6 Let's Devise New Ways to Calculate

Mental Calculation

Textbook	pp. 68 to 69	Suggested number of lessons: 2

1 Goal of the Unit

- By the end of this unit, students will understand how to mentally perform addition and subtraction between 2-digit numbers, and will be able to use these skills in everyday life as well as in their studies.

Interest, Motivation, and Disposition

- Students recognize the benefits of mental calculations, and attempt to use them in everyday life and in their studies.

Mathematical Reasoning

- Students devise methods of mental calculation and express them using an adaptable view of numbers as a basis.

Skills and Procedures

- Students mentally calculate addition and subtraction between 2-digit numbers.

Knowledge and Understanding

- Students observe the structures of numbers as well as the properties of addition and subtraction, and understand how to perform mental calculations.

2 Unit Outline

Suggested number of lessons: 2

Sub-Units	Lesson	Textbook Pages	Primary Learning Content
1. Mental Calculation	1	68	• Mentally add 2-digit numbers.
	2	69	• Mentally subtract 2-digit numbers.
Extending Mathematical Thinking	1	70-71	• Use tape diagrams to increase problem-solving abilities.

3 Explanation of the Mathematics

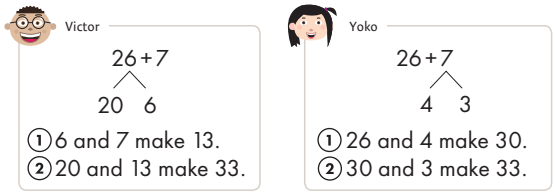
1 Goals:

In this unit, students will learn how to mentally calculate 2-digit numbers + 2-digit numbers (including situations in which there is regrouping to the tens place) and 2-digit numbers − 2-digit numbers (including situations in which there is regrouping). Students have studied how to perform these calculations in the algorithm format in Grade 2. However, there are many everyday situations in which it is necessary to mentally calculate simple addition and subtraction. The goal of this unit is for students to acquire the ability to mentally perform addition and subtraction calculations in these simple everyday situations when necessary, using the structures of numbers and properties of addition and subtraction as a basis.

2 What students have learned previously:

As noted above, students have studied how to calculate 2-digit numbers ± 2-digit numbers by hand, the commutative property of addition, and the relationship between addition and subtraction in Grade 2 Unit 3, "Addition Algorithm" and Unit 4, "Subtraction Algorithm." Additionally, in Grade 2 Unit 8, "Better Ways to Calculate," students studied the associative property of addition as well as the calculation of 2-digit numbers ± 1-digit numbers by focusing on the structures of numbers, decomposing the addend and/or augend, and calculating results by manipulating the figures mentally.

Grade 2, Unit 8, p. 84



These are foundational to learning the content of this unit, but mental manipulation of numbers is an area in which there are large differences in student performance, and therefore, the teacher should provide thorough instruction according to students' needs.

3 Ideas to be emphasized:

♦ Improving students' sense of numbers

In Grade 2 Unit 8, "Better Ways to Calculate," students studied mental calculations of addition and subtraction, but the term "mental calculations" was purposely avoided. This is because the main objective of Unit 8 was not for students to acquire the ability to perform "mental calculations" (such as beginning calculations starting from the largest place values), but to improve and make use of their sense of numbers through the mental manipulation of the digits.

This unit informs students of "mental calculations," which are processed in the mind, in contrast to "algorithm calculations," but it is also important to include the goal of improving students' sense of numbers. Consequently, instead of increasing proficiency in one technique, students should be able to recognize various methods of calculation and adopt them depending on the numerical values in each calculation.

♦ Relationship to future studies

In algorithm addition and division calculations that students will study in the future, they will become confused if they are not capable of mentally calculating sums of 2 digits + 1 digit. The content itself is identical to that of Grade 2, so it would be prudent to refer back to it and confirm student understanding in this unit.

This is the last unit in which addition and subtraction calculations of whole numbers are explored as a whole unit. At this stage, there will be cases in which there are considerable differences in technical ability among students, so it would be beneficial to assess students' levels of proficiency and provide supplementary practice.

4 Instructional points to consider:

In Sansu Math™, with the exception of foundational calculations in Grade 1, all calculations are first done by hand. This is because algorithm calculations have a more intelligible process and can be managed in a dependable manner.

In contrast, mental calculations occur within the mind, so there are considerable differences in individual capabilities. How each student grasps numbers affects how they manipulate numbers in their minds, which results in a greater difference in their structural understanding of numbers.

However, mental calculations explored in this unit will be used frequently within the processes of calculating multiplication and division in future studies. In that sense, the mental calculations in this unit are foundational. Moreover, they have a high degree of application in everyday contexts such as shopping. Therefore, it is important to devise ways of instruction that allow as many students as possible to acquire and become proficient in mental calculations.

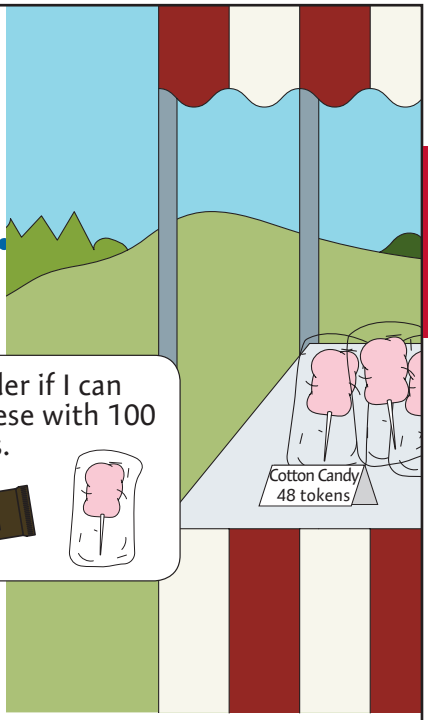
Support

Accommodations for students who are struggling:

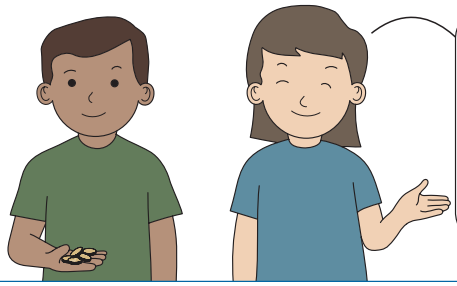
The difference between students who have strong calculation abilities and those who do not tends to be obvious among children with special needs. In particular, the instruction of mental calculations causes severe distress in children with dyscalculia, which can lead to an increased dislike of mathematics. Therefore, depending on the circumstance, use special methods such as instructing in parallel with calculations done by hand to aim for acquisition of the contents of the unit in small steps.

Mental Calculation

6 Let's Devise New Ways to Calculate



1 Yassin and Haley won 100 tokens at a carnival. They want to use their tokens to get some treats.



2 **1** Think about how to calculate 48 + 36 in your head.

3

Jayla

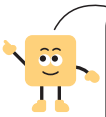
48 + 36
40 8 30 6
First, 40 + 30...

Sam

48 + 36
30 6
48 + 30 = 78
then 78 + ...

Yoko

48 + 36
50 40
If you think about 48 as 50 and 36 as 40...


Think about calculating the way you think is the easiest.

There tends to be considerable differences in individual ability to perform mental calculations, so advise students that they can use methods that they find the easiest to use.

Answer: 84

4
Think about other ways to calculate.

5 **1** Calculate the following in your head.

① 27 + 42 69 ② 15 + 35 50 ③ 57 + 38 95 ④ 19 + 64 83

Once they complete the problems using one method, encourage students to think of another method they can use.

2 Look at the picture above and think about what you can get with 100 tokens. In your head, calculate the total number of tokens needed to get those items.

< Example >
Popcorn and licorice
53 + 12 = 65

Lesson 1 of 2

Goal

- Students understand how to mentally calculate addition between 2-digit numbers, and carry out the calculations.

Materials

T Enlarged copy of the top of pp. 68-69 (for display)

1 Grasping the problem

- Students look at the illustration, discuss the situation, and develop an interest in the problem situation.

Hatsumon Let's think of combinations of treats that we can get with 100 tokens.

2 Independent problem solving

- Students read and understand Problem 1.
- Students think about how to mentally calculate 48 + 36.

Hatsumon Let's say we want to get a cotton candy and a chocolate bar. Let's calculate the total in our heads.

[Anticipated responses]

➔ See board organization example.

3 Discussion

- Clarify the focus of the discussion.

Hatsumon What were the similarities in the ideas that were presented?

[Anticipated responses]

a. All of them split either the number that is being added to or the number that is being added.
b. All of them are calculated from the tens place.

- Students discuss the benefits of each idea.

4 Summary

- Summarize what the ideas have in common and the benefits of each. "If we split the number that is being added or the number that is being added to into 10s and the remainder, or if we think about what amount we need to reach the next 10, we can easily calculate the answer without writing out the calculation process by hand."

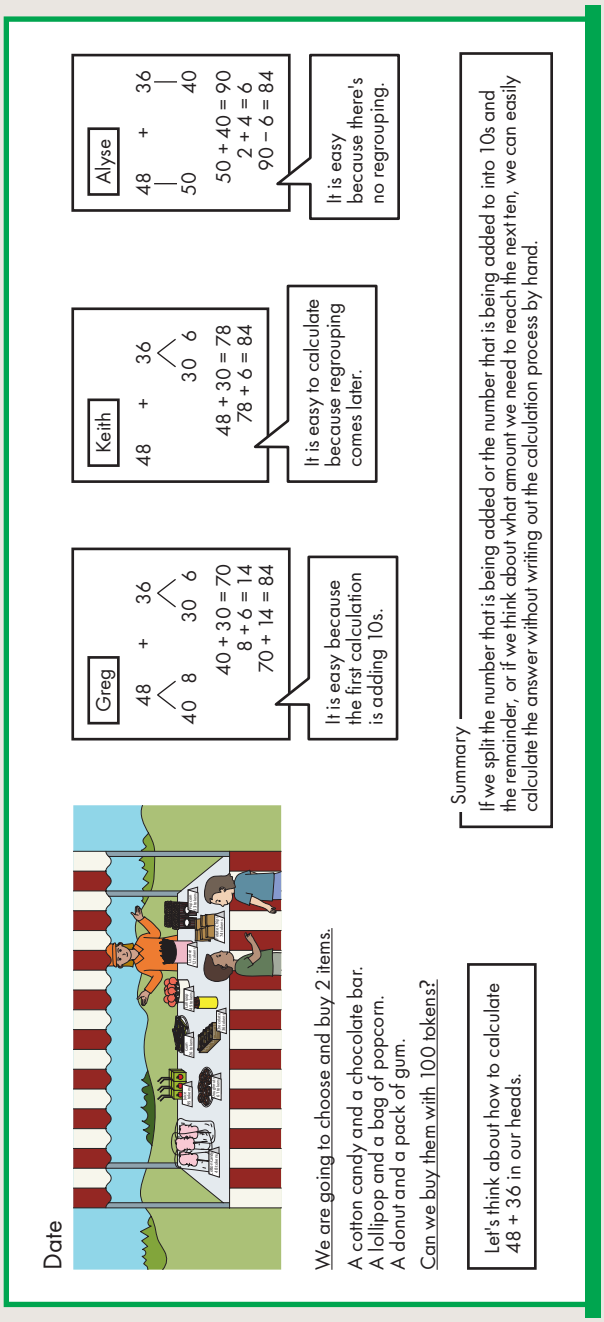
K&U Students notice the structures of numbers and the properties of addition, and understand how to mentally calculate addition between 2-digit numbers. (Presentation)

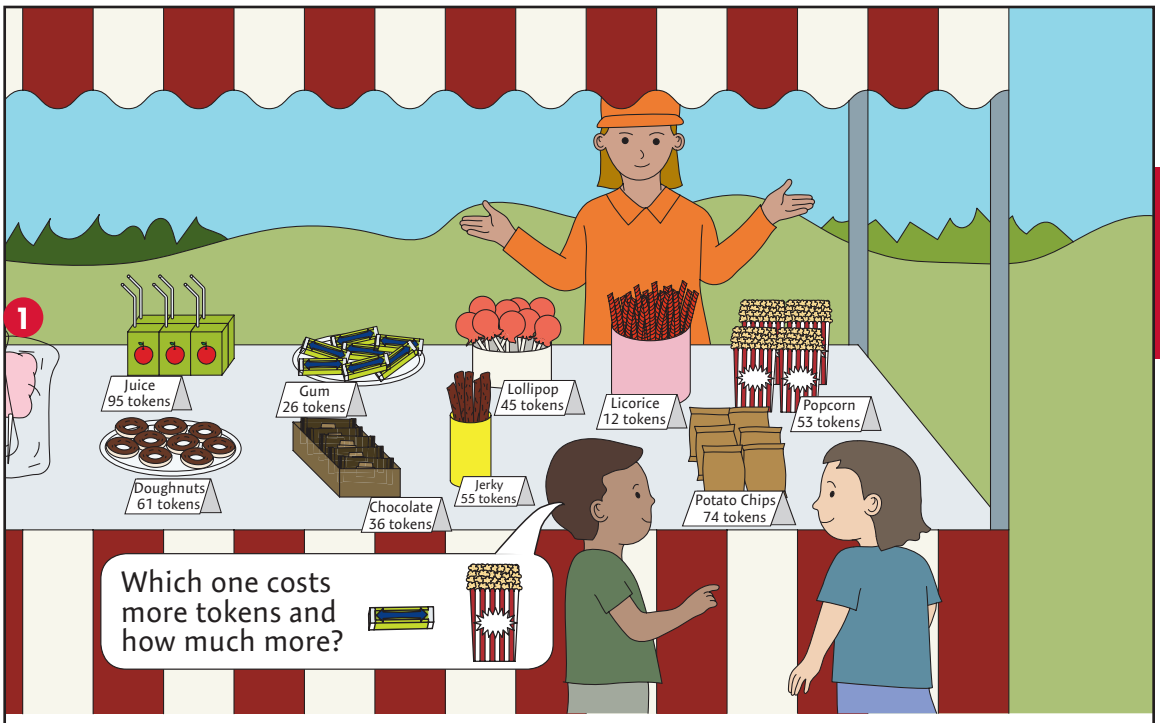
5 Application problems

- Students solve Problems 1 and 2.

IMD Students recognize the benefits of mental calculations and are attempting to apply them to their studies and everyday life. (Observation, Statement, Notebook)

Example of board organization (Lesson 1)





1

2 Think about how to calculate 53 − 26 in your head.

2

Govind: If you think about 26 as 20 and 6, first you can do 53 - 20 and...

Eliza: If you think of 53 as 50 and 3, 50 - 26 is...

3

Victor: If you think about 26 as 30...

Answer: 27

4

Calculate it the way you think is the easiest.

Inform students that they do not have to rush and that they can use the method they find the easiest to use.

5

3 Calculate the following in your head.

① 85 − 13 72 ② 51 − 36 15 ③ 72 − 49 23 ④ 94 − 47 47

4 From the picture above, choose 2 items, juice and licorice, for example, and calculate the price difference in your head.

Sam: How much more expensive are the potato chips than the lollipop?

74 − 45 = 29

Answer: 29 more tokens

Lesson 2 of 2

Goal

- Students understand how to mentally calculate subtraction between 2-digit numbers, and carry out the calculations.

Materials

T Enlarged copy of the top of pp. 68-69 (for display)

1 Grasping the problem

- Students look at the illustration, discuss the situation, and develop an interest in the problem situation.

Hatsumon Let's choose two snacks and think about which one costs more tokens, and how many more.

2 Independent problem solving

- Students read and understand Problem 2.
- Students think about how to mentally calculate 53 − 26.

Hatsumon Between a popcorn and a pack of gum, which one costs more tokens, and how many more? Let's do the calculation in our heads.

[Anticipated responses]

a.	53 − 20 = 33	33 − 6 = 27	(Govind's idea)
b.	53 − 30 = 23	23 + 4 = 27	(Eliza's idea)
c.	50 − 26 = 24	24 + 3 = 27	(Victor's idea)
d.	53 = 40 + 13	26 = 20 + 6	
	40 − 20 = 20	13 − 6 = 7	
	20 + 7 = 27		

MR Using an adaptable view of numbers as a basis, students devise ways to calculate subtraction between 2-digit numbers and explain their ideas. (Observation, Statement, Notebook)

3 Discussion

- Clarify the focus of the discussion.

Hatsumon Were there any similarities among the ideas that were presented?

4 Summary

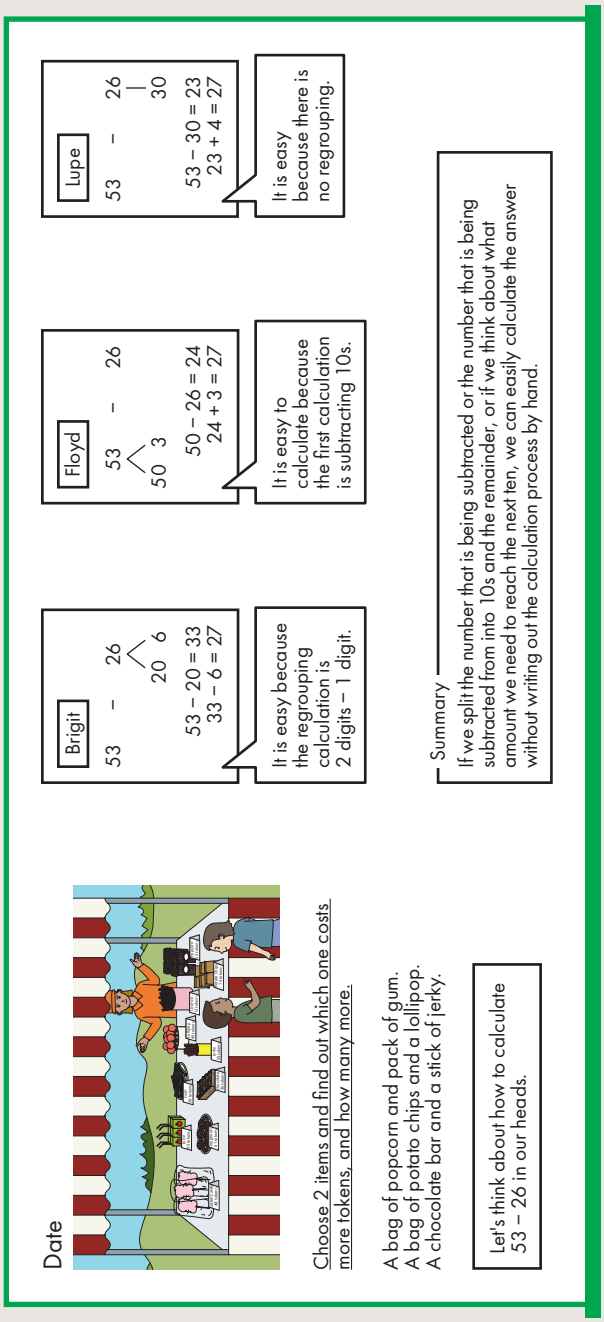
- Summarize what the ideas have in common and the benefits of each. "If we split the number that is being subtracted or the number that is being subtracted from into 10s and the remainder, or if we think about what amount we need to reach the next ten, we can easily calculate the answer without writing out the calculation process by hand."

5 Application problems

- Students solve Problems 3 and 4.

S&P Students can mentally calculate the subtraction of 2-digit numbers. (Notebook)

Example of board organization (Lesson 2)

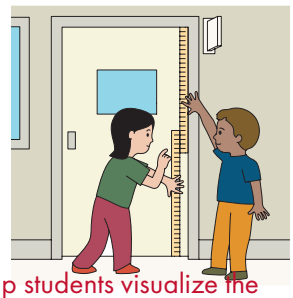


Extending Mathematical Thinking

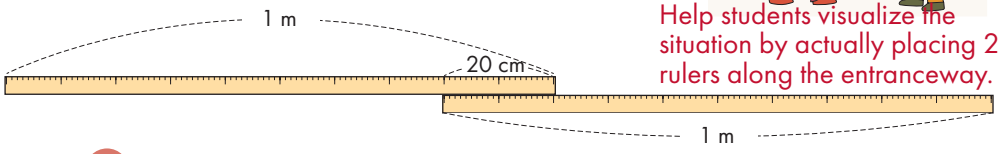
Pay Attention to Overlap

● Let's Think using Diagrams

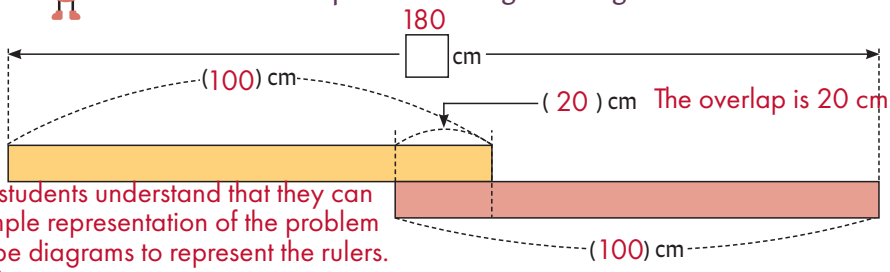
1 **1** Two students measured the height of the doorway to their classroom using 2 metersticks. The measurements are shown in the diagram below.
What is the height of the doorway to the classroom?



Help students visualize the situation by actually placing 2 rulers along the entranceway.



Think about the problem using the diagram.



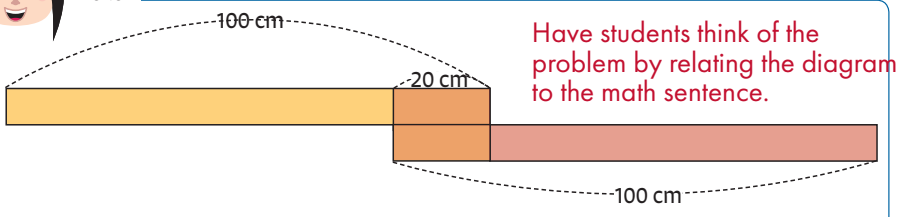
Ensure that students understand that they can create a simple representation of the problem by using tape diagrams to represent the rulers.

1 Fill in the ( ) with the appropriate numbers.

1 m = 100 cm

2 **2** Find out the height of the doorway using the diagram above.

Yoko



Have students think of the problem by relating the diagram to the math sentence.

100 + 100 − 20 = 180

Answer 180 cm

Lesson 1 of 1

Extending Mathematical Thinking

Goal

Through solving problems in which students must pay attention to the overlapping part of two quantities, students will increase their ability to solve problems.

Materials

S A copy of the tape diagram on pp. 70 - 71

1 Grasping the problem

- Students read and understand Problem 1.

Hatsumon	What do we already know? Also, what must we find?

[Anticipated responses]

a. We already know the lengths of the rulers and the overlapping part.
b. We need to find the height of the entranceway.

- 1 Students fill the appropriate values into the ( ) only on the tape diagram.

2 Independent problem solving

- 2 Students look at the tape diagram and think about how to solve for the answer.

Hatsumon	Let's use the diagram to think about how to find the height of the doorway.

[Anticipated responses and support]

a. Students are at a loss and do not know where to begin.
➡ Show students what the situation would look like if 2 rulers are used and there is no overlap as well as what it would look like with 20 cm of overlap through manipulation, and lead them to realize how to manage the overlap.
b. Add the lengths of both rulers, then subtract the overlapping part. (Yoko's idea)
100 + 100 − 20 = 180
c. Add the lengths of the rulers that remain after subtracting the overlapping part, and add the overlapping part at the end.
80 + 80 + 20 = 180
d. Add the length of one ruler and the what remains of the length of the other ruler after subtracting the overlapping part. (Govind's idea)
100 + 80 = 180

- Ensure that students can explain their solutions using diagrams and math sentences.

IMD Students realize that expressing numerical quantities as tape diagrams makes them easier to understand, and attempt to use tape diagrams to solve problems. (Observation)

▪ The benefits of expressing problems as diagrams

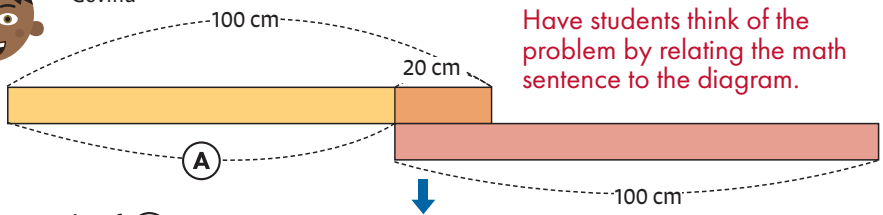
In problem situations that lacked overlapping parts, there was little need for diagrams. One goal of this textbook is for students to understand the benefits of expressing problems in diagrams through solving problem situations in which there are overlapping parts.

▪ Students pay attention to the overlapping part

Showing students diagrams and having them express problems as diagrams are both concrete means to lead students to pay attention to the overlapping part.

Through solving Problems 1 and 2, students should realize that even though the part they must solve for is different, the structure of the problem is the same. Provide support for solving Problem 2 by discussing the statements of students who are aware of its similarities to Problem 1. ("The same diagram as the previous problem," "It looks similar," "It's overlapping again," etc.)

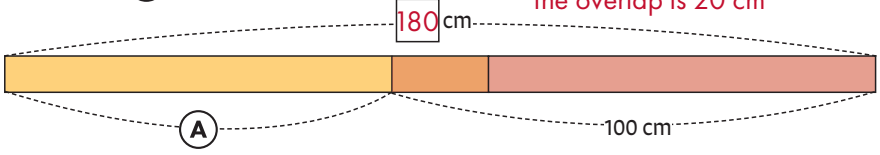
Govind



Have students think of the problem by relating the math sentence to the diagram.

Length of (A) 100 − 20 = 80

The overlap is 20 cm



80 + 100 = 180

Answer 180 cm

3

3 Explain the following 2 students' ideas.

4

Sam

Yoko's idea is
100 + 100 = 200
In the 200 cm, there is an overlapping part ...

so the overlap must be subtracted.

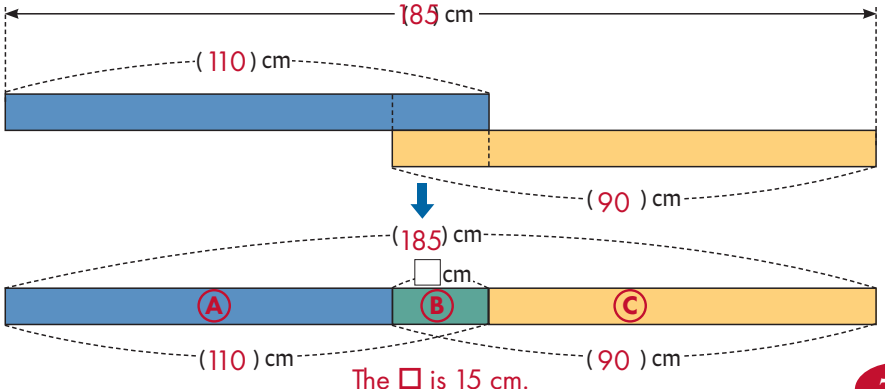
Govind's idea is, first he figured out the length of (A), then ... added the length of the other ruler.

Jayla

Continue Sam's and Jayla's explanations.

5

2 We will connect a 110 cm tape and a 90 cm tape together. We want to make the total length of the tape 185 cm. By how many cm should the tape overlap?



The □ is 15 cm.

71

3 Discussion

- Students use diagrams and present their methods to each other. Ensure that they can explain their reasoning as well.
- Use phrases that the students use, such as "overlap," "where they're the same," and "where it's added twice" as keywords, and focus their attention on the overlapping part.

MR Students pay attention to the overlapping part and explain how to solve for the answer. (Observation, Statement)

- Clarify the focus of the discussion.

Hatsumon How can we find the length when there is an overlap?

[Anticipated responses]

a. We can add the 2 lengths together and subtract the length of the overlap.
b. We can subtract the overlap from one and add the length of the other.
c. There is a difference between subtracting beforehand or doing it later, but either way, the overlapping part is being subtracted.
d. If we represent the problem as a diagram, the relationship is easier to understand.

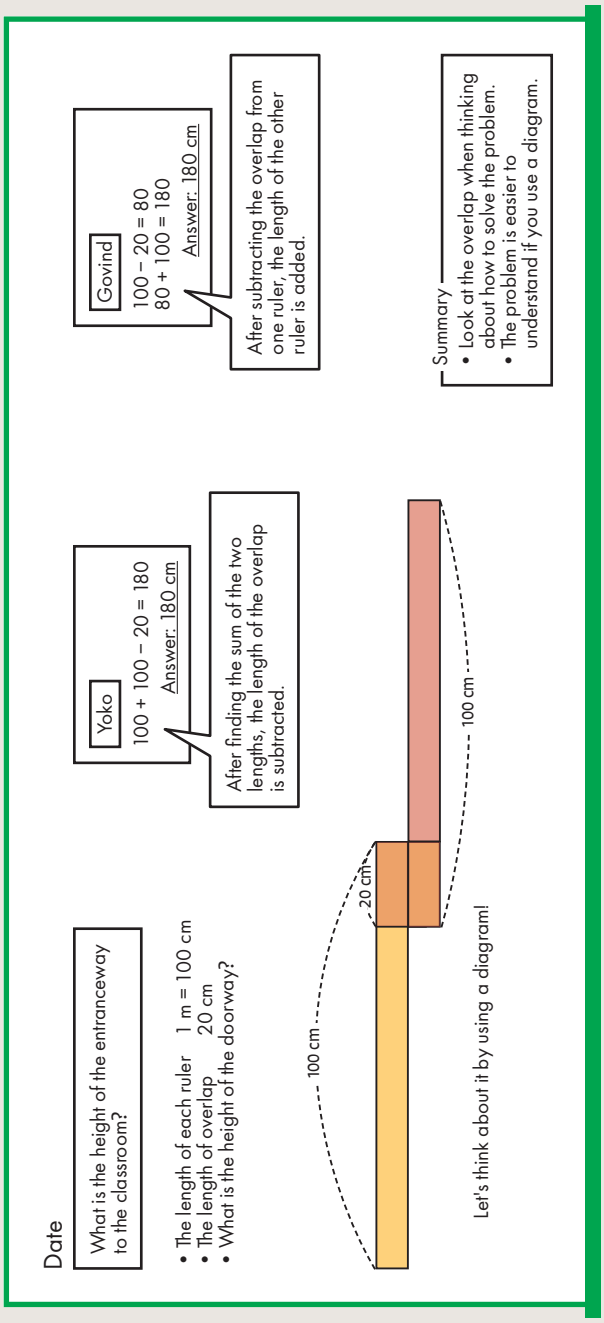
4 Summary

- Confirm how to solve for lengths that involve overlap by relating diagrams to the corresponding math sentences.

5 Application problems

- Students solve Problem 2.
- After solving for the length, ensure that students can explain using a diagram.

[Anticipated responses]

a. After adding 2 lengths as though they do not have an overlap, subtract the length of the total.
110 + 90 = 200
200 − 185 = 15
b. Find the lengths in order: First part C in the diagram, then part B.
185 − 110 = 75
90 − 75 = 15
c. Find the lengths in order: First part A in the diagram, then part B.
185 − 90 = 95
110 − 95 = 15

Example of board organization (Lesson 1)