GRADE 12

Mathematics

ADVANCED FUNCTIONS 12

ON EDITION

COMPLETE GRADE 12 MATH CURRICULUM

- questions ranging from easy to advanced
- step by step guided examples
- chapter tests included

DYNAMICMATH

more than just a book (see inside)

DYNAMIC CLASSROOM

math-help.ca

GRADE 12 MATHEMATICS (ON EDITION) ISBN: 978-1-988243-81-8

WHY DYNAMIC MATH?

Dynamic Math workbooks are written by teachers directly for each province. This ensures that you are getting the exact same material that is being taught in the classroom. Our teachers also produce engaging online content to further support and enhance learning.

DYNAMIC CLASSROOM

Suite 207 8501 162nd Street
Surrey, BC V4N 1B2

604.592.9309

sales-inquiries@dynamic-classroom.ca

www.dynamic-classroom.ca

Contributing Authors: Alan R. Taylor, Ed.D. & Bill Kokoskin, M.A.

Dear Parents,

Helping kids understand and apply mathematics knowledge and skills is a collective responsibility of parents, teachers, and principals.

Students need to learn mathematics in a way that will serve them throughout their lives. Understanding mathematics can provide our students with many job and career opportunities.

This is why students need to know why mathematics works the way it does, how to use it with confidence and competence when solving problems.

Understanding mathematics enables us to:

- Solve problems, make sound decisions and perform calculations with ease
- Explain how we solved a problem and why we made a particular decision
- Understand patterns and trends so that we can make predictions
- Understand Financial Literacy to manage time and money
- Handle everyday situations that involve numbers and feel confident

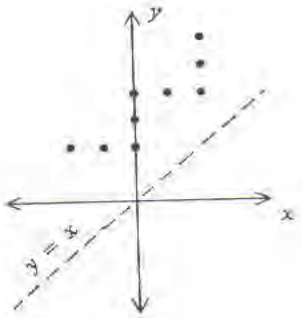
Before your child can learn mathematics, he or she needs to believe in his or her ability to do so. That's where you come in!

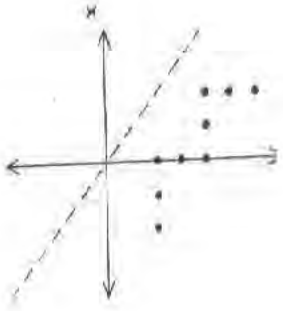
Parents, you are your child's first role model for learning. When you engage with your child in a supportive, relaxed atmosphere, your child will enjoy exploring the world of mathematics.

Dynamic Math is committed to helping parents and students. We understand that not everyone learns the same way, and not everyone feels the same about math. This is why we are continually working to create math resources that help students of all abilities, while supporting the many learning styles and varying levels of enthusiasm towards math.

From our clear concise instructions and straightforward guided examples to our additional practice material and tests, there's something to suit everyone. Combined with our video tutorials, students will be able to get a tutor-like experience from anywhere and at a fraction of the cost of standard tutoring or after-school help programs.

Table of Contents
Advanced Functions 12





CHAPTER 1

TRANSFORMATIONS

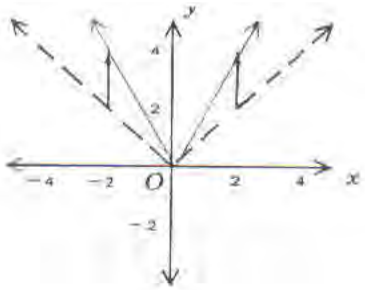
1.1 Relations and Functions (Review)

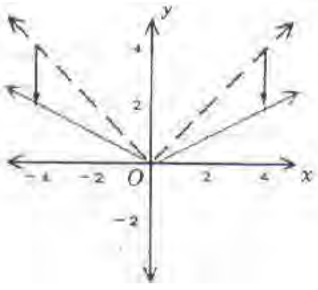
1.2 Even and Odd Functions

1.3 Vertical and Horizontal Translations

1.4 Compressions and Expansions

1.5 Reflections and Inverses





1.1 Relations and Functions (Review)

Before we work with transformations of functions and their related equations, it is important to review what is meant by a **relation** and a **function**. In this section, we will look at relations and functions, in addition to their corresponding equations and graphs.

Relation

A relation is a set of ordered pairs. As a result, every graph is a relation. A relation can be thought of as a set of ordered pairs. All first elements in the set of ordered pairs are called the domain while all second elements determine the range.

Relations can be shown in several ways, among them the following: as a word description, a set of ordered pairs, an equation or inequality, or as a graph. An example of a relation shown in each of these ways is next.

Word description

An output number is determined by multiplying an input number by 2 and then adding 1.

Equation

$y = 2x + 1$

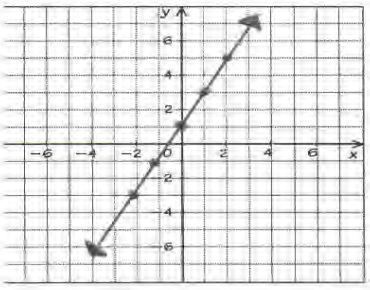
Set of ordered pairs

$(0, 1), (-\frac{1}{2}, 0), (1, 3), (2, 5), (-1, -1), (-2, -3), \ldots$

Table of values

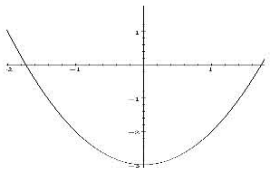
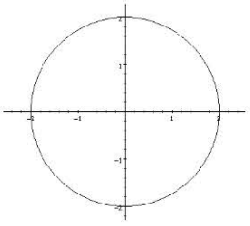
x	0	$-\frac{1}{2}$	1	2	-1	-2	-	-
y	1	0	3	5	-1	-3	-	-

Graph

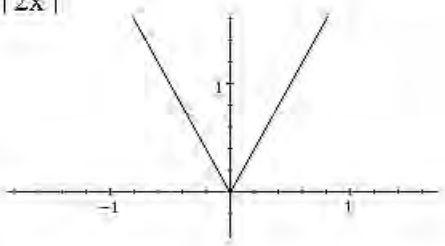


In the preceding graph of $\mathbf{y} = 2\mathbf{x} + 1$
↑ ↑
Output Input

x is the independent variable.
y is the dependent variable.
The domain is all real numbers (all possible values of x).
The range is all real numbers (all possible values of y).

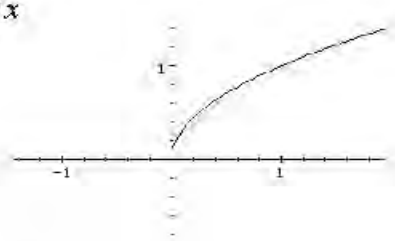
Examples of Relations	**Domain**	**Range**
Ordered Pairs		
(2, 3), (3, 5), (4, 6), (5, 8)	2, 3, 4, 5	3, 5, 6, 8
$(-2, 3), (-\frac{1}{2}, 4), (0, \frac{9}{2}), (2, 6)$	$-2, -\frac{1}{2}, 0, 2$	$3, 4, \frac{9}{2}, 6$
Equations		
$y = 3$	all real numbers	3
$x = y$	all real numbers	all real numbers
$y = \lvert x \rvert$	all real numbers	all reals ≥ 0
$x^2 + y^2 = 1$	$-1 \leq x \leq 1$	$-1 \leq y \leq 1$
Graphs		
$y = x^2 - 3$ 	all real numbers	all real numbers ≥ -3
$x^2 + y^2 = 4$ 	$-2 \leq x \leq 2$	$-2 \leq y \leq 2$

$y = |2x|$



All real numbers $y \geq 0$

$y = \sqrt{x}$



$x \geq 0$ $y \geq 0$

Function

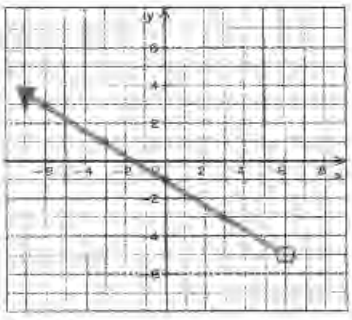
A **function** is a special kind of relation that exists between numbers or variables with the following rule: for every valid input number, there is a single output number. In other words, for every x, there is only one y.

Example: $y = x^2$ is a function since only 1 value of y results from every value of x.
$x = y^2$ is <u>not</u> a function, for example if $x = 4$, y could equal either 2 or -2.

The **domain** of a function is the set of all possible input numbers called the independent variable (all values of x that work).

The **range** is the set of all possible output numbers, called the dependent variable (all values of y that work).

Example: What is the domain and range of the following function?

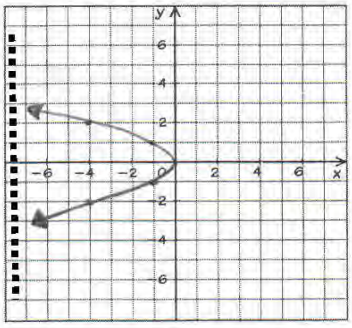


Domain, set of "x's"
∴ all real numbers < 6

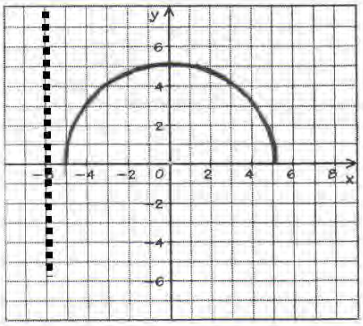
Range, set of "y's"
∴ all real numbers > -5

A quick way to determine whether a relation is a function or not is to do a vertical line test on the graph of the relation. If only one value of y can be found for every value of x, then we know that the relation is function.

Example: Which of the following are graphs of functions?



The above graph is **not** a function since the vertical dotted line intersected it in more than one location.



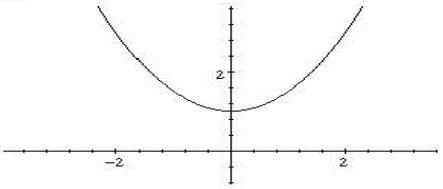
The above graph is a function since any vertical line will intersect it in only one location.

Several examples of relations are shown next, some are functions and others are not.

Examples of Relations	Function?	Reason
Ordered Pairs		
(2, 3), (3, 5), (4, 6), (5, 8)	Yes	Each value in the domain has a unique value in the range.
(1, 3), (2, 4), (3, 5), (1, 6)	No	When x = 1 there is more than one value of y (e.g. 3 and 6).
Equations		
$y = 2x + 1$	Yes	Each value in the domain has exactly one value in the range.
$x^2 + y^2 = 1$	No	There is more than one value of y for all but two values of x (when x = 0, $y = \pm 1$).

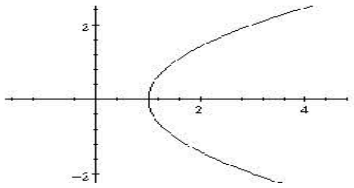
Graphs

$y = \frac{1}{2}x^2 + 1$



Yes — There is exactly one value of y for each value of x.

$x = \frac{1}{2}y^2 + 1$



No — There are two values of y for each value of $x > 1$.

In our review of functions and relations, we stated that a **function** is a special kind of relation that exists between numbers or variables with the following rule: for every valid input number, there is a single output number. In other words, for every x, there is only one y.

To quickly distinguish between a function and a relation when using an equation we use a special notation for a function. It is called **f of x notation,** as described next.

Using f(x) Notation

Functions are usually denoted using "f of x" notation (f(x)), which is another name for the dependent variable, y.

Examples:

f (2) = 3	means that function f maps 2 to 3
f(-1) = 2	means that function f maps -1 to 2
f(x) = y	means that function f maps x to y

Examples with Solutions

f(x) = 2x – 7	f (3)	2(3) – 7 = -1
	f(-5)	2(-5) – 7 = -17
	f (x+1)	2(x + 1) – 7 = 2x – 5

$f(x) = x^2 + x - 1$	$f(-2)$	$(-2)^2 + (-2) - 1 = 4 - 2 - 1 = 1$
	$f(x + 1)$	$(x + 1)^2 + (x + 1) - 1 = x^2 + 3x + 1$
	$f(-x)$	$(-x)^2 + (-x) - 1 = x^2 - x - 1$
	$-f(x)$	$-(x^2 + x - 1) = -x^2 - x + 1$
$g(x) = \sqrt{x+2} - 1$	$g(7)$	$\sqrt{7+2} - 1 = \sqrt{9} - 1 = 3 - 1 = 2$
	$g(-1)$	$\sqrt{-1+2} - 1 = \sqrt{1} - 1 = 1 - 1 = 0$
$h(x) = \dfrac{x^2 + x}{x - 1}$	$h(2)$	$\dfrac{2^2 + 2}{2 - 1} = \dfrac{6}{1} = 6$
	$h(-1)$	$\dfrac{(-1)^2 + (-1)}{(-1) - 1} = \dfrac{1 - 1}{-2} = \dfrac{0}{-2} = 0$
	$h(x - 1)$	$\dfrac{(x-1)^2 + (x-1)}{(x-1) - 1} = \dfrac{x^2 - 2x + 1 + x - 1}{x - 2} = \dfrac{x^2 - x}{x - 2}$

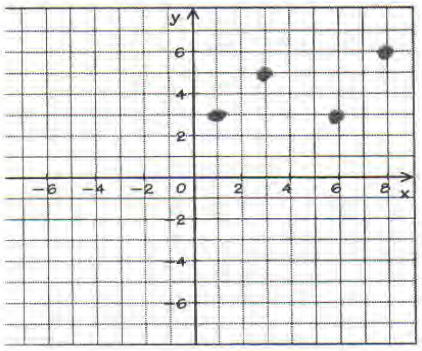
Exercises 1.1

1. Find the domain and range of each function.

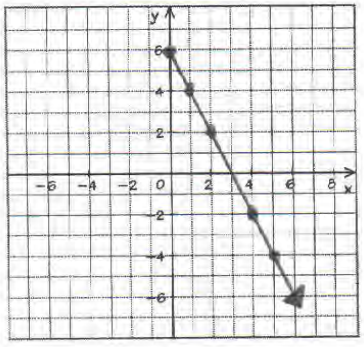
a. (1, 5), (2, 7), (3, 7), (5, 8)

b. (-2, 4), (0, 3), (0.5, 4), (1.5, 4)

c.



d.



e. $y = x^2$

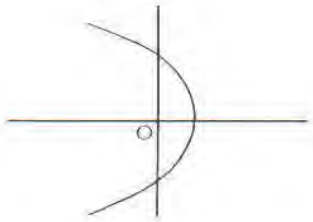
f. $y = |x| + 1$

2. Which of the following relations are functions?

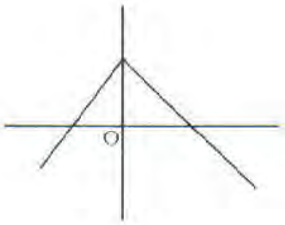
a. (1, -1), (1, 2), (7, 7), (8, 8)

b. (5, 3), (6, 3), (7, 7)

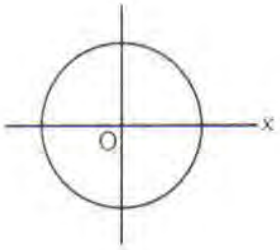
c.



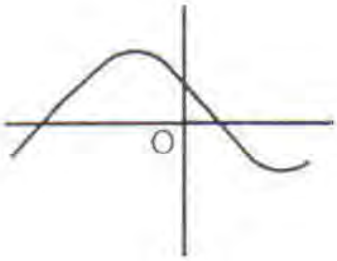
d.



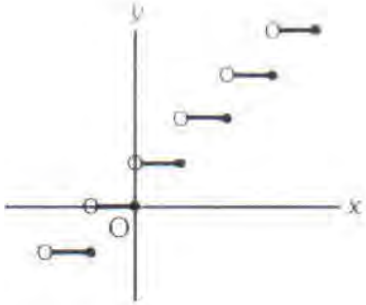
e.



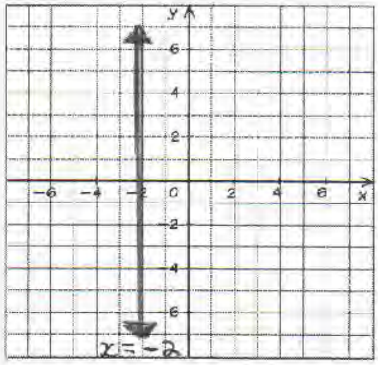
f.



g.



h.



3. If $f(x) = (x + 5)^2$, find

a. the domain

b. the range

c. f(2)

d. f(-2)

e. f(2x)

f. f(x + 1)

4. If $f(x) = -2(7 - x^2)$ find

a. f(-2)

b. f(0)

c. $f(\sqrt{2})$

d. f(x + 1)

5. If $f(x) = 2x^2 - \frac{1}{2}$, find

a. the domain

b. the range

c. f(-2)

d. $f(\frac{1}{2})$

ANSWERS TO EXERCISES AND CHAPTER TESTS

CHAPTER 1

Exercises 1.1 (page 7)

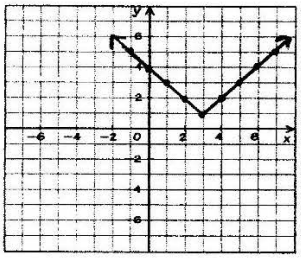
1. a) Domain: 1, 2, 3, 5; Range: 5, 7, 8
b) Domain: -2, 0, 0.5, 1, 5; Range: 3, 4
c) Domain: 1, 3, 6, 8; Range: 3, 5, 6
d) Domain: $x \geq 0$; Range: $y \leq 6$
e) Domain: all reals; Range: $y \geq 0$
f) Domain: all reals; Range: $y \geq 1$
2. a) No **b)** Yes **c)** No **d)** Yes **e)** No **f)** Yes
g) Yes **h)** No **3. a)** Domain: all reals
b) Range: $y \geq 0$ **c)** 49 **d)** 9 **e)** $4x^2 + 20x + 25$
f) $x^2 + 12x + 36$ **4. a)** -6 **b)** -14 **c)** -10
d) $2x^2 + 4x - 12$ **5. a)** Domain: all reals
b) Range: $y \geq -\frac{1}{2}$ **c)** 7.5 **d)** 0

Exercises 1.2 (page 13)

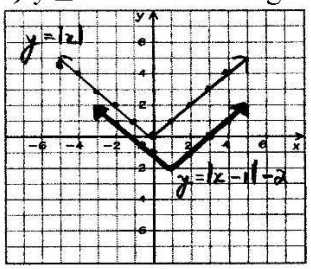
1. a) odd **b)** even **c)** neither **d)** even
e) neither **2. a)** neither **b)** even **c)** neither

Exercises 1.3 (page 16)

1. $y = f(x - 4)$ **2.** $y = |x - 1| - 1$
3.

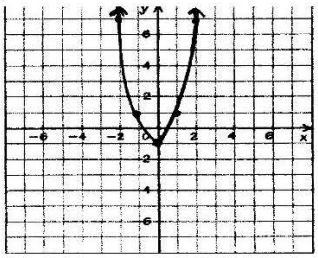


4. 5 right and 9 down **5.** 1 down
6. a) (-3, 7) **b)** (0, -2) **c)** (-4, 1) **d)** (-5, 1)
7. $y = |x - 3|$ **8. a)** 3 units left, 3 units down
b) $y \geq -2$ **9.** 1 unit right and 2 units down



Exercises 1.4 (page 22)

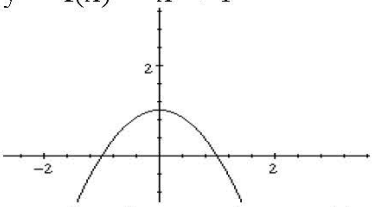
1. a) Replace x with $\frac{1}{3}x$. **b)** Replace x with $\frac{3}{7}x$.
2. a) Replace x with 4x. **b)** Replace x with $\frac{5}{2}x$.
3. a) horizontal expansion by a factor of 2
b) horizontal compression by a factor of $\frac{1}{3}$
4. $y = 9x^2 + 4$
5.



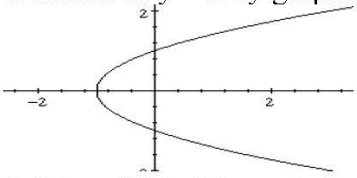
6. a) vertical expansion by a factor of 3
b) vertical compression by a factor of $\frac{1}{4}$
c) horizontal compression by a factor of $\frac{1}{4}$
d) horizontal expansion by a factor of 3
e) vertical expansion by a factor of 3 and a horizontal compression by a factor of $\frac{1}{2}$
f) vertical compression by a factor of $\frac{1}{2}$ and a horizontal expansion by a factor of 3
7. a) $y = 2|x|$ or $y = |2x|$ **b)** $y = \frac{1}{2}x^2$
c) $y = 2(x + 2)$ **d)** $y = 0.5x^3$ **e)** $y = 2(\frac{4}{x^2+1})$
8. a) 0, -1, 2 **b)** 0, -2, 4

Exercises 1.5 (page 30)

1. $y = -(2x^2 + 3x - 5) = -2x^2 - 3x + 5$
(change to the graph of $y = -f(x)$)
2. $y = 5(-x)^2 + 3(-x) - 2 = 5x^2 - 3x - 2$
(change to the graph of $y = f(-x)$)
3. $x = 4y^2 + 3y - 1$ (change to $x = f(y)$)
4. reflect in x-axis by graphing
$y = -f(x) = -x^2 + 1$



5. reflect in $y = x$ by graphing $x = y^2 - 1$



6. It is reflected through the y-axis.
7. It is reflected through $y = x$.

Dynamic Math Videos

The Dynamic Math Program supports the teacher to confidently teach their students with:

- a curriculum aligned **Math book**
- **online teacher resources** such as worksheets and extra tests with answer keys
- a **digital flipbook**
- **a library of videos** to help the student in their math journey.

Our program is designed to have the videos support our Math books which are clear, concise and 100% aligned to the provincial curriculum.

Many schools purchase class sets of our Math books and licenses to provide access to the videos. The videos are designed to help the student get 'unstuck' when they are doing their homework. Each video matches the related section in the Math book so that it is easy for the student to find.

Through the licensing process, access to all available grades is offered to each subscriber, allowing students to go back to previous years to review and enhance their understanding or work ahead for advanced learning.

The video program is not designed for each student to watch all available videos for their grade level. Some students may only need to watch some videos throughout the year as they already have a strong grasp of the math concepts being taught in the classroom. Students can do this at home for homework and will not fall behind as they have some extra guidance.

This combination of book and video instruction improves learning outcomes and makes students more confident.

In summary, please consider the following:

Video Benefits for you and your students

- They will help support you in teaching Math to your students.
- The videos will help your students to reinforce what you taught them in the classroom.
- They will be more confident when doing their homework.
- They will equip parents to help the student at home.
- They will dramatically increase the success of your students to learn math.

At a Glance

- Videos available for Grades 4-10, 50-80 Videos per grade, 2 to 7 minutes in length
- The videos match the chapters and units in our Math books and help instruct students learn how to complete the questions in the books.
- Videos are 100% aligned to the provincial curriculum.

MORE THAN JUST A BOOK!

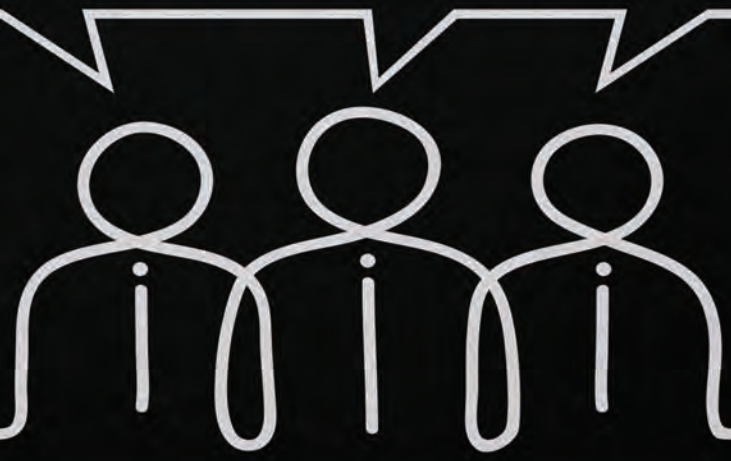
- ✓ Continuously expanding set of online resources
- ✓ Extra practice for struggling students
- ✓ Develop critical thinking skills
- ✓ Engage learning with fun "puzzlers" and games

www.math-help.ca

"My son would never have passed grade 10 without Dynamic Math. Thank you!

My daughter loves your math books because she can work through them on her own. We'll definitely be buying the next grade in the fall.

I was so happy to finally find a math workbook that was exactly what my son was doing in class."



DYNAMIC DC CLASSROOM

ON Grade 12 Mathematics

$34.95 CAD

ISBN 978-1-988243-81-8

9 781988 243818