Contents

A Letter From the Author

Dear Reader-Colleagues:

The Miquon Math Lab Materials were developed by the children of the Miquon School and myself during the early days of the post-Sputnik mathematics education reform. I was fortunate at the time to be teaching in a school atmosphere where creativity and inquiry were encouraged; where children and subject matter were genuinely respected.

At that time laboratory "hands-on" techniques for teaching mathematics were rare in America. Few good structural teaching aids existed. No one had heard of the British Primary School reforms.

Those of us who were dissatisfied with the prevailing mathematics teaching methods had to steer our own course and travel far to find allies. What we wanted to change, we had to invent and test. (We couldn't buy it.) What we wanted to incorporate of the traditional curriculum, we had to vitalize. For me, as I developed materials at the time, *the children were my teachers.* They let me know clearly when they were bored, confused or didn't understand and were frustrated. The curriculum that emerged was developed according to the signals they gave me.

I was also fortunate at the time to receive generous encouragement, knowledge and support from some leading mathematicians and mathematics educators who were becoming aware of the ease with which young children could grasp basic mathematical notions that had previously been presented only to older children. A new era of optimistic reform seemed to be opening up as sophisticated topics from college and high school curricula found their way into the elementary school.

Coached by the experts, I, too, tried many of these ideas. Some worked. Others confused and depressed me as well as the children. The "new ideas" to which the children responded positively became the collection of materials known as the Miquon Math Lab Materials. They also contained enough practice papers in basic skills to assure cautious parents that arithmetic computation was still held important. In this way my teaching and writing became a fabric jointly woven by children and parents on the one hand and inspiring professional mathematicians on the other.

Fifteen years have passed since then. Those years produced many diverse and sometimes pedagogically unsound innovations in mathematics teaching. The swing of the pendulum has now moved back to the new-old overconcern with "basic" skills. I feel good after restudying my materials in 1977. Their development in a classroom setting made for a common sense synthesis of both points of view.

What *are* then some of the significant aspects of the program that made it stand the test of time?

- Few word reading skills are required in the materials, but the program builds up a large vocabulary of mathematical reading skills: notation, chart reading, map reading, diagram reading. The materials allow the non-word-reader a fair chance at success.
- Most concrete learning aids suggested in these materials are teacher or student made or are easily collected from the everyday environment. Only a few, carefully selected, commercially available aids are suggested. (I still consider the Cuisenaire® rods the best all-purpose aid.)
- Both metric and common English units of measurement are used. No other American series did that a generation ago. Cuisenaire® rods, based on the centimeter scale, are used from the start to measure in metric units.
- Number lines are used in a variety of ways that range from ordering numbers (whole numbers, fractions, integers) to beginning functions. 10 by 10 arrays are used with similar variety, from counting to making tables for basic operations.
- Practice in computing is given through the creation and analysis of varied number patterns and series. Also included are instructions for card games, lotto, number line races, spinner and dice games.
- All four arithmetic operations and work with fractions are introduced in the first year. Experience in building models for concepts precedes all written work. The development of mathematical notation arises out of the children's own needs to record the results of experiments.
- Lab sheets do not have a commercial appearance and are not overcrowded. The pages are often handwritten or typed and deliberately varied. There is space for children to add their own problems. This serves as an encouragement for teachers and children to create their own additional materials. If pages are too slick, home-made materials can't compete with them.

- Ideas about sets are introduced with fun activities. Sorting games, for example, are used to introduce Venn diagrams. Many textbooks introduce an artificial and stilted formal language of sets in the early grades—only to give up any reference to sets by grades 4 and 5. This is ridiculous!
- Our base ten system of numeration is shown in many ways without overburdening the children by formal references to expanded notation.
- A variety of types of open sentences are introducted early in the series and continued throughout.
  5 + 3 = □ (Five plus three equals what?)
  □ = 5 + 3 (What equals five plus three?)
  8 = 5 + □ (Eight equals five plus what?)
  5 + □ = 8 (Five plus what equals eight?)
- Perforations in the workbook pages allow greater flexibility in their use. Children can remove lab sheets for correction or to take home. Lab sheets can also be arranged in different sequences and stapled together as units. Extra sheets can go into a classroom collection for use by other children. The progress charts on the inside back covers of the workbooks can be used to keep track of the completed work of the children.

Over the years I have had the opportunity to keep in close touch with many of the children I taught and whose mathematical education started with these materials. It gives me pleasure and satisfaction to know that a high proportion of students kept their love for the subject and experienced success in mathematics throughout their schooling. Many chose careers requiring advanced mathematics. They still recall with pleasure episodes from our math lab. My personal experience has been reinforced by my discovery this year that many of the schools that participated in the first large tryouts years ago are still using the books—and still consider them the best materials available.

Although I have learned much about teaching mathematics since 1960, I do not hesitate to sign my name again to them in 1977.

Lore Rasmussen

Philadelphia, Pennsylvania
June 1977

Lab Sheet Annotations

This book, *Lab Sheet Annotations,* contains a complete set of the 650 pupil lab sheets (reduced in size) that appear in the workbooks of the Miquon Math Lab Materials. The intervening text indicates the objectives of the sheets and ways in which the pupils may use them. Included are preliminary activities, teaching suggestions, and follow-up activities for many of the lab sheets. The annotations provide questions that the teacher can ask to reinforce the concepts underlying the problems on the sheets. The text also suggests activities for practice in applying those concepts. Where the procedures and objectives are apparent at a glance, lab sheets are not annotated.

Using the annotations

Following are some recommendations for the use of *Lab Sheet Annotations:*

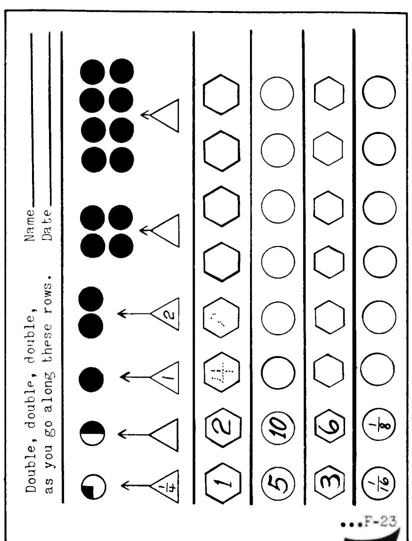
1 Before beginning work on a topic with the children, the teacher should read the mathematics background material on that topic included in *Lab Sheet Annotations.* The teacher should also continually refer to *First-Grade Diary* for teaching suggestions and games.

2 Answers to only the most difficult or the trickiest problems are provided in the annotations. The teacher should use an extra set of the workbooks and work through *all* of the problems in order to be

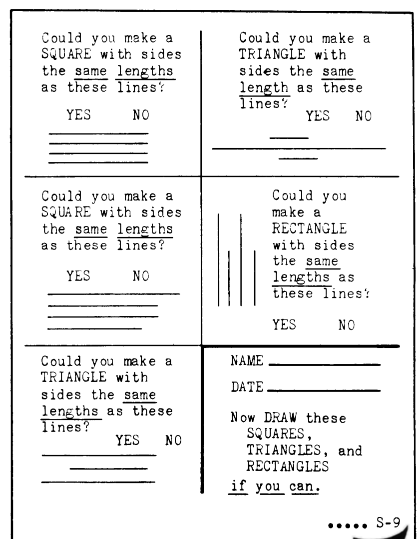
familiar with the concepts involved and the relative difficulty of the sheets. Thus teachers become their own first pupils as they transform their personal copies of the workbooks into answer books.

3 The divisions between Levels 1 to 6 are somewhat arbitrary. The teacher should feel free to dip into any level at any time that this is warranted for a particular pupil or group of pupils. No child should necessarily have to finish every sheet in a given level before advancing to the next level. Since the divisions between levels are artificial, all levels have been combined in *Lab Sheet Annotations.* This allows the teacher to develop a topic beyond the pages available in a particular workbook. It also allows each teacher to get a complete overview of the entire sequence of learning tasks for children. First grade teachers can look ahead, third grade teachers can look back.

4 The teacher should understand the organization of the material in *Lab Sheet Annotations.* Each student lab sheet is labeled with a dot-letter-number sequence. The dots indicate the lab sheet's level of difficulty and locate it in a particular workbook. The letter identifies the topic of the lab sheet. The number indicates the position of the lab sheet within its topic. Hence "••• F23" means that this lab sheet is in Level 3 (The Blue Book) and is the twenty-third lab sheet on Topic F, Multiplication. Similarly, "••••• S9" means that this lab



Level 3, The Blue Book
Topic F, Multiplication
23rd lab sheet in topic



Level 5, The Yellow Book
Topic S, Geometric Recognition
9th lab sheet in topic

sheet is in Level 5 (The Yellow Book) and is the ninth lab sheet on Topic S, Geometric Recognition. *Lab Sheet Annotations* is organized according to topic, not according to level. The Lab Sheet Level Chart that follows provides a quick reference guide for the levels and topics of lab sheets.

Lab Sheet Level Chart

Section	Topic	Level 1 ● Orange Book	Level 2 ●● Red Book	Level 3 ●●● Blue Book	Level 4 ●●●● Green Book	Level 5 ●●●●● Yellow Book	Level 6 ●●●●●● Purple Book
A	Counting	A1-A24					
B	Odd–Even		B1-B12	B13-B20			
C	Addition	C1-C11	C12-C25	C26-C34	C35-C36	C37-C46	
D	Subtraction	D1-D4	D5-D12	D13-D16	D17-D18	D19-D22	D23-D44
E	Addition and Subtraction	E1-E25	E26-E41	E42-E49		E50-E53	E54-E57
F	Multiplication	F1-F12	F13-F22	F23-F42	F43-F46	F47-F50	F51-F56
G	Addition, Subtraction, and Multiplication	G1-G8	G9-G12	G13-G20			
H	Fractions	H1-H6	H7-H24	H25-H42		H43-H56	H57-H64
I	Addition, Subtraction, Multiplication, and Fractions		I1-I8	I9-I16			
J	Division		J1-J12		J13-J18	J19-J26	J27-J32
K	Addition, Subtraction, Multiplication, Fractions, and Division		K1-K8	K9-K16		K17-K22	
L	Equalities and Inequalities	L1-L4	L5-L10		L11-L16	L17-L20	L21-L24
M	Place Value				M1-M16	M17-M20	M21-M24
N	Number Lines and Functions	N1-N4			N5-N10	N11-N18	
O	Factoring		O1-O4		O5-O6	O7-O12	
P	Squaring				P1-P16		P17-P18
Q	Simultaneous Equations					Q1-Q8	
R	Graphing Equations						R1-R30
S	Geometric Recognition	S1-S4		S5-S8		S9-S11	
T	Length, Area, and Volume	T1-T8			T9-T20	T21-T28	
U	Series and Progressions				U1-U6	U7-U12	
V	Grid and Arrow Games			V1-V6		V7-V8	V9-V10
W	Mapping				W1-W12		
X	Clock Arithmetic	X1-X10			X11-X14	X15-X16	X17-X18
Y	Sets						Y1-Y10
Z	Word Problems						Z1-Z13

A Counting

A robin will come back to its nest when one of four of its eggs has been removed. A robin will desert its nest when one of three eggs has been removed. Can a robin count to three?

A pre-schooler rattles off number words from one to twenty with lightning speed. Does this pre-schooler necessarily know how to count?

What is involved in counting? Is counting the first rung in man's—or the child's—climb up the mathematical ladder?

Counting is not the very beginning of man's mathematical development. Counting implies that we can extract a common feature, "threeness," from three elephants and three years. It also implies that we consider this "threeness" as *less* than the "fourness" embodied in four mice and four minutes.

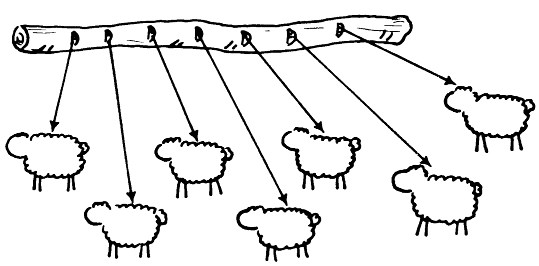
The robin recognized a group of "more than two." The pre-schooler memorized a "word sequence." Neither activity is enough to be called counting. The ideas of one-to-one correspondence and number sequence, and a special generalized vocabulary to express the two ideas, are prerequisites for counting. Let us illustrate them.

Matching, or one-to-one correspondence

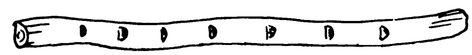
We see many examples of one-to-one correspondence in our everyday experience. We enter a lecture hall with friends and all get seats. Others come into the hall and take seats. When all seats are taken, the number of seats matches the number of people. For every person in the audience, there is a corresponding chair. For every chair, there is a corresponding person. If one person is without a chair, the "set of chairs" is one less than the "set of persons"—whether there are 50 chairs, 10 chairs, or 300 chairs. We know this without having to count the chairs or the people. If one chair is without a person, the set of persons is one less than the set of chairs. Again, we do not have to count to know that this is true.

Children do this kind of matching when they pass out candy, pencils, paper, and other objects. Will there be enough? Let's give each child a piece of candy. If the supply of candy is exhausted at the same time that all of the children have gotten their candy, then the groups (candy, children) are matched and are equal. We do not need number names, nor do we need to count. We either "have enough," "have more than enough," or "do not have enough."

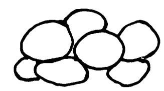
Early man used matching or tallying in this way to keep track of his herds and other possessions. The word *tally* comes from the Latin word *talea,* which means "stick" or "cutting," and probably refers to the cutting of notches into sticks to make a record of some possession. For example, seven notches refer to seven sheep let out to pasture.



The word *calculate* is from the Latin word *calculus,* meaning "pebble," and is a reminder of our ancestors' use of piles of pebbles to keep track of their possessions. Therefore a tally



or a pile of pebbles

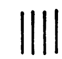


served as a check of the herd. If the herd, upon its return, did not match with the tally or the pile of pebbles, then the missing animal(s) had to be sought. The herd was intact only when the set of tally marks or pebbles was in one-to-one correspondence with the set of animals.

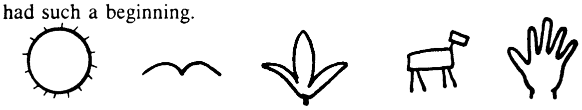
Fingers—and toes—have been man's most convenient matching device. The fact that our own system of numeration (that is, system of counting) is a base-ten system is an outgrowth of the early use of fingers and toes for matching. Other systems of numeration have been used, such as base five (number of fingers on one hand), base twenty (number of fingers and toes), base twelve (still used in our dozens and in inches in a foot), etc. The modern computer uses base two, the same system we use in measuring dairy products (cup, pint, quart, half gallon, gallon).

From the crude beginnings of tallying, pebbling, or fingering, a shorter method of grouping tallies, pebbles, or fingers arose.

 was probably easier to read than 

 was probably easier to read than 

Later, picture numerals and picture words were used instead of repeated tallies. Picture symbols, such as those below, were used to refer to different numbers of things. From these beginnings simpler numerals developed. Our own number words and numerals probably had such a beginning.



sun	wings	leaf	deer	hand
1	2	3	4	5

From directly matching two groups of things (for example, matching pebbles with sheep), matching could be extended. Groups could be matched indirectly.

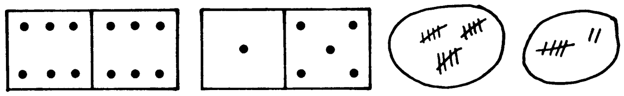
As an example, each child is given a pencil. The set of children in the group is thereby matched with the set of pencils. The children go out for recess but leave their pencils on their desks. Can the set of children be matched, during their absence, with a set of paper? Yes. We can match a set of paper with the set of pencils on the desks and thereby indirectly match the paper with the set of children.

A set of children	is matched with	B set of pencils
B set of pencils	is matched with	C set of paper

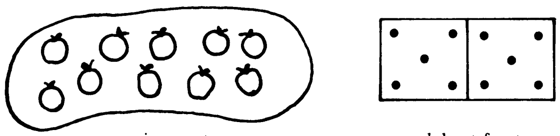
therefore,

A set of children	is matched with	C set of paper

There are certain sets whose elements are easily counted. Examples are a formation of dots on a domino or the set of lines in a tally.



By establishing a one-to-one correspondence between such a "model" set and an arbitrary given set, we can determine the number of elements in the given set.



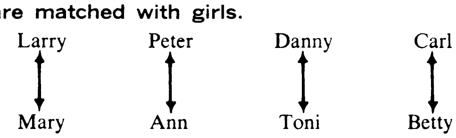
given set model set for ten

In the picture above, by matching each apple with each dot on the domino, we can prove that altogether there are ten apples.

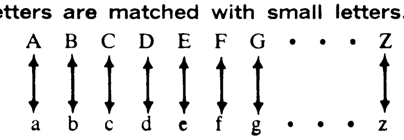
Suggested activities

The following nine activities are ones that can be used to suggest to children the concept of one-to-one correspondence. Use any collection, or set, of things: children, leaves, geometric solids, blocks, books, letters of the alphabet, etc.

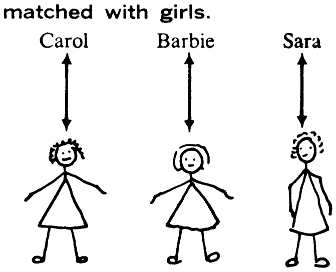
1 Boys are matched with girls.



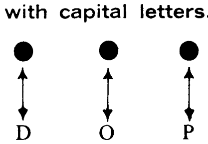
2 Capital letters are matched with small letters.



3 Names are matched with girls.

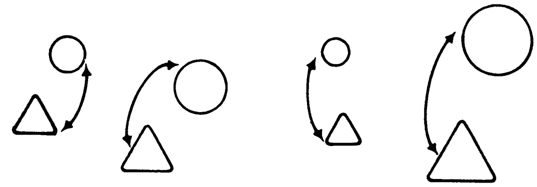


4 Dots are matched with capital letters.



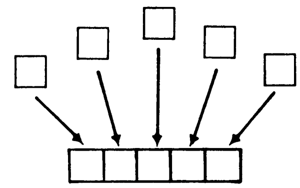
The number of the sets of dots matches the number of the set of capital letters.

5 Geometric shapes are matched.



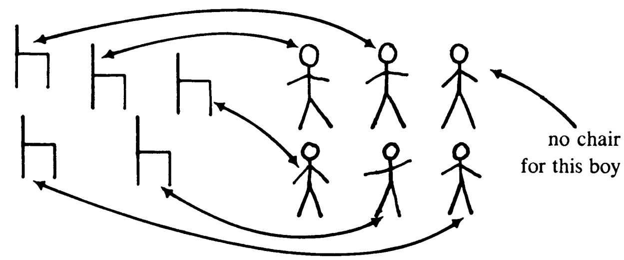
The number of the set of circles matches the number of the set of triangles.

6 Individual shapes are matched with a row of identical shapes.



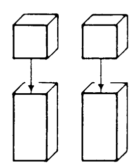
The number of the set of single squares matches the number of the row of squares.

7 Children are matched with chairs.



The set of chairs does *not* match the set of boys.

8 Rods of different lengths are matched.

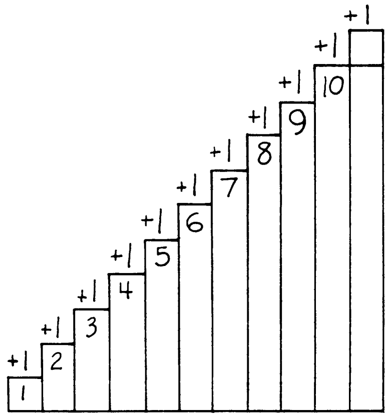


The number of the set of white rods matches the number of the set of red rods.

Number sequence

We may have many of the kinds of "model" sets which we found on dominoes and still not have a number system. To establish a number system, we must have a systematic way of arranging the model sets in the order of their magnitude. When we develop a way to obtain a successor to each number, we have established a counting system. The usual way in which to obtain the successor to a number is by adding one to it.

In the Mathematics Laboratory program, the Cuisenaire® rods are recommended as a basic model-making device for giving children experiences in matching and counting. Once a child has selected a model for the set of one, he can pass on to models for the sets of two, three, four . . . and on and on. The "rod stairs" illustrate this.



Thus,

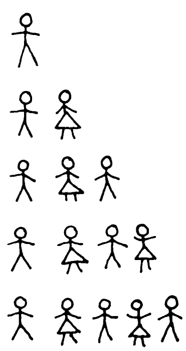
$1 = 1$
$(1) + 1 = 2$
$(1 + 1) + 1 = 3$
$(1 + 1 + 1) + 1 = 4$ (etc.)

From this experience, the child perceives that every number has a successor. The successor to a number is found by adding one to the number.

Suggested activities

Below are sketchy outlines for activities that introduce children to the notion of number sequence.

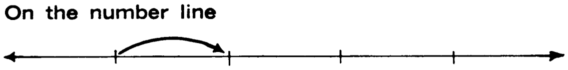
1 With children



1	one child
1 + 1	and one more
1 + 1 + 1	and one more
1 + 1 + 1 + 1	and one more
1 + 1 + 1 + 1 + 1	and one more (etc.)

Whenever we stop, we can always go backward, getting one less until "all are gone." When "all are gone," then the number of children or objects is zero, written '0.'

2 On the number line



one jump

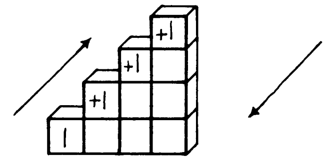
two jumps

three jumps

four jumps

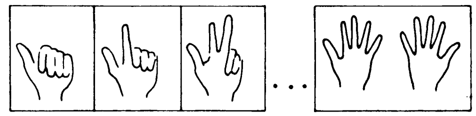
. . . and then going backward to "and one less." 

3 With rods



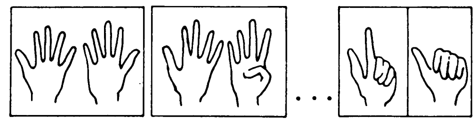
. . . and one more and one more, or backward to "and one less."

4 With fingers



1, 2, 3 . . . 10

and backwards



10, 9 . . . 2, 1

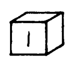
Number vocabulary and numerals

After the preceding activities, it will not be difficult to associate number words (one, two, three, etc.) and numerals (1, 2, 3, etc.) with groups of discrete objects and length.

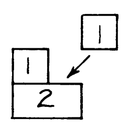
Suggested activities

Repeat many of the previous activities, using children, jumps on the number line, blocks, or fingers. However, this time make a verbal and visual association between number words, numerals, and objects.

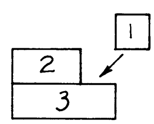
1 Use of rods



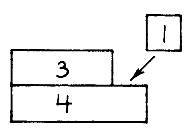
$1 = 1$



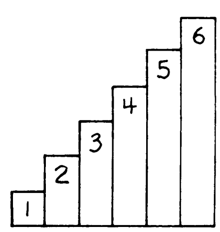
$1 + 1 = 2$



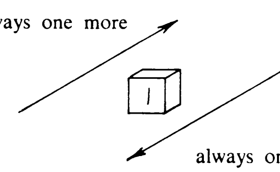
$2 + 1 = 3$



$3 + 1 = 4$



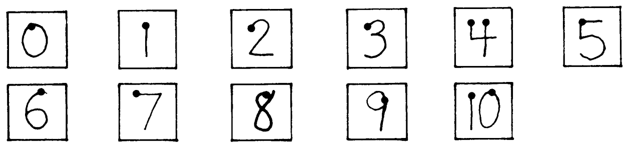
always one more



always one less

2 Use of numeral cards

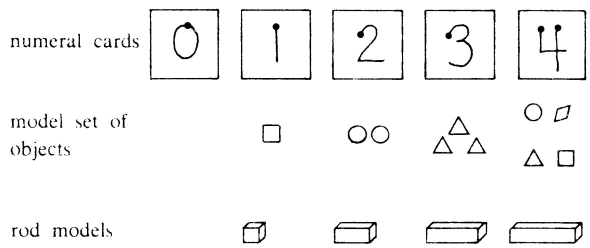
Each child should have a deck of numeral cards for the numbers 0 through 10. They may eliminate the need for early writing, or they can assist the child who is learning to write numerals.



They can be made from 3 x 5″ file cards. Dots should be put at the spot where each stroke is started as the numeral is written. The child then can put these cards at the top of his desk to serve as models when he writes. He can place his finger on the dot and trace the numeral first when he is confused about how to write it.

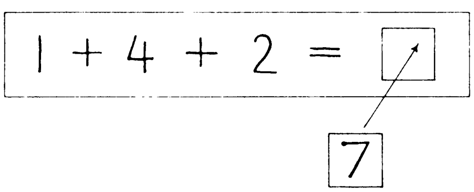
These numeral cards may be used in such activities as the following:

a. Children match number of things with cards.



b. Children arrange numeral cards in counting sequence: 1, 2, 3, 4, 5, . . . [etc.] or 10, 9, 8, 7, . . . [etc.].

c. Children use numeral cards as answer cards for large-scale single problems the teacher has made on strips of oaktag.



Transition from activity to writing

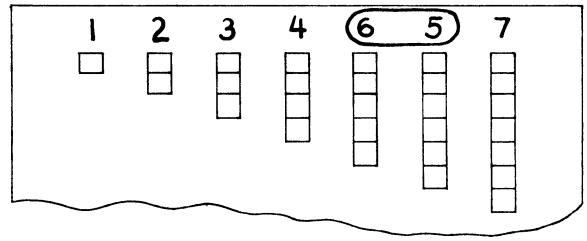
The lab sheets in Section A will require the children to write for the first time.

Not all children will make the transition from model building to written work at the same time. Some reasons for this are the children's varying degree of fine-muscle control for writing, reversal tendencies, varying rates of committing new symbols to memory, differing backgrounds, and varying intelligence.

It is therefore very important that teachers do not press children too hard and too early on matters of neatness, correct numeral form, and quick memorization of symbols.

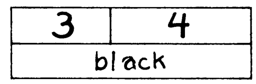
Following are some examples of children's work which show that they *understand* the task at hand but have trouble because of unfamiliarity with numerals.

1 John built this model of the counting sequence 1 through 7. He placed the rods on a sheet of paper on which he had written:



John had the right mathematical idea but mixed up the numerals 5 and 6. He will learn their proper placement soon!

2 Ann made a fine model for 3 + 4 = □

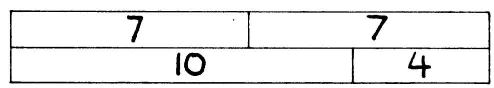


and then called out, "It's the black rod! It's the black rod! What is its name?" The teacher asked her to line up the white rods under it and find out. "Oh, it's seven, but how do you make a seven?"

3 Gary made up problems of his own and was very proud of them. Here is one:

7 + 7 = 41

What was his mistake? The teacher asked Gary to read his problem out loud to her. Gary read, "Seven plus seven equals fourteen. And I'll show you with my rods!"



Gary is a good thinker. He wrote:

41

(four teen)

It was not his fault that our language and our numerals do not match! The teacher showed him from his model that we write:

1 4

(one ten) (four ones)

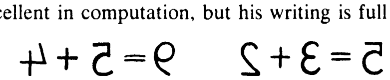
Note: It might be fun to make a game of inventing new names for ten, twenty, thirty, as follows. Such words would be consistent with their numerals.

. . . eight, nine, onety, onety-one, onety-two, etc.

8	9	10	11	12

twoty 20	twoty-one 21	twoty-two, etc. 22
threety 30	threety-one 31	
fourty 40	fourty-one 41	

4 Louis is excellent in computation, but his writing is full of reversals.



The teacher has to be a detective to tell he knows the answers. But she gives him the benefit of the doubt.

Work with children during their *handwriting lesson* on writing numerals. On arithmetic papers do not make the children take time out to correct their reversals or sloppy rendering of numerals.

•A1–A3

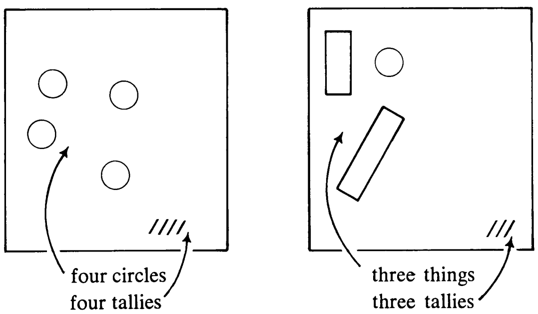
These lab sheets are arranged in order of complexity. Each sheet contains nine collections (sets) of objects which vary in number from one item to nine items (elements).

A-1 All sets contain circles of uniform size and in an orderly arrangement.
A-2 Sets contain elements in a variety of shapes and sizes.
A-3 Elements are less neatly arranged and vary in shape and size.

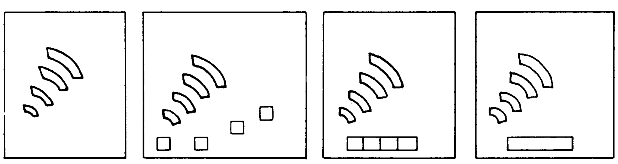
Suggested uses of lab sheets

1 Pupils match each element in each set (picture group) with an object such as a rod or a bead.

2 Pupils check off each element in a set with a tally mark.



3 Pupils match each element in a set with a white rod to get a model set. Then they arrange the rods in a row and exchange the row of white rods for a single rod of that length.



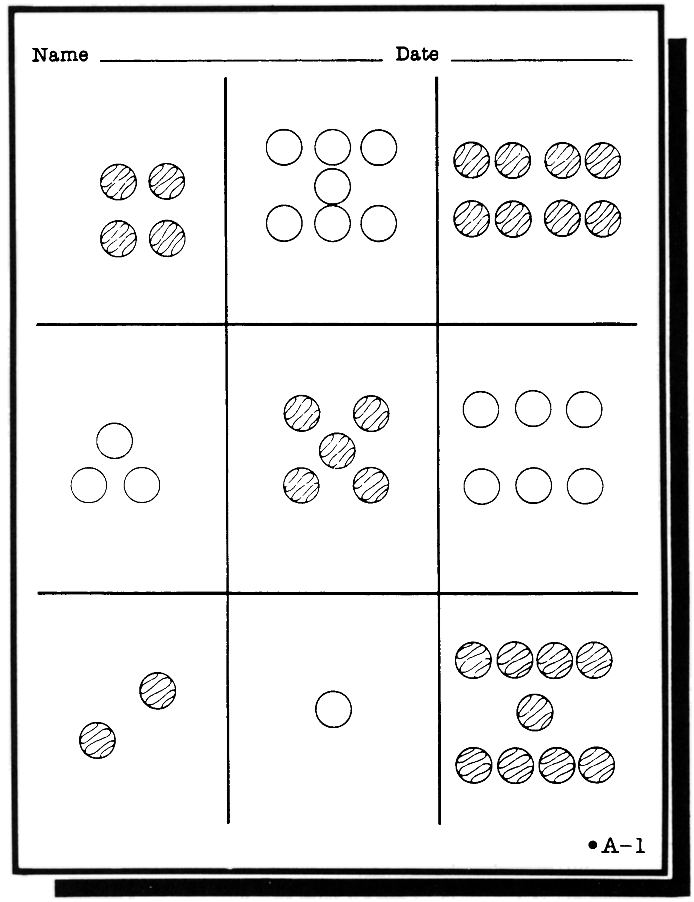
Set of 4 elements	is matched with 4 white rods.	White rods are placed in a row, and	white rods are exchanged for 1 purple rod.

The purple rod now represents the original set of 4 curved lines.

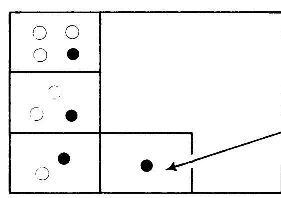
4 Pupils count the elements in each set and match each set with a numeral card.

5 Sample questions appropriate for all three pages:

Which collection (group, set)
. . . is the smallest (the least)?
. . . is the largest (most)?
. . . has one more than three things?
. . . has twice as many as two things?

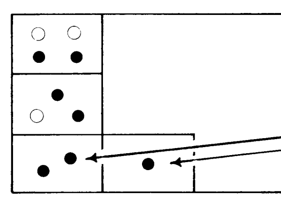


6 Coloring activity for lab sheet A1.



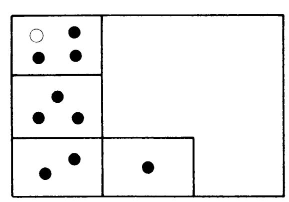
Color *one* circle in each set. Which set(s) are all colored?

[The one set is all colored.]



Color *one more* circle in each set. Which set(s) are all colored now?

[one and two]



Color one more circle in each set. Which sets are all colored now?

[one, two, three]

Continue until all sets are colored.

7 Lotto
Each child is given one sheet (either A1, A2, or A3). A "caller" holds up a Cuisenaire® rod—for example, a yellow rod. The children find on their sheets the set containing five elements and lay a yellow rod on that picture. The game is over when all the sets are matched with rods. The same game should also be played with numeral cards instead of rods.