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Introduction

Mathematical activity is a thought process in abstract relationships, growing out of human observation and experience in the physical world. It is a faculty which extracts patterns from specific problems and experiences and applies those patterns to the solution of other "not yet understood" problems. The joy and excitement of mathematics lie in the discovery that many apparently unrelated problems have basically the same mathematical structure. Mathematical analysis, then, is one of man's most powerful tools for understanding this unity.

A first step in mathematical analysis is to start with a simple idea that lends itself to concrete observation. Beyond this, however, man has devised mathematical symbols as more than a tool for recording his observations of the physical world. Through the preciseness and brevity of his symbolism, he can spotlight relationships so that they can be interpreted instantaneously. Word language, in contrast, appears slow and vague. Mathematical notation, although originating as a language for describing physical reality, takes on a life of its own and, through its clarity, suggests to the human mind new patterns in the physical world.

During the past century, revolutions in the sciences and in industry have created new uses for old ideas in mathematics, and have introduced new ideas and new topics in this expanding science. The new scope and new emphasis in mathematics naturally require changes in scope and emphasis in school mathematics, particularly in the elementary grades.

Schools, from the elementary grades through graduate levels; must provide mathematics programs that stress understanding of fundamental mathematical concepts, provide exposure to new concepts, and encourage creative thinking and creative use of ideas. Such programs should lead to the formulation of ideas and the discovery of mathematical relations and properties by the student himself.

Too often, we have tended to underestimate the ability of small children to discover, understand, and use basic mathematical concepts from the beginning of their school experience. This error in judgment has been built into present mathematics curricula for the elementary school. Emphasis has been placed almost solely on arithmetical computation skills, largely learned by rote, thus sacrificing the delightful experience of teacher and children thinking abstractly about the relationships that hold this arithmetic together.

When the school-age child first comes to the teacher, he already has a storehouse of mathematically interpretable life experiences. These experiences are much more complex and complete than traditional first-grade arithmetic programs lead us to expect. The first-grader does lack the conventional mathematical vocabulary and symbolism to communicate his insight verbally or in writing. He needs to be helped in school to acquire these tools. However, this must be done with full utilization of his continuing mathematical thinking and intuition, and not as a replacement for them.

Notations such as numerals and operational symbols come alive only through the active human mind. Yet, arithmetic is often taught as if these symbols were the only way in which certain relationships can be expressed. Our numerals and other notations are arbitrary choices of preceding generations in our culture to express man's thinking about numerical and non-numerical relationships among physical phenomena. Every generation of thinkers in a dynamic, forward moving culture must scrap, supplement, and revise this symbolism in order to express with more precision and power its new understanding of and mastery over universal relationships. Therefore, it is of utmost importance that even on the most elementary level of mathematics education, an atmosphere is fostered in which ideas may be explored and new symbols may be invented.

Teaching mathematics to young children cannot be merely a situation of verbal dialogue. It must be a doing, discovering, puzzling-out situation in which children are guided to pose questions, as well as to answer them in a variety of ways.

The Miquon Math Lab Materials

These primary-grade materials are the result of one elementary school's attempt to reshape its primary mathematics program. The materials grew out of daily teaching and were developed by the teacher and her assistant over a period of years. All of the materials have been tried out with six- and seven-year-olds and have been revised and edited with the help of the children. They were not developed to prove a point or to champion a preconceived method—except insofar *as we learn children's capabilities and interests directly from them.*

Many people helped to develop these materials. Robert Hightower, teacher at the Miquon School*, worked with the author over a period of four years and contributed greatly by his participation in the teaching and writing of the materials. He worked closely with many of the tryout schools and conducted numerous workshops for teachers and parents. Lee Evans and Clara Gitlin served as the author's assistants during various stages of the project. Peter Rasmussen assisted significantly in the writing of the manuscript during three summers while he was a graduate student in mathematics. Paul Moulton, mathematics teacher at the University of Chicago Laboratory School, contributed invaluable theoretical advice during the preparation of the final version of the materials. The Miquon teachers and staff, and many others, worked tirelessly to help make this work possible.

The author used freely what she could adapt from the experimental work of others, especially that of David Page of the University of Illinois Arithmetic Project, Max Beberman of the University of Illinois Committee on School Mathematics, Franz Hohn of the University of Illinois, William Hull of the Shady Hill School, Cambridge, Massachusetts, and Caleb Gattegno, mathematician and educator—to name only those with whom she has had prolonged professional contact.

Other ideas seeped in through extensive reading. All of these ideas went through the inevitable transformations and adaptations that a daily confrontation with eager, curious children requires of a teacher.

Lore Rasmussen
January 1964

Introduction to the second edition

Thirteen years have passed since the publication of the first edition of the Math Lab Materials. These years have produced many innovations in mathematics teaching using a variety of pedagogical approaches. The pendulum is now swinging back toward a greater concentration on mastery of basic skills.

I feel good after restudying and updating these materials in 1977, for their development in a classroom setting made for a common sense synthesis of both paint of view. The author's views about children and how they learn and the role of the teacher have not changed. Neither have her views regarding written work as being secondary to experiences. Appendix E of these Notes to Teachers has been expanded, and a selective list of ten paperback references has been added as Appendix F. Otherwise, except for minor corrections, they have been left unchanged.

Lore Rasmussen
July 1977

*The Miquon School, at Miquon, Pennsylvania, is an independent school for children ages 3 through 12.

Why these materials?

Some observations and insights

The teacher, whose mathematics materials you are beginning to share, considers her work with the children at the Miquon School one of her greatest learning experiences. What has produced this profound impression?

1. She has realized that children learn quickly and easily when there exist model-making experiences by means of which they can teach themselves. Gradually, she has learned to "talk less" so that the children may "learn more." (This has not been easy.)

2. She has observed over and over again that little children do not make senseless mistakes or random guesses if they are not pushed into making them. Hours spent analyzing children's errors revealed that the errors "made sense." Often they were answers to a different problem than the one posed, or were close approximations of what had been asked for. In an atmosphere where the learner may leave undone a task he cannot do at present, he does not resort to nonsense. (See Appendix C, page 35.)

3. She has become very cautious about typing a child as either a rapid learner or a slow learner. The records of her pupils reveal that their growth in problem-solving, their mastery of skills, and their attention span do not grow at easily observed, steady rates. Some children seem to grow at consistent rates. Others leap and stop and leap again, while still others move very slowly, only to suddenly overtake all others. (See Appendix D, page 37.)

4. She has noticed, of course, that most little children chatter and wiggle and move about while they work. When freedom of movement is accepted, then work periods may be extended far beyond what has been recommended for this age group. As long as the work is individualized and the "rate of expectation" is set by the children themselves, these long informal work sessions *do not* deteriorate into chaos.

Because of these observations and insights, the laboratory approach to teaching was adopted. These primary mathematics materials are an outgrowth of the mathematics laboratory environment at Miquon School.

The Miquon Mathematics Laboratory

One classroom at the Miquon School is reserved exclusively for teaching and learning mathematics. All children in the three primary grades come to the room regularly, although they do not receive *all* of their mathematics instruction in this room. The room is equipped with a great variety and quantity of *concrete materials,* such as Cuisenaire® rods, Stern blocks, Dienes multibase blocks, geometric blocks, blocks from the Soviet Union, and geoboards; *measuring devices,* including metersticks, yardsticks, Celsius and Fahrenheit thermometers, balances and scales; *number lines and number plane arrays; geometric and drafting tools;* a large variety of *graph paper; a library* of puzzles and mathematics books; and most important, a file of thousands of *problems* and hundreds of lab sheets all developed at the Miquon School over a period of years.

Children have free access to the materials in the room, and they use them as they feel a need to demonstrate to themselves "how something works" or to prove that their conjectures are correct. These materials are not conceived of as "crutches" to learning, but as the child's version of the laboratory behavior of the adult researcher who makes direct observations, sets up experiments, collects data, and generalizes from them—*before he is willing to accept ideas as facts*.

Classroom sessions vary greatly in the way they are conducted. Some classes are informal play sessions which are completely undirected—where the children explore the properties of materials, puzzle out for themselves a use for them, engage in complex experiments of their own, or sometimes do nothing of apparent mathematical value. By observing carefully the free activity of children in this situation, the teacher can collect ideas, topics, and problems that the children are interested in.

Other classes are highly organized question-and-answer sessions with the focus on the chalkboard or on other teacher-centered, teacher-dominated activity. Even in these situations, the children may also use models and other structured materials.

Most sessions start with a short teacher-pupil planning activity in which certain problems are posed. The children then work individually or in small groups for the rest of the period, checking their work by use of any materials or resources available in the room (which, of course, includes other children and the teacher). In such independent work periods, which take up the bulk of the time, the children are free to choose from a great variety of problems the ones they want to work on. (The *First-Grade Diary* gives many examples of these styles of teaching.)

A cumulative notebook is kept of every child's work. An analysis of the difficulties encountered and the comments made by the children as they are working supply the teacher with clues as to how to improve her questions, games, and lab sheets. Sheets are sometimes composed to meet the needs of a single child. Every sheet is designed to present the child with some new and difficult (for him!) problem. The teacher does not expect perfect responses on the lab sheets. New types of problems are presented without their having been previously explored. Children are not pressured to complete every problem; therefore, some children will skip any difficult material until they are interested enough to ask about it.

II

What are these materials?

Learning materials for children

The 649 individual *Children's Lab Sheets* included in the Mathematics Laboratory Materials were developed for six-, seven-, and eight-year-old children at the Miquon School. The sheets have been used by these children and revised (often as suggested by the children) as a result of their use. Many of the lab sheets were made up by the children themselves.

These children's materials are varied, not only as to content covered, but also as to visual appeal, length of task per page, and complexity. *In each section,* the sheets are numbered consecutively. For example, in Section C, Addition, the sheets are numbered C1 through C46, reflecting a steady increase in complexity. Most of the pages contain two lab sheets (one on the front, a second on the back), while others are printed on one side only.

The total collection of lab sheets can be used as a skeleton program for three years, taking care of the slow learner as well as the exceptionally gifted child. Teachers and children are encouraged to make their own supplements or to incorporate more traditional materials if they so desire.

The format of the lab sheets is intentionally informal and lacking in standardized size and style of lettering. The drawings are deliberately simple and child-like. This informality is an invitation to teachers and children to create additional materials for their collection, just as the children in the Miquon School contributed to this one.

On page 10 is a list of the topics covered, the section reference for each topic, and the number of lab sheets included in each topic. Following this is a chart showing how the lab sheets are grouped into six levels of difficulty. This multilevel arrangement is based on the experience of the author and the teachers who participated in the tryouts—though of necessity any such classification is arbitrary and does not hold true for every child.

Chart 1: Mathematical Topics in the Laboratory

Section	Topic	Number of Lab Sheets
A	Counting	24
B	Odd–Even	20
C	Addition	46
D	Subtraction	44
E	Addition and Subtraction	57
F	Multiplication	56
G	Addition, Subtraction, and Multiplication	20
H	Fractions	64
I	Addition, Subtraction, Multiplication, and Fractions	16
J	Division	32
K	Addition, Subtraction, Multiplication, Fractions, and Division	22
L	Equalities and Inequalities	24
M	Place Value	24
N	Number Lines and Functions	18
O	Factoring	12
P	Squaring	18
Q	Simultaneous Equations	8
R	Graphing Equations	30
S	Geometric Recognition	11
T	Length, Area, and Volume	28
U	Series and Progressions	12
V	Grid and Arrow Games	10
W	Mapping	12
X	Clock Arithmetic	18
Y	Sets	10
Z	Word Problems	13
26		649