Math

A Manual for Early Childhood (Ages 3 to 6+)

Instructor's Album

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Introduction

This book is a manual to assist adults in their work with the young child. The lessons included herein assume that the trained adult already has foundational experience offering practical life and sensorial lessons. **The Practical Life and Sensorial Manuals are a prerequisite for this one.** For example, the lessons in this manual do not include the details of all the steps in every lesson yet those must be followed.

All lessons must include the following steps:

Beginning the lesson:

1. Invite the child to work with you. Say something like, "I would love to give you a lesson. Are you free?"
2. Go together to the work. Touch the work and name it.
3. If this is a new type of material, say, "This is how we carry it." Model how you lift and carry it and return it to the shelf. Then, let the child carry it.
4. Say, "You may take this to any table you like." (If a rug is needed, say, "We'll need a rug for this work. You can put it wherever you like.")
5. Follow the child to the work area and sit on their dominant side.
6. Say, "I'll have a turn and then you can have a turn."
7. Present the mode of activity.
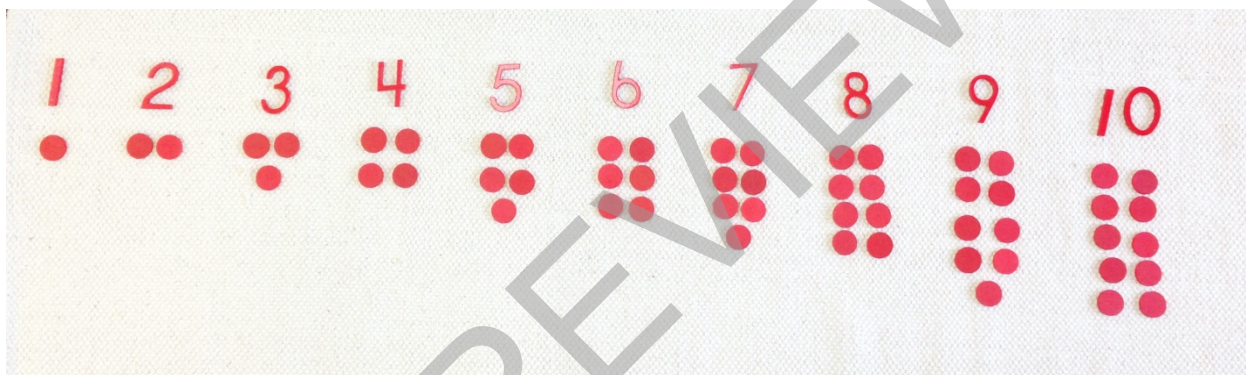8. Give the child a turn.

Completing the lesson:

1. Fade and observe, offering to take a turn if re-presentation is needed.
2. Encourage repetition.
3. When they are finished, go back to the shelf together to return the materials.
4. Free them to repeat. Say, "You can work on this [name the activity] anytime it is available."

The activities included herein are suggestions of what may be appropriate for children in North American, English speaking early childhood environments (ages 3 to 6+). Most of the lessons presented here have been effectively used for decades around the world with little change. In all cultures, the general progression of math exercises is the same:
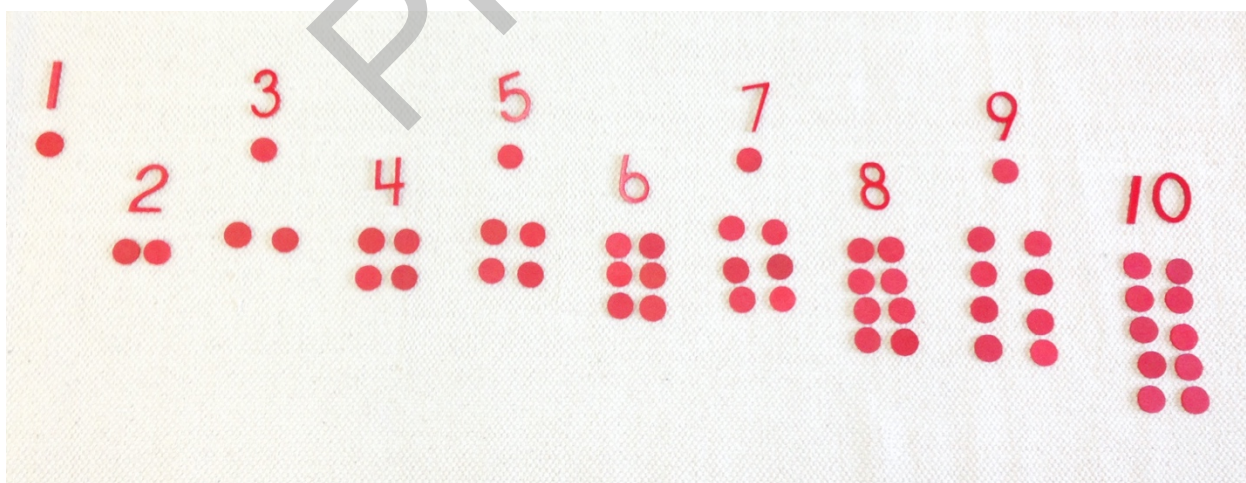
Cards and Counters

Leaving space during layout

Layout

Odds versus evens

Cards and Counters

Age	*Prerequisite Lessons*
4 and up	Spindle Boxes

Purpose:

- To help the child recognize the numerals from 1 to 10 and their correct sequence
- To understand how many separate units form each number
- To create a visual and muscular impression of odd and even numbers
- To indirectly prepare for the divisibility of numbers and, therefore, of multiples

Materials:

- Numbers from 1 to 10 printed on cardstock or wood (you can also use cut-out numerals as an extension if it is the right level of challenge for your children)
- 55 identical counters (e.g., red wooden discs)

Presentation: Counting

1. Begin the lesson. Invite the child to do the work with you. Guide the child to prepare the work rug. Go together to get the numerals and counters and bring them to the rug.

2. Take the numerals out of their container and place them randomly (but from the correct perspective) on the rug (if you are using cut-out numerals, put the 1 0 together for 10).

3. Say, "First we need these numbers in the proper sequence."

4. Place the 1 in the middle, left of the rug.

5. Find the 2. Pick it up with your right hand and place your left hand directly to the right of the 1. Place the 2 directly to the right of your left hand. (The left hand creates the necessary space between the numerals). Remove your hands and repeat for the 3.

6. Guide the child to step in and do the rest.

7. Point to the 1 and ask, "What is this? Let's put one counter underneath it." Line it up in the counter beneath the card.

8. Point to the two and ask, "What is this? We should put two counters underneath it." Line one up on the left and the other on the right.

9. Continue for the 3. Line one up on the left, one on the right, and the third centered underneath.

10. Invite the child to continue to put counters under all the numbers, keeping odd ones in the center under two even columns. Fade and observe.

11. When all counters are out, admire the work.

12. Encourage repetition by mixing up the numerals or counters and giving the child a turn.

13. Clean up by carefully returning the numerals and counters to their box, one at-a-time.

14. Complete the lesson.

Exercise #1: Odds and evens

1. Begin as in the presentation (proceed directly from the presentation if the child is ready).

2. When all numerals and counters are lain out, take your right first two fingers and slide the single counter beneath the 1 up so that it pushes the 1 above the level of the 2.

3. Proceed to the 2 and slide your finger straight up between the empty column. Since no counter slides up, the numeral stays put.

4. Repeat for the 3 and 4 and invite the child to do the rest.

5. When all odd numerals are raised, point to the row of odd numerals and say, "These are odd numbers.

6. Point to the row of even numbers and say, "These are even numbers."

7. Continue the three-period lesson for odds and evens. Say, "Point to an odd number." "Point to an even number." "Put the odd numbers over here." etc.

8. Complete the lesson.

Common Core Standards

• Counting & Cardinality: Know number names and count the sequence; Compare numbers; Count to tell the number of objects

Control of error:
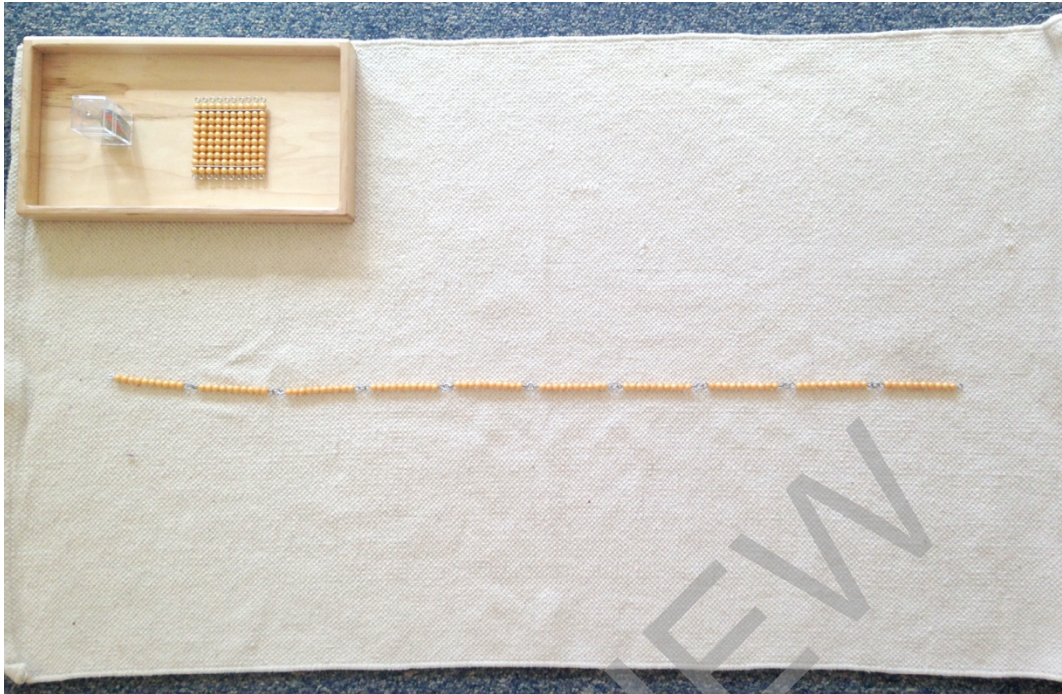
• The exact number of counters

Accommodations:

• Some children may need practice laying out the numerals and counters before they are ready for the odds/evens language lesson. Adapt to the needs of the child if necessary and give the language lesson as an extension, after plenty of manipulation of the materials.

Notes:
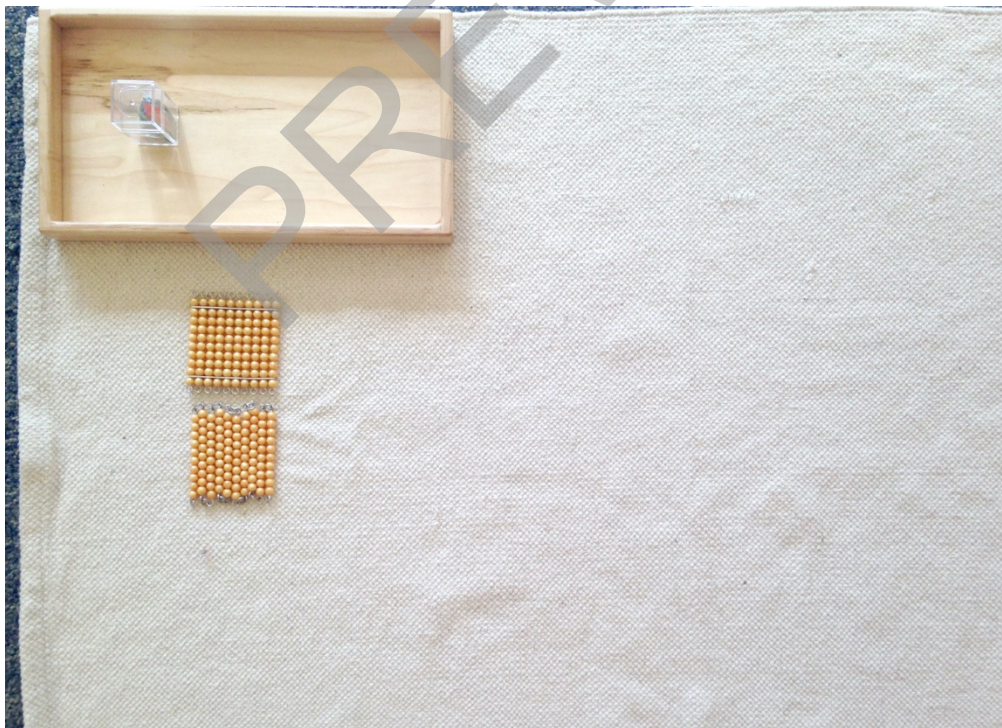
Linear Counting: 100 and 1,000 Chains

Laying out the chain
Photo by Veritas Montessori Training Residency Residents (2018)

Folding the chain
Photo by Veritas Montessori Training Residency Residents (2018)

Linear Counting: 100 and 1000 Chains

Age 4.5 and up	***Prerequisite Lessons*** • <u>Presentation</u>: Tens boards with beads: Counting to 100 • <u>Number roll</u>: 1,000 chain; Recording process (Paper)

Purpose:

- To solidify the child's knowledge of counting
- To indirectly prepare the child for squaring and cubing operations

Materials:

- *100 Chain*: Golden 100 chain of ten 10 bars, one 100 square of golden beads, Labels for 100 chain (units 1 to 9 in green; tens 10 to 90 in blue; 100 in red); A tray large enough to contain all of the labels lain out
- *1,000 Chain*: Golden 1,000 chain of 100 10 bars, 10 hundred squares of golden beads, one 1,000 cube of golden beads, Labels for 1000 chain (units 1 to 9 in green; tens 10 to 90 in blue; hundreds 100 to 900 in red; 1000 in green); A tray large enough to contain all of the labels lain out
- *Number Roll*: A roll of paper (e.g., for a printing calculator), scissors, a pencil, a container of paper clips; a basket/tray to hold all items; work drawers for storing in-progress work

Presentation: 100 Chain

1. Begin the lesson. Invite one child to do the work with you and ask him/her to prepare the work rug.
2. Go together to the bead cabinet. Say, "Today I will give you your first lesson on the bead chains. I know you have been waiting for this! These bead chains are truly precious. We must handle them with great care. I'll show you how."
3. Touch the 100 chain and say, "Today we'll use the short 10 chain."
4. Use your right hand to pick up the copper ring at the right end of the chain. Use your left hand to pick up the copper ring at the left end of the chain. Stand up and hold the chain so it makes a U in front of you. Place it back with great care, left end first, and straighten it.
5. Let the child carry it to your work rug and lay it out horizontally, to its full length.
6. Go back to the bead cabinet with the child to collect the 100 square, number tickets, and small tray (place items on the small tray). Have the child carry the tray back to the rug.
7. Place the 100 square just above the left end of the chain. Place the tray with number tickets in their container above the square, on the top left of the rug.
8. Grasp the far-left bead on the chain with your left index finger and thumb. Grasp the far-right bead in that first 10 bar on the chain with your right index finger and thumb. Slide the left side up so the first bar on the chain is now vertical and the rest horizontal.
9. Switch hands so your left is now holding the bottom bead of the first bar and your right is on the right bead of the next bead bar. Slide the second bar up so it is parallel and touching the first bar (they are both now vertical).
10. Repeat for one more bar.

11. Give the child a turn to do the rest.
12. Place the 100 square on top of the folded result and compare them. Say, "These are both the same. How many is this? 100. So this one must be how many? 100. Let's do it again to make sure we're right."
13. Straighten out the chain and have the child try it.
14. Once the child sees for himself that they are the same, straighten out the chain. Say, "Let's just make sure there are really 100 beads here."
15. Move the tray so it is in front of the child. Guide them to help you layout all the labels on the small tray, oriented to the child's perspective.
16. Say, "Let's count these beads." Place your left finger on the first bead and say, "One". Keep your left finger on the bead while you look on your tray and find the "1" number ticket. Use your right hand to place it just beneath the bead.
17. Move your left finger to touch the next bead and say, "Two." Keep your left finger on the bead while you find the "2" ticket and place as above.
18. Ask the child to step in and continue. Be sure they touch and maintain a finger as a placeholder on the bead of interest.
19. When they get to 11, notice that there is no "11" number ticket. Say, "Now, we'll only put a ticket at the beginning of each bar." Stay with them while they count to 20 and find/place the "20" ticket.
20. Fade and observe while they continue, staying ready to guide as they find the 30, etc.
21. When they are finished, ask, "Are there really 100 beads? Let's read the tickets to make sure."
22. Guide them to point to each ticket as they read it aloud, all the way to 100. If a mistake is discovered, just help them fix the labels gently, don't say anything negative.
23. Encourage repetition by turning the tickets for 1 to 9 upside-down and then have them read it from 10 to 100 and 100 to 10. Then, turn the 10-ticket upside-down and read them from 10 to 100. Then, have them turn the 20-ticket upside-down and read it from 10 to 100. Guide them to continue in this fashion until all of the tickets are face down and they recite the numbers from memory.
24. Say, "So this chain of beads is the same as this square of beads? They are both 100? Wow."
25. To clean up, put the tickets back in their box and place them on the tray with the 100 square. Return these to the cabinet, then come back for the 100 chain, guiding the child to carry it precisely as you did to begin the lesson.
26. Guide them as needed as they place it carefully back in the cabinet.
27. Encourage repetition by saying they may want to do it with a partner next time. Complete the lesson.

Exercise #1: 1,000 Chain

1. Begin the lesson and invite one or two children to do the work with you. Go together to the bead cabinet, touch the chain and name it. Say, "Today I will show you the long 10 chain. I know you have been waiting for this! This is such a big work that we need a special rug." Show them the special, long rug you'll need and help them roll it out straight in the designated location (without obstructing walkways; the rug must be straight, not bent or curved in anyway).
2. Go back to the bead cabinet and say, "There is a special way to carry these long chains. I'll show you and then you can have a turn."
3. Place your left hand out straight, palm parallel to the bead cabinet with the thumb pointing up. Use your right pincer grip to lift the far left, top copper ring of the long chain off of its

hook and place it on your left hand, sliding back near the thumb. Repeat for the rest of the chain until it is hanging neatly on your left hand just as it hung in the cabinet.

4. Stand up straight and use your right hand to secure the chain below your left hand. Say, "Now I'll put it back so you can have a turn. Watch closely."

5. Place your palm in front of the empty hooks and parallel with the cabinet. Take the farthest outside ring off of your left hand and place it on the far-right hook. Repeat for all until it is neatly hanging in the cabinet.

6. Guide them to carefully remove the chain and carry it to the long rug in the same fashion. Lay it folded on the left end of the rug so that one end of the chain is on the bottom left.

7. Place your right hand on the bottom end of the chain to secure it. Guide the child to lift the right end and slowly stretch the chain out to its full length on the rug.

8. Return to the cabinet and guide the child to help you place the 1,000 cube, ten 100 squares, and number tickets on the tray. Let the child carry it to the rug and place it on the top left of the rug.

9. Guide the child to fold the chain into squares of 10 bars as you did for the 100 chain. When you complete one square, place a 100 square on top of it to compare and verify that they are the same. Then, place the 100 square just above the folded 100.

10. When you are finished, say, "So this long chain has the same number of beads as these little 100 squares? What if we stacked the 100s up?" Guide the child to stack up the 100 squares and compare to the 1,000 cube to verify that they are the same.

11. Demonstrate that the 100s stacked up are the same as the cube. Have the child hold the two together and really examine them.

12. Once the child is convinced, straighten out the chain. Move each 100 square so it is just above each 10th bar. Say, "Let's count these beads to make sure there are really 1,000 beads on this chain."

13. Layout the labels on the tray, organizing them by 100s.

14. Say, "Let's count the beads and place these tickets." Take a few turns to get them started and then guide them to take over.

15. Fade and observe.

16. When they are finished, say, "Tell me what you've done. Read each of these number tickets to me."

17. Guide them to point to each ticket as they read it to you, all the way to 1,000 and then back down again to 1. If a mistake is discovered, just help them fix the labels gently, don't say anything negative.

18. Encourage repetition as with the 100 chain by guiding them to read the numbers forwards and backwards and then recite the numbers with more and more tickets flipped face-down. You don't have to flip over all the tickets, just as many as they feel they'd like to do.

19. To clean up, put the tickets back in their box and place them on the tray with the 100 squares and the 1,000 cube. Return these to the cabinet, then come back for the 1,000 chain. Guide the child to place every other copper ring on their left palm (thumb up) until it is hanging as when you began. Carry it and re-hang it precisely as when you began.

20. Encourage repetition by saying they may want to do it with a partner next time. Complete the lesson.

Exercise #2: 100 & 1,000 Chains

1. Begin as above but have the child lay out the 1,000 chain and label it and then lay out the 100 chain next to it and label that too.

Exercise #3: Number Roll

1. Begin the lesson and invite one child to work with you. Go together to gather the materials. Say, "I'm going to show you how to make a number roll."
2. Guide them to carry the materials to a table.
3. Ask them to hold the number roll. Say, "You stay right there. I'm going to gently pull this paper out so we make a very long piece. When I'm far enough away, I'll nod and you can cut the paper there."
4. Pick up the end of the paper and start walking carefully backwards until the paper extends about six feet. Nod to indicate that they should cut the paper.
5. Beckon for them to come over to you.
6. Show them how to roll the paper back into a roll. Let them take over after you have started it.
7. Go back to the table and sit down.
8. Stretch about a foot of paper out from the roll. Use the paper clip to clip the rest of the roll in place.
9. Take the pencil and write number "1" on the bottom right of the paper, number "2" just above it, etc. Invite them to step in as soon as they are ready. Say, "You can keep writing numbers…forever."
10. Stay close as they first begin and guide them carefully to maintain the columns for each hierarchy (to clarify place value).
11. When they have used all the exposed paper, guide them to roll the open part up, clip it with a paper clip, unclip the empty paper and unroll a new blank portion for writing. Reclip the empty roll and continue. Say, "You can keep going for as long as you like. You could even fill this entire paper with numbers! If you do, we'll tape more paper onto this so you can make it as long as you want!"
12. Complete the lesson when they are finished that day.

Control of error:

- The adult guide
- The limited quantity of labels, their color, and size

Accommodations:

- If the child needs it, before presenting Exercise 1, do a preliminary lesson to demonstrate how to place the long chain on your hand, walk around the room, and return it to its hooks. You may then do a second preliminary exercise where you spread out the chain, layout the squares/cube, and fold the beads but don't use the tickets.
- If the tickets are overwhelming for the child, use small rubber bands to bundle them by groups of 10 (10 to 90, 100 to 190, 200 to 290, etc.). You may then do a preliminary exercise where you just layout the number tickets and put them away without using the chain at all.

Common Core Standard

- Counting & Cardinality: Know number names and count the sequence

Notes:

This is a peaceful activity which is great for allowing the child to focus and develop some inner peace. Be sure you or your co-teacher checks the tickets after they've been used (easy to misplace

them). If you have to stop in the middle of this time consuming long chain work, do not let it be cleaned up. The child can come back to it when they return. If you must pick it up (e.g., for the janitor), don't let the child know and go ahead and put it away after school but come in early and set it up again before the child returns.

The other children will likely be curious about this work. Be sure to help them learn how to walk around or over the rug gracefully and with great care so as not to disturb this enormous work! Let anyone who is free know to be extra careful because there is a lot of hard work going on with these beads.

Number roll: They may wish to write each number individually or skip count as they do with the number tickets. Invite them to continue for as long as they like that day. They can pick it up on any day to do some more. This work does not have to have a set end and may continue throughout the year. Periodically help them admire their work by unrolling their number roll across the room to demonstrate how much work they've done.
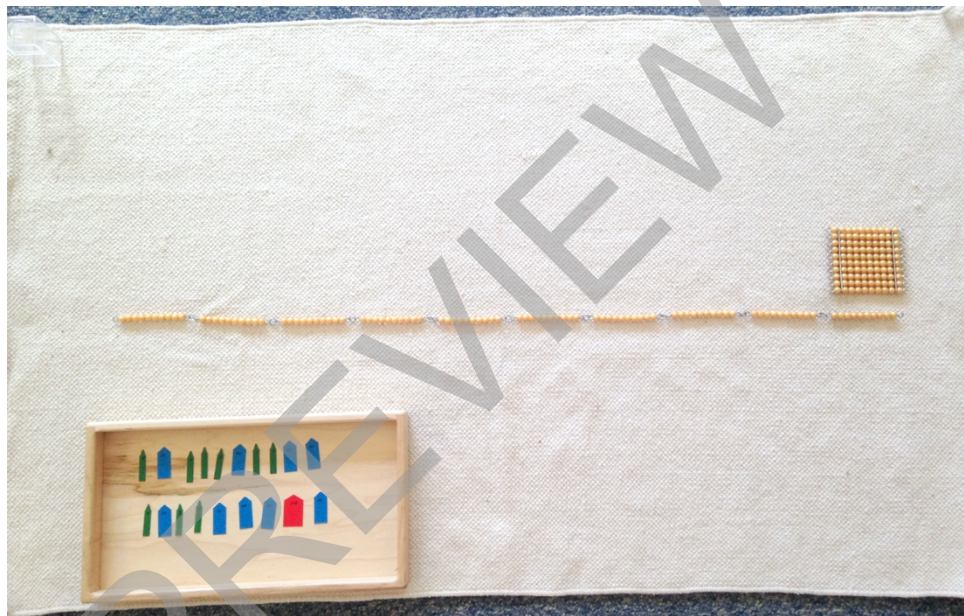
Preparing to count the 100 chain
Photo by Veritas Montessori Training Residency Residents (2018)
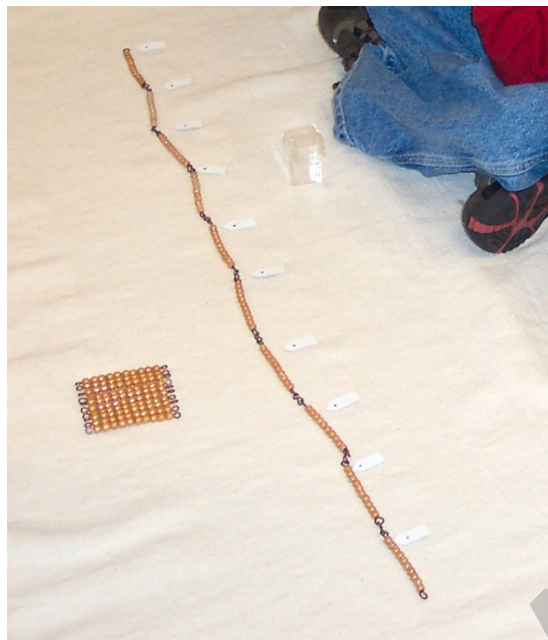
Labelled 100 chain after counting

Carrying the long chain
Photo by Veritas Montessori Training Residency Residents (2018)

Folding the chain
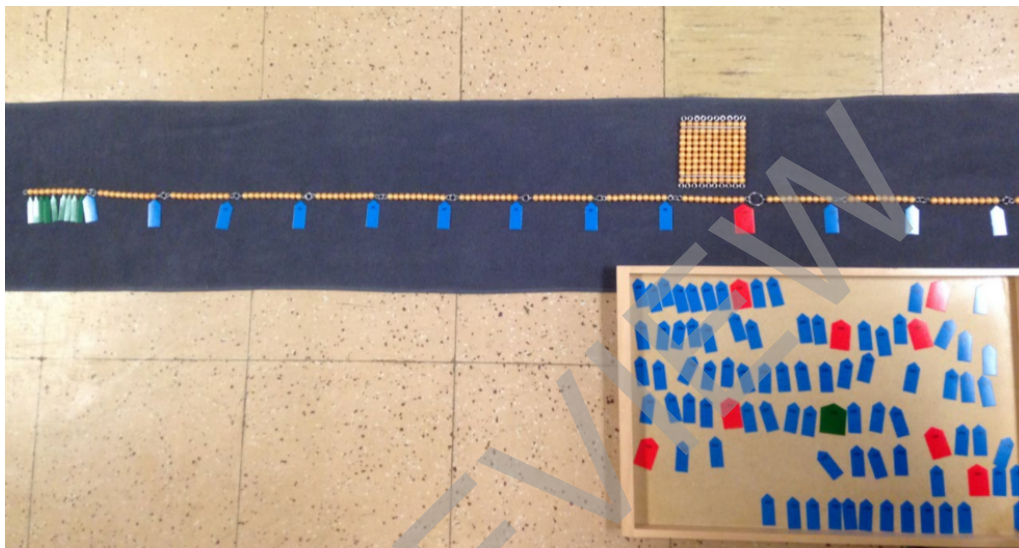*Photo by Veritas Montessori Training Residency Residents (2018)*

Comparing the fold to the squares
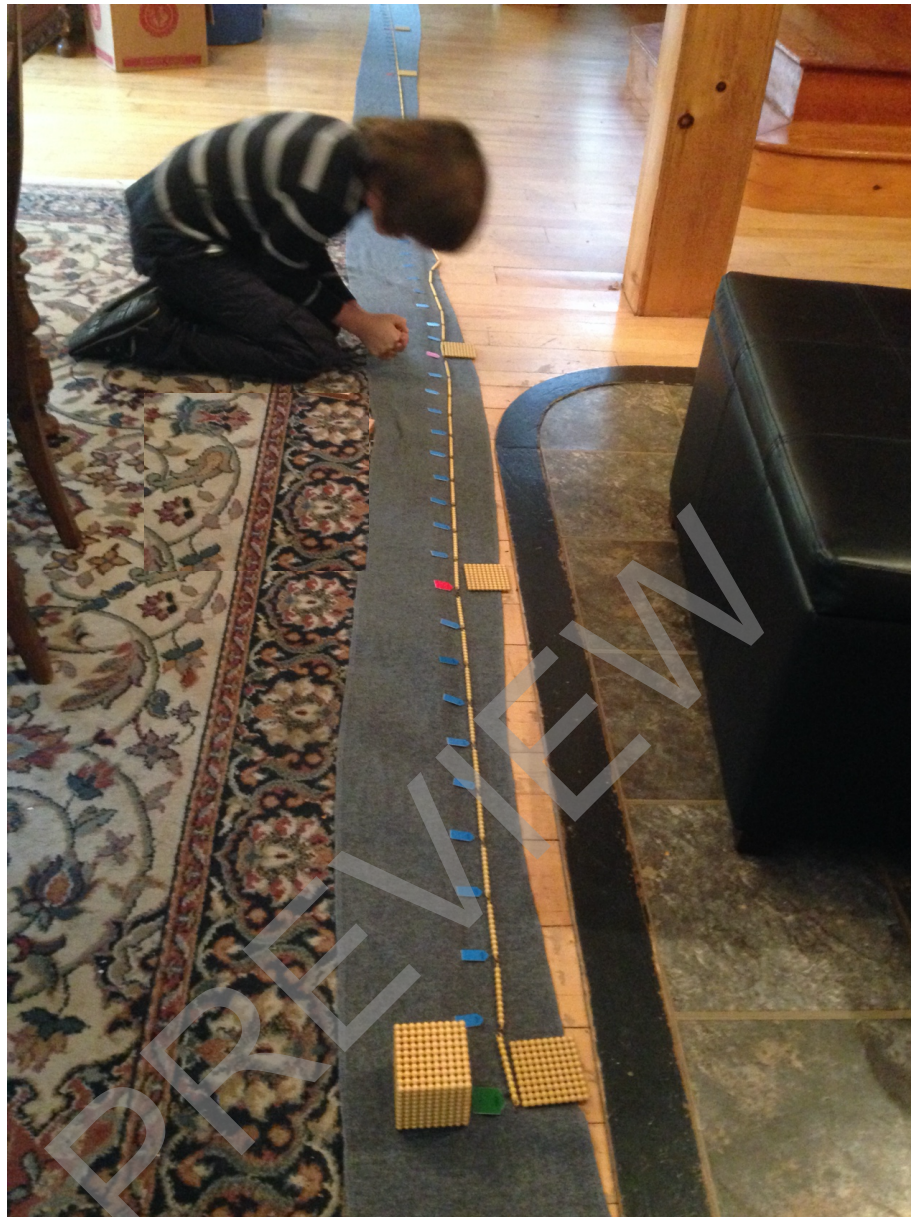*Photo by Veritas Montessori Training Residency Residents (2018)*

Comparing the squares to the cube
Photo by Veritas Montessori Training Residency Residents (2018)

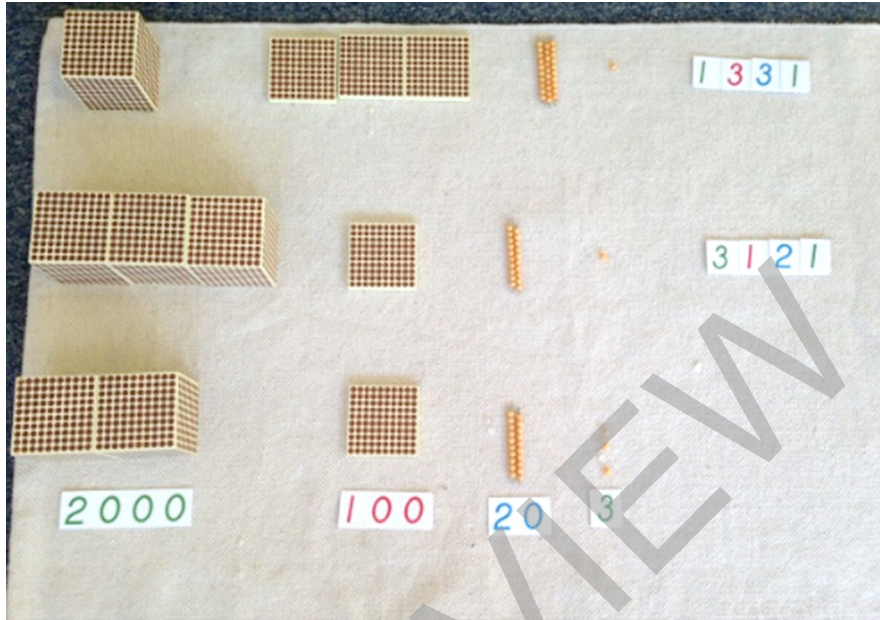***Layout for the first 100***
Photo by Veritas Montessori Training Residency Residents (2018)

Counting the 1,000 Chain

Bank Game: Addition

Collective Exercises

Static addition card & bead layout: *Addend plus addend plus addend*
Photo by Veritas Montessori Training Residency Residents (2018)

Bank Game: Addition
Collective Exercises

Age	*Prerequisite Lessons*
4.5 and up	Formation of complex numbers with beads and cards

Purpose:

- To give the children an impression of the nature of addition
- To show how numbers can carry from one hierarchy to the next

Materials:

- Golden beads (50 units, 50 10 bars, 50 100 squares, 12 1,000 cubes)
- One set of large number cards (1 to 9,000)
- 3 sets of small number cards (1 to 3,000)
- 3 small trays with unit bowls (felt lined is preferred)
- A sum tray (lined with a felt mat) including a unit dish and a large, sturdy red scarf

Sample equations:

    <u>Static</u>   <u>Dynamic</u>

- 3123 + 2213 + 1331   •   3565 + 2678 + 1899
- 4314 + 3473 + 1212   •   2676 + 2787 + 2878
- 1255 + 2511 + 3121   •   1987 + 1896 + 1789

Presentation: Static

1. Begin the lesson and invite two or three children to do the work with you. Say, "Let's play the Bank Game Addition. First, you each need to layout a set of small number cards." Each card set needs its own table or rug. When one child finishes, have them help you lay out the large number cards and another child can layout the work rug and bring three trays (each with a unit cup). You will also need the sum tray and red scarf.
2. When all cards are lain out, gather together at your work rug. Say, "For addition, you each get to be an addend. Jenay, use your tray to get the small cards and beads for 3123. Dawn, use your tray to get the small cards and beads for 2213. Christina, use your tray to get the small cards and beads for 1331."
3. Wait for all children to return to the mat with their trays and then ask each one in turn, "What did you bring? Let's count it out together." Always name the hierarchy as you count

them out "one unit, two units, three units," etc. Be sure the child picks up the beads and physically counts them out onto the work rug. Start with the first child's units (place on the top right of the rug but leave room for the number cards to the right of the beads), then tens (place just to the left of the units), then hundreds (place just to the left of the tens), and then thousands (place just to the left of the hundreds).

4. Place the first child's number cards directly to the right of their beads. As you do, ask them, "How many beads did our first addend bring?" Continue to set up the beads and number cards in this manner so that the numbers make a visual equation (with the addends lining up in rows).

5. Say, "Today we're playing addition. Addition means put it all together. What does it mean? Let's put all these beads together on the sum tray. For addition, we always start with the smallest thing."

6. Place the sum tray just below the bottom addend. Place the red scarf beneath the tray (in position so it can lift the tray when it is full).

7. Starting with the units, guide the children to transfer all beads onto the sum tray. When all beads are on, say, "Wow, that is so much more than we started with! I bet its heavy now." Stand up and lift the four corners of the scarf so that the scarf gently raises the tray holding all the beads off of the floor. Let each child have a turn lifting the scarf so they can experience its size via the baric sense.

8. Say, "Let's count and see how many we have. Who will count the units?" When the units are counted, ask the child who counted them to retrieve the large number card for that quantity. Say, "We have 6 units. Jenay, go and get us the big number card from the rug that says 6 units." Then, place the card to the right of the sum tray (lined up beneath the addend cards).

9. When all categories have cards, review the total and then slide the cards together.

10. Say, "Let's tell the story of our addition work. Jenay brought 3 thousand, 1 hundred, 2 tens, and 3 units. Dawn brought 2 thousands, 2 hundreds, 1 ten, and 3 units. Christina brought 1 thousand, 3 hundreds, 3 tens, and 1 unit. We put them all together to make our sum of 6 thousands, 6 hundreds, 6 tens, and 7 units."

11. Say, "That was fun! Do you want to do another one?"

12. If they want to or if they are done, you must first reorganize the number cards and put all beads back in the bank. Say, "Let's put the number cards away first. Then, come back to this rug to put the beads away."

13. Guide them as necessary and stay nearby to support completion. Fade and observe.

14. Complete the lesson.

Exercise #1: Dynamic

1. Begin as for static addition, assigning each child to be one of the two or three addends. When all beads and cards are laid out, review the rules for addition. "Today we are doing addition which means…what do we do with the beads?" Guide the children to say that you'll put them all together. "And which beads do we count first?" Guide them to say the smallest ones.

2. Say, "Today we have a new rule to remember, when we count the beads, stop when you get to 10. What do you do when you get to 10? Stop!"

3. Guide the children to put all beads together and move down to the bottom of the mat in their hierarchical columns. Guide one child to count the units. When they get to 10, make sure they stop!
4. Say, "How many units do you have there? 10? What can we do with 10 units…" Guide them to discover that they already know they can make an equal exchange.
5. Go with them to make the exchange and bring back one 10 bar. Place the 10 bar just above and to the left of the ten's column of beads (to show the carry in a visual equation)
6. Continue for the other quantities and equal exchanges. Fade and observe.
7. Guide the children to get their sum number cards and encourage them to tell the story of their work (see static addition).
8. Encourage repetition. Complete the lesson.

Exercise #2: Independent work

1. Each time you play the bank game with the children, fade back more and more as their capacity grows. Ultimately, you want to be needed only to assign each child (the addends) their numbers (so as to keep the sum below 10,000).

Control of error:

- The intention is to give children an impression of the nature of addition, NOT to extract exact results. However, once they are experienced with bank game addition and bank game subtraction, show them how to subtract the addends from the sum in order to check their own work.

Accommodations

- If a child has working memory difficulties, you can place the cards on their tray rather than asking them to memorize their number.
- Partner a struggling older child with a friend who is capable. Have the struggling child help in any way they can (e.g., by gathering one category of bead, by lifting the sum tray)

Notes:

The scarf is only needed for the first presentation of static addition to give the children the concrete baric experience of a large sum. However, they may want to use it every time!

Give the children many opportunities to repeat static equations before moving to dynamic ones.

The focus is on the concept/experience/procedures of addition, not exact results. You can snitch an extra bead when they are not looking if necessary to maintain precision but don't worry about correcting them. The materials will self-correct as the children gain experience.

When telling the story of the work, after the first few times, you can add some fun by saying, "Let's pretend that these aren't beads but that they are chocolate chips!" Choose whatever

substance the children are most interested in and have fun. Encourage them to choose the objects in the future.
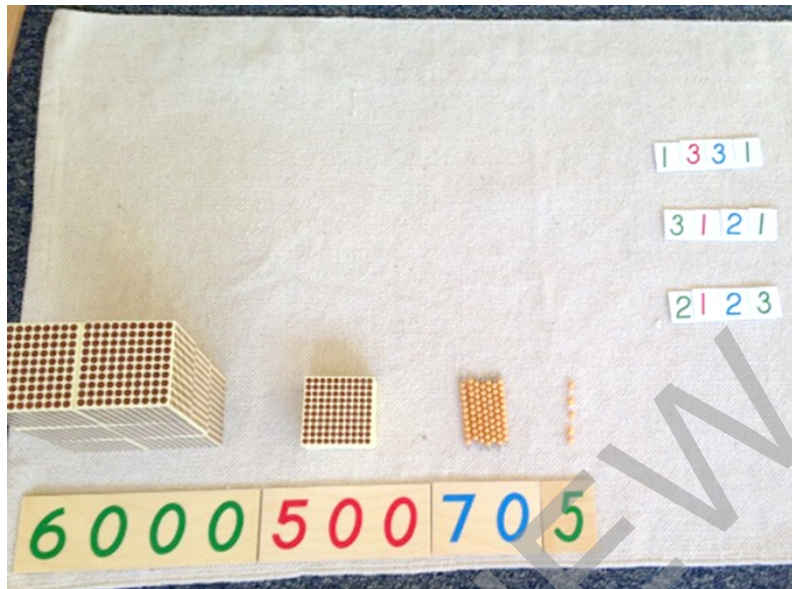
Static addition solution: *Addend plus addend plus addend equals sum*
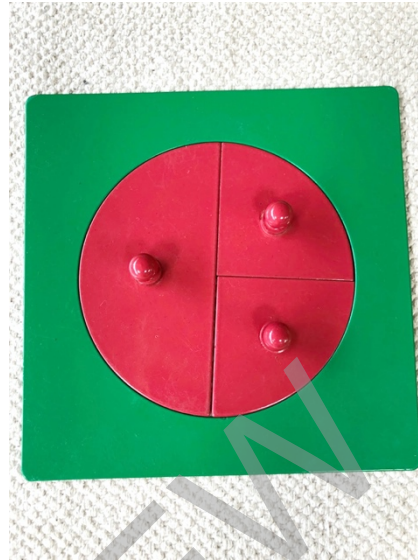*Photo by Veritas Montessori Training Residency Residents (2018)*
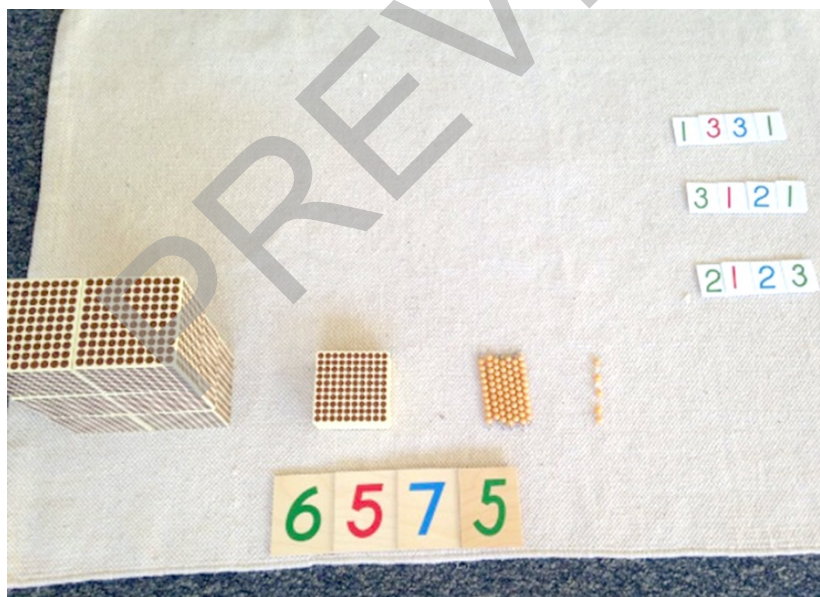
Sliding the sum cards together
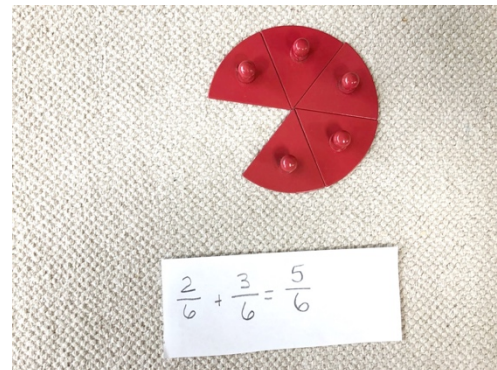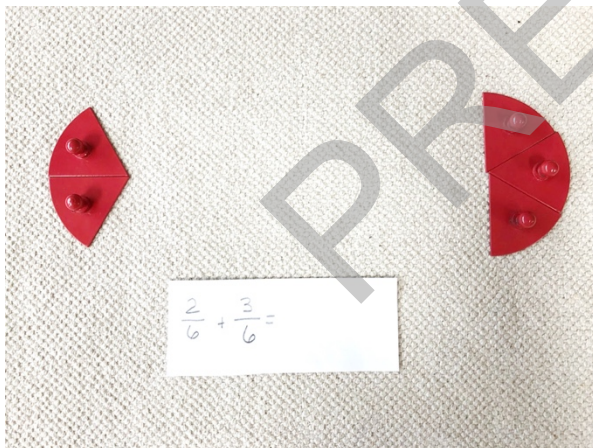*Photo by Veritas Montessori Training Residency Residents (2018)*

Weighing the sum on the sum tray
Photo by Veritas Montessori Training Residency Residents (2018)

Operations with Fractions

Equivalence: *Two-thirds (left) doesn't equal one-half but one-half and two-fourths are equivalent*
Photo by Veritas Montessori Training Residency Residents (2018)

Addition (same denominator)
Photo by Veritas Montessori Training Residency Residents (2018)

Operations with Fractions

	Prerequisite Lessons
Age 5.5 and up	Equivalence: Introduction to Fractions (Prepared slips) Addition: Addition strip board Subtraction: Subtraction strip board Multiplication: Multiplication bead board Division: Unit division board

Purpose:

- To allow the child to discover fraction equivalence and the possibility of substitution (e.g., 2/4 for 1/2)
- To introduce the child to conducting operations with fractions

Materials:

- Fraction insets
- Writing tray with pencil and slips of paper
- Prepared equation cards (color-coded by operation) for:
    - Adding fractions with like denominators (e.g., 3/8 + 2/8 =)
    - Subtracting fractions with like denominators (e.g., 7/9 − 4/9 =)
    - Multiplying by a whole number (e.g., 1/3 x 2 = , 2/7 x 3 =)
    - Dividing by a whole number (e.g., 3/3 ÷ 3 = , 6/8 ÷ 2 =)
- Skittles (for division)

Presentation: Equivalence

1. Begin the lesson and invite one or two children to work with you. Bring both fraction trays to your rug.
2. Say, "I want to see if we can substitute some fractions for others."
3. Take out the frame and insets for one-half. Remove one of the 1/2 insets and set it aside.
4. Pick up a 1/3 inset. Place it in the empty half. Take a second 1/3 inset and discover that it does not fit in the remaining space in the half. Say, "These don't work. I'll try the next one." Put both 1/3 insets back.
5. Pick up a 1/4 inset and place it in the empty half. Take a second 1/4 inset and discover that it fits perfectly! Say, "Look, 1/2 is the same as 2/4s! They are equivalent. I'm going to write that down." Write it down on a slip of paper 1/2 = 2/4.
6. Replace the 1/4 insets and guide the child to step in and take over, writing it down anytime it works.
7. Fade and observe. Encourage repetition. Complete the lesson.

Exercise #1: Addition (same denominator)

1. Begin the lesson and invite one child to work with you. Go together to bring the materials to your work area.
2. Say, "Today I'll show you how to add fractions."
3. Write down 2/6 + 3/6 = on a piece of paper. Place it in front of the child.
4. Say, "First, let's take out our addends." Work with the child to place two sixth insets on the left and three sixth insets on the right.
5. Ask, "Tell me what addition means…put it all together, that's right. Go ahead and count how many sixths we have when we put them all together.
6. Write down your sum of 5/6 to complete your equation. Read the whole equation aloud.
7. Say, "Let's do another one." Create another problem and this time let the child do the work. Fade and observe.
8. After a few examples, look over your equations and notice that the fractions always have the same denominator. Say, "Look at that, all of these always have the same denominator. That's a rule when you add fractions. They have to have the same denominator or it won't work."
9. Encourage repetition by introducing the prepared equation cards. Complete the lesson.

Exercise #2: Subtraction (same denominator)

1. Begin as in Exercise #1 but say, "Today I'll show you how to do subtraction with fractions."
2. Write down a subtraction equation with like denominators (e.g., 4/6 − 1/6 =).
3. Say, "First let's take out the minuend, 4/6s." Guide the child to take out four sixths and place them on the rug.
4. Say, "Now, what do we do when we subtract…that's right, we take something away. Let's take 1/6 away." Take the one sixth inset and put it behind the inset tray.
5. Ask, "How many do we have left? That's right, 3/6. Let's write it down."
6. Guide the child to write down the difference and then read the whole equation aloud.
7. Say, "That's great. Let's do some more."
8. Guide the child to repeat with more equations. Fade and observe.
9. After a few examples, look over your equations and notice that the fractions always have the same denominator. Say, "Look at that, all of these always have the same denominator. That's a rule when you subtract or add fractions. They have to have the same denominator or it won't work."
10. Encourage repetition by introducing the prepared equation cards. Complete the lesson.

Exercise #3: Multiplication by a whole number

1. Begin as in Exercise #1 but say, "Today I'll show you how to do multiplication with fractions."
2. Write down a multiplication equation such as 2/7 x 3 = .
3. Say, "First let's take out the multiplicand, 2/7. What does multiplication mean…that's right, add the same thing together so many times. How many times do we need to take 2/7? That's right, three times."
4. Place the two sevenths insets in groups in rows on the rug (i.e., two sevenths in a top row, two sevenths below that and two sevenths below that).
5. Say, "Now let's put it together and see what we have." Count up the result and guide the child to write down the answer, 6/7 and read the entire equation aloud.
6. Say, "That's great. Let's do another one."

7. Guide the child through a few more until you're sure they have the process.
8. Encourage repetition by introducing the prepared equation cards. Complete the lesson.

Exercise #4: Division by a whole number

1. Begin as in Exercise #1 but say, "Today I'll show you how to do division with fractions."
2. Write down a division equation such as $4/9 \div 2 =$.
3. Say, "First let's take out the dividend." Place four ninths on the rug.
4. Say, "How many times do we share it out? Two times…we need two skittles. Do you remember where they are?"
5. Lay two skittles out on the bottom of the rug, beneath the dividend.
6. Say, "Go ahead and start sharing, make sure everyone gets the same amount."
7. When finished, say, "What's the answer? 2/9. Write that down.
8. Say, "Let's do another one."
9. Guide the child through a few more until you're sure they have the process.
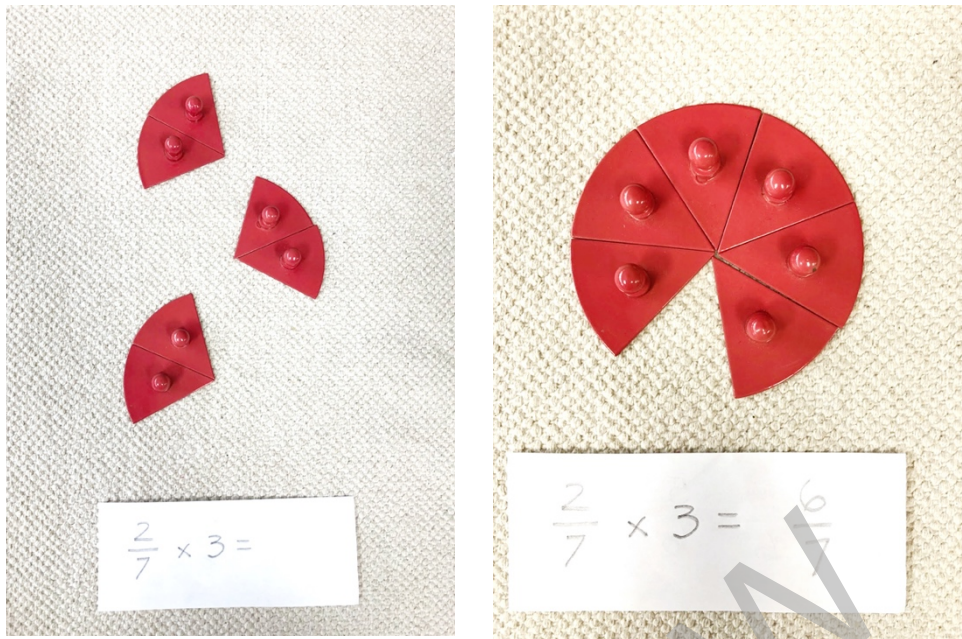10. Encourage repetition by introducing the prepared equation cards. Complete the lesson.

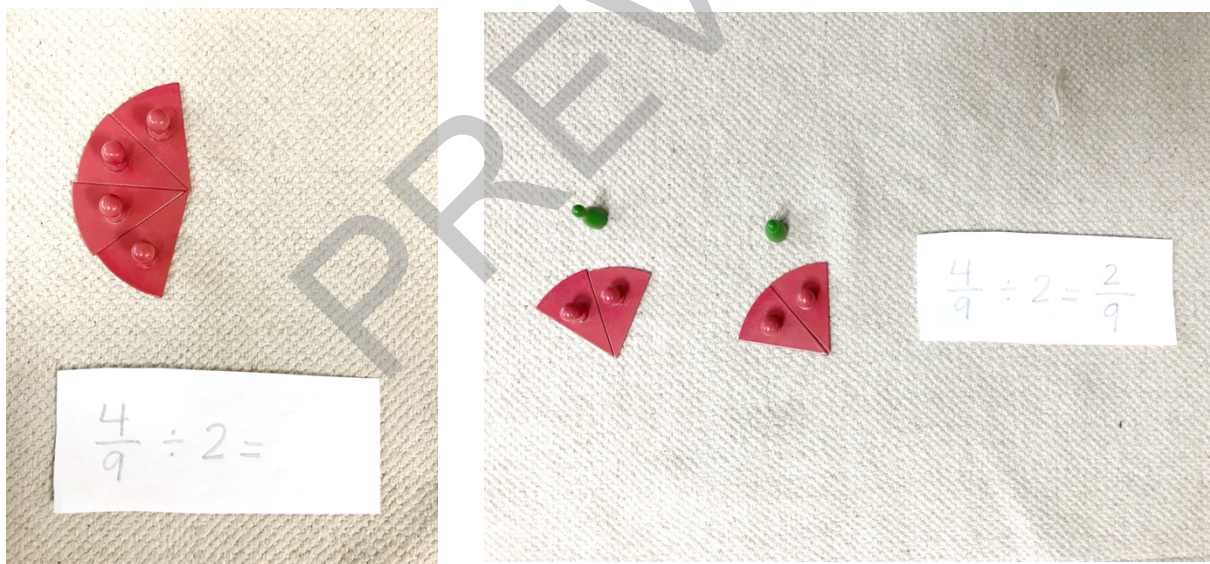### *Control of error:*

- The child's growing knowledge

Notes:
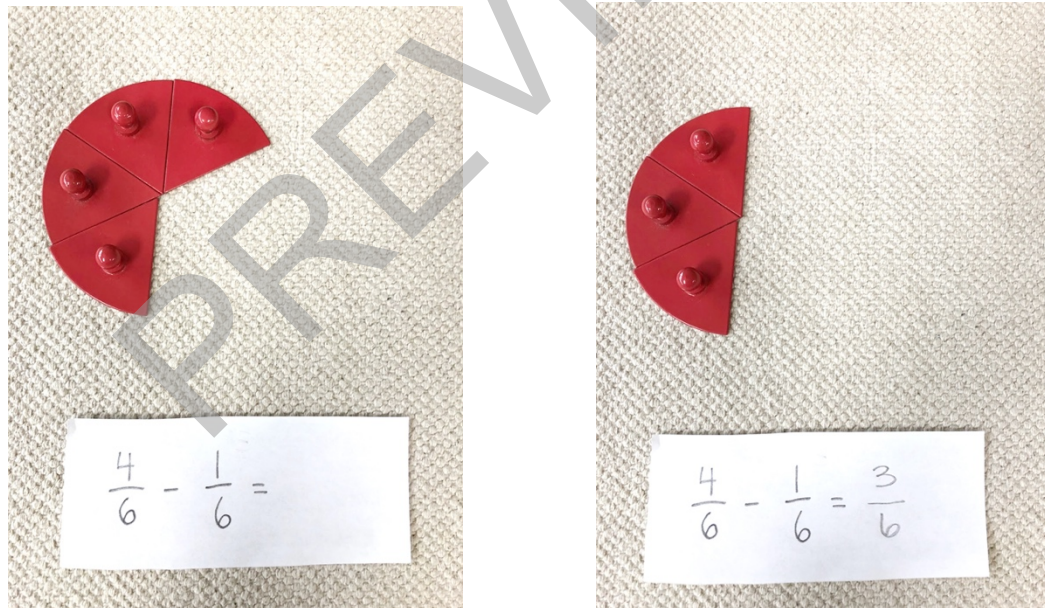
Subtraction (same denominator)
Photo by Veritas Montessori Training Residency Residents (2018)

Multiplication by a whole number
Photo by Veritas Montessori Training Residency Residents (2018)

Division by a whole number
Photo by Veritas Montessori Training Residency Residents (2018)