for 1

If students are having difficulty with the textbook's diagram due to its small size, provide an enlarged copy and have them count the number of squares in the columns and rows.

There are 23 rows of squares with 1-mm sides and 36 columns of squares with 1-mm sides.

The total number of squares is 23 × 36 = 828, 828 squares.

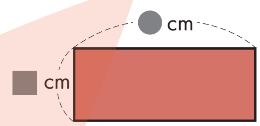
**6** Let's see if the properties of operations we found for whole number calculations also work for decimal number calculations.

Ⓐ ■ × ● = ● × ■
Ⓑ (■ × ●) × ▲ = ■ × (● × ▲)
Ⓒ (■ + ●) × ▲ = ■ × ▲ + ● × ▲
Ⓓ (■ − ●) × ▲ = ■ × ▲ − ● × ▲

If needed, review the properties using whole numbers.

**1** Check to see if Ⓐ is true by putting 2.5 in the ■ and 3.4 in the ●.

2.5 × 3.4 = 8.5    3.4 × 2.5 = 8.5

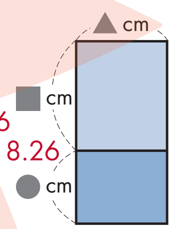



**2** Choose decimal numbers to put in ■, ● and ▲ in Ⓑ. Then, check to see if the rule is still true.

(Example) (3.4 × 2.5) × 1.4 = 11.9
3.4 × (2.5 × 1.4) = 11.9

**3** Choose decimal numbers to put in ■, ● and ▲ in Ⓒ. Then, check to see if Ⓒ is still true. Check Ⓓ in the same way.

(Example)
(3.4 + 2.5) × 1.4 = 8.26
3.4 × 1.4 + 2.5 × 1.4 = 8.26




(Example) (3.4 − 2.5) × 1.4 = 1.26
3.4 × 1.4 − 2.5 × 1.4 = 1.26

**SUMMARY**

The properties of operations we found for whole numbers also apply to calculation of decimal numbers.

Have students present how they simplified the calculations.

**9** Think about easy ways to calculate each of the following problems, then calculate.

You may want to think about using properties Ⓐ - Ⓓ above.

(Examples)

① 4.8 × 4 × 2.5 = 4.8 × (4 × 2.5) = 4.8 × 10 = 48

① 4.8 × 4 × 2.5    ② 2.4 × 1.8 + 2.6 × 1.8

② 2.4 × 1.8 + 2.6 × 1.8 = (2.4 + 2.6) × 1.8 = 5 × 1.8 = 9

③ 15.3 × 4    ④ 9.8 × 3

Since 15.3 = 15 + 0.3, ...

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③ 15.3 × 4 = (15 + 0.3 ) × 4 = 15 × 4 + 0.3 × 4 = 60 + 1.2 = 61.2

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④ 9.8 × 3 = (10 − 0.2) × 3 = 10 × 3 − 0.2 × 3 = 30 − 0.6 = 29.4

Lesson 7