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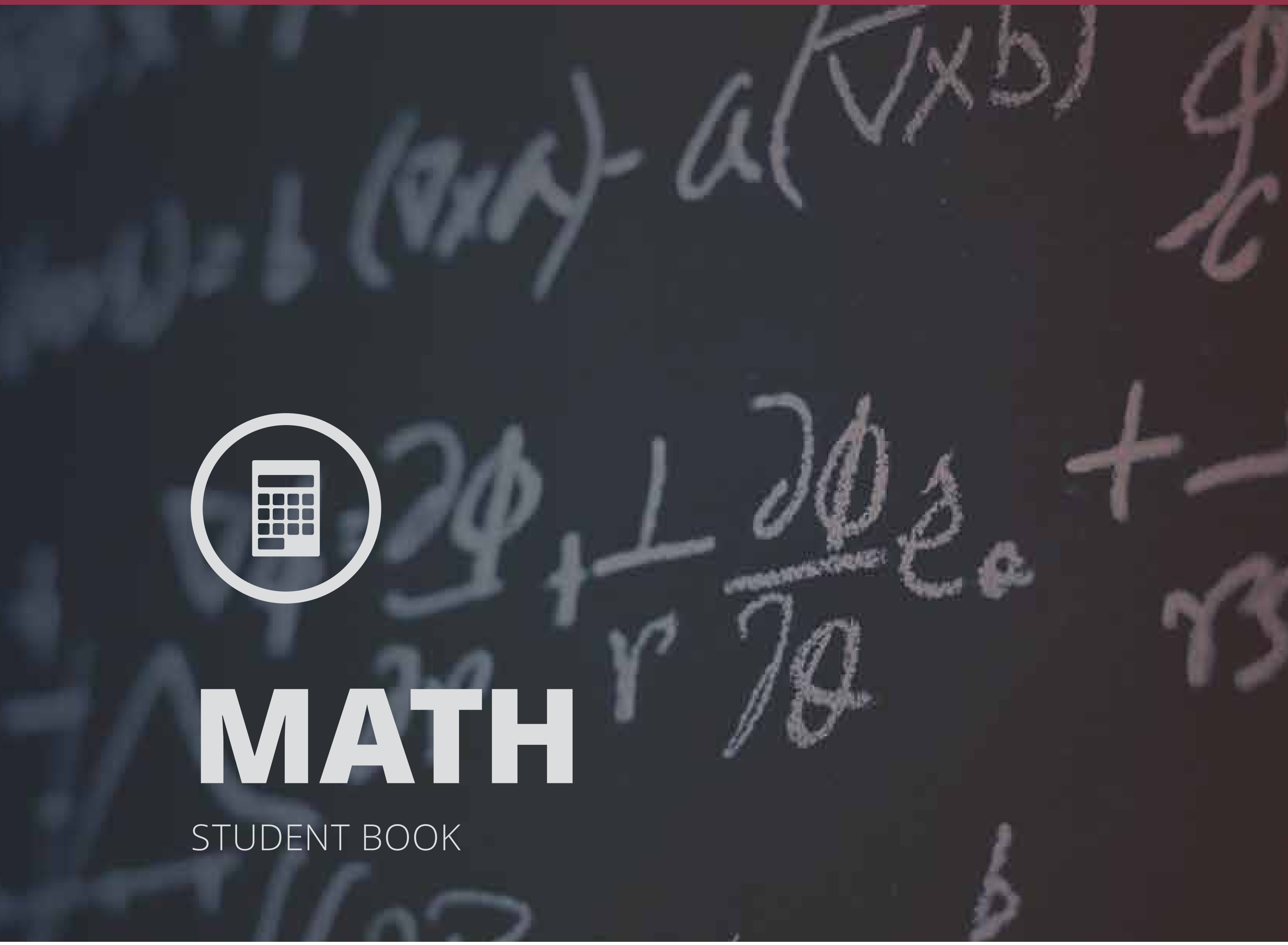

MATH

STUDENT BOOK

9th Grade | Unit 10

Alpha Omega
PUBLICATIONS

Math 910

Quadratic Equations and a Review of Algebra

LIFEPAC Test is located in the center of the booklet. Please remove before starting the unit.

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Quadratic Equations and a Review of Algebra

INTRODUCTION

This LIFEPAC® is the final LIFEPAC in the first-year study of the mathematical system known as algebra. In this LIFEPAC you will learn how to solve equations that involve second-degree polynomials; these equations are called *quadratic equations*. Then you will review some representative exercises and problems from each of the LIFEPACs in this course of study.

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC. When you have finished this LIFEPAC, you should be able to:

1. Identify quadratic equations.
2. Write quadratic equations in general form.
3. Solve quadratic equations by completing the square, by the quadratic formula, and by factoring.
4. Work representative problems of the first-year algebra course.

1. QUADRATIC EQUATIONS

In this section you will need to apply many skills that you have acquired in previous LIFEPACs, while you learn about a new type of equation. After an introduction to quadratic equations, you will learn three methods for solving them. Finally, you will learn to solve verbal problems that require the use of quadratic equations.

OBJECTIVES

Review these objectives. When you have completed this section, you should be able to:

1. Identify quadratic equations.
2. Write quadratic equations in general form.
3. Solve quadratic equations by completing the square, by the quadratic formula, and by factoring.

IDENTIFYING QUADRATIC EQUATIONS

First you must be able to recognize quadratic equations. Learn these two basic definitions.

DEFINITIONS

quadratic equation—an equation that can be written as $Ax^2 + Bx + C = 0$, where A is not zero.

In this LIFEPAC, you will consider an equation to be in **general form** when A is positive and when A, B, and C are integers whose greatest common factor is 1.

Model 1: $2x^2 + 3x - 4 = 0$ is a quadratic equation in general form, where A is 2, B is 3, and C is -4.

Model 2: $x^2 + 7 = 0$ is a quadratic equation in general form, where A is 1, B is 0, and C is 7.

Model 3: $-5x^2 + x = 0$ is a quadratic equation. Its general form can be found by multiplying both sides of the equation by negative one.

$$-1[-5x^2 + x] = -1[0]$$

$$5x^2 - x = 0$$

Then A is 5, B is -1, and C is 0.

Model 4: $3x - 11 = 0$ is not a quadratic equation since it does not contain a second-degree term.

Model 5: $x^3 - 2x^2 + 1 = 0$ is not a quadratic equation since it contains a third-degree term.

Model 6: $\frac{1}{3}(x + 2)(x - 7) = 5$ is a quadratic equation that can be rewritten.

$$3[\tfrac{1}{3}(x + 2)(x - 7)] = 3[5]$$

$$(x + 2)(x - 7) = 15$$

$$x^2 - 5x - 14 = 15$$

$$x^2 - 5x - 29 = 0$$

Then *A* is 1, *B* is -5, and *C* is -29.

Model 7: $0.7x^2 = 1$ is a quadratic equation. Its general form can be found by multiplying both sides of the equation by 10.

$$10[0.7x^2] = 10[1]$$

$$7x^2 = 10$$

$$7x^2 - 10 = 0$$

Then *A* is 7, *B* is 0, and *C* is -10.

Model 8: $2x^2 - 4x + 6 = 0$ is a quadratic equation. Its general form can be found by dividing all the terms by 2.

$$\frac{2x^2}{2} - \frac{4x}{2} + \frac{6}{2} = \frac{0}{2}$$

$$x^2 - 2x + 3 = 0$$

Then *A* is 1, *B* is -2, and *C* is 3.

Indicate (by yes or no) whether each of the following equations is quadratic. If so, give the values of *A*, *B*, and *C* from each equation's general form; if not, tell why.

1.1 $3x^2 + 5x - 7 = 0$ ____________________

1.2 $2x - 1 = 0$ ____________________

1.3 $2x^2 - 1 = 0$ ____________________

1.4 $-4x^2 + 2x - 1 = 0$ ____________________

1.5 $5x^2 + 15x = 0$ ____________________

1.6 $(x - 3)^2 = 0$ ____________________

1.7 $\frac{1}{4}x^2 + 5 = 0$ ____________________

1.8 $2x^3 - x = 0$ ____________________

1.9 $(x + 1)(2x + 3) = 4$ ____________________

1.10 $\frac{2}{3}(x - 4)(x + 5) = 1$ ____________________

1.11 $6x - 1 = 4x + 7$ ____________________

1.12 $6x^2 - 1 = 4x + 7$ ____________________

1.13 $6x^3 - 1 = 4x + 7$ ____________________

1.14 $1.3x^2 + 2.5x - 1 = 0$ ____________________

1.15 $(4x - 1)(3x + 5) = 0$ ____________________

1.16 $-2x^2 - 3x = 5$ ____________________

1.17 $(5 + x)(5 - x) = 7$ ____________________

1.18 $\frac{x^2}{2} = 7x$ ____________________

1.19 $\frac{1}{5}x = \frac{2}{3}x^2 - 2$ ____________________

1.20 $x(x + 1)(x + 2) = 3$ ____________________

METHODS OF SOLVING QUADRATIC EQUATIONS

We shall now look at three ways to solve quadratic equations. The first two methods may be used with any quadratic equation. The third method may be used only with quadratic equations that have factorable polynomials.

COMPLETING THE SQUARE

Let us begin by considering the equation $x^2 = 16$. You know that 4^2 and $(-4)^2$ equal 16; therefore, 4 or -4 are the two roots for this quadratic equation. These roots can be written in a *solution set* as {4, -4}.

Now consider the equation $x^2 = 17$. Since $(\sqrt{17})^2$ and $(-\sqrt{17})^2$ equal 17, the solution set for this quadratic equation is $\{\sqrt{17}, -\sqrt{17}\}$. Similarly, the equation $x^2 = 18$ is satisfied by $\sqrt{18}$ or $-\sqrt{18}$. These radicals can be simplified to $\sqrt{9}\sqrt{2}$ or $3\sqrt{2}$, and $-\sqrt{9}\sqrt{2}$ or $-3\sqrt{2}$.

Therefore, the solution set for this quadratic equation is $\{3\sqrt{2}, -3\sqrt{2}\}$.

The equation $x^2 = -18$ has no real roots since no real number exists whose square is negative.

In general, to solve equations that contain squares of binomials, we use the following property.

PROPERTY

If $X^2 = N$ and N is not negative, then $X = \sqrt{N}$ or $X = -\sqrt{N}$. This property is called the *Square Root Property of Equations*.

Study these models to see how this property can be applied to solve equations that contain squares of binomials.

Model 1: $(x + 1)^2 = 16$

4^2 is 16, so	$(-4)^2$ is 16, so
$x + 1 = 4$	$x + 1 = -4$
$x = 3$	$x = -5$

The solution set is {3, -5}

Model 2:

$$(x - 1)^2 = 17$$

$(\sqrt{17})^2$ is 17, so	$(-\sqrt{17})^2$ is 17, so
$x - 1 = \sqrt{17}$	$x - 1 = -\sqrt{17}$
$x = 1 + \sqrt{17}$	$x = 1 - \sqrt{17}$

The solution set is $\{1 + \sqrt{17}, 1 - \sqrt{17}\}$

Model 3:

$$(3x + 2)^2 = 18$$

$(\sqrt{18})^2$ is 18, so	$(-\sqrt{18})^2$ is 18, so
$3x + 2 = \sqrt{18}$	$3x + 2 = -\sqrt{18}$
$3x = -2 + \sqrt{18}$	$3x = -2 - \sqrt{18}$
$x = \frac{-2 + \sqrt{18}}{3}$	$x = \frac{-2 - \sqrt{18}}{3}$
$x = \frac{-2 + 3\sqrt{2}}{3}$	$x = \frac{-2 - 3\sqrt{2}}{3}$

The solution set is $\{\frac{-2 + 3\sqrt{2}}{3}, \frac{-2 - 3\sqrt{2}}{3}\}$.

Apply the Square Root Property of Equations to find the solution set for each of the following equations.

1.21 $x^2 = 25$

1.22 $x^2 = 26$

1.23 $x^2 = 27$

1.24 $x^2 - 80 = 0$

1.25 $x^2 + 80 = 0$

1.26 $(x + 2)^2 = 36$

1.27 $(2x - 5)^2 = 11$

1.28 $(x - 4)^2 = 12$

1.29 $(3x + 1)^2 - 100 = 0$

1.30 $(4x - 3)^2 - 50 = 0$

You should know how to find the root for quadratic equations that are written in the form $X^2 = N$. In some instances you may be able to write an equation in this form immediately by factoring $Ax^2 + Bx + C$ to the square of a binomial.

Model: $x^2 + 6x + 9 = 0$

$(x + 3)(x + 3) = 0$

$(x + 3)^2 = 0$

$0^2 = 0$

Therefore, $x + 3 = 0$

$x = -3$

The solution set is {-3}.

Note: This quadratic equation has only one root since 0 is the only number whose square is 0.

When $Ax^2 + Bx + C$ does not factor to the square of a binomial, a procedure known as *completing the square* may be used to write a quadratic equation in the form $X^2 = N$. The following steps can be used to accomplish this goal.

1. Write the quadratic equation in general form and identify A and B.
2. Multiply the terms of the equation by $4A$.
3. Isolate the constant term on the right side of the equation.
4. Add B^2 to each side of the equation.
5. Factor the left side of the equation to the square of a binomial.

These steps will be indicated by number in the solutions to the next three models.

Model 1: Solve $x^2 + 7x + 12 = 0$ by completing the square.

1. A is 1 and B is 7.
2. $4A$ is 4: $4[x^2 + 7x + 12] = 4[0]$
   $4x^2 + 28x + 48 = 0$
3. $4x^2 + 28x = -48$
4. B^2 is 49: $[4x^2 + 28x] + 49 = [-48] + 49$
   $4x^2 + 28x + 49 = 1$
5. $(2x + 7)(2x + 7) = 1$
   $(2x + 7)^2 = 1$

Now solve:

1^2 is 1, so	$(-1)^2$ is 1, so
$2x + 7 = 1$	$2x + 7 = -1$
$2x = -6$	$2x = -8$
$x = -3$	$x = -4$

The solution set is $\{-3, -4\}$.

Model 2: Solve $5x^2 = 3x$ by completing the square.

1. $5x^2 - 3x = 0$; A is 5 and B is -3.
2. $4A$ is 20: $20[5x^2 - 3x] = 20[0]$
   $100x^2 - 60x = 0$
3. The equation contains no constant term.
4. B^2 is 9: $[100x^2 - 60x] + 9 = [0] + 9$
   $100x^2 - 60x + 9 = 9$
5. $(10x - 3)(10x - 3) = 9$
   $(10x - 3)^2 = 9$

Now solve:

3^2 is 9, so	$(-3)^2$ is 9, so
$10x - 3 = 3$	$10x - 3 = -3$
$10x = 6$	$10x = 0$
$x = 0.6$	$x = 0$

The solution set is $\{0.6, 0\}$.

Model 3: Solve $4x(x - 3) = 2$ by completing the square.

1. $4x^2 - 12x = 2$
   $4x^2 - 12x - 2 = 0$
   $2x^2 - 6x - 1 = 0$; A is 2 and B is -6.
2. $4A$ is 8: $8[2x^2 - 6x - 1] = 8\ [0]$
   $16x^2 - 48x - 8 = 0$
3. $16x^2 - 48x = 8$
4. B^2 is 36: $[16x^2 - 48x] + 36 = [8] + 36$
   $16x^2 - 48x + 36 = 44$

5. $$(4x - 6)(4x - 6) = 44$$
$$(4x - 6)^2 = 44$$

Now solve: $(\sqrt{44})^2$ is 44 and $(-\sqrt{44})^2$ is 44, so

$$4x - 6 = \pm\sqrt{44}$$
$$4x = 6 \pm \sqrt{44}$$
$$4x = 6 \pm 2\sqrt{11}$$
$$x = \frac{6 \pm 2\sqrt{11}}{4}$$

The solution set is $\{\frac{3 + \sqrt{11}}{2}, \frac{3 - \sqrt{11}}{2}\}$.

Note: The ± symbol is read "plus or minus."

Solve each quadratic equation by completing the square.

1.31 $x^2 + 9x + 8 = 0$

1.32 $x^2 - 4x - 7 = 0$

1.33 $2x^2 = 7x$

1.34 $3x^2 + x = 0$

SELF TEST 1

Indicate (by yes or no) whether each of the following equations is quadratic. If so, give the values of *A*, *B*, and *C* from the general form of each equation; if not, tell why (each part, 3 points).

1.01 $2x^2 - 4x + 1 = 0$ ________ ; ____________________

1.02 $x(x^2 + 1) = 0$ ________ ; ____________________

1.03 $5(4x + 2) = 3$ ________ ; ____________________

1.04 $(x + 3)(x + 4) = 5$ ________ ; ____________________

1.05 $-\frac{3}{4}x^2 + 2 = 0$ ________ ; ____________________

Apply the Square Roots Property of Equations to find each solution set (each answer, 3 points).

1.06 $x^2 = 9$

1.07 $(3x - 1)^2 = 5$

1.08 $x^2 + 5x + 1 = 0$

1.09 $2x(x - 1) = 3$

Solve each quadratic equation by using the quadratic formula. If the roots are irrational, give both the exact value and the rational approximations to the nearest tenth (each problem, 4 points).

1.010 $3x^2 + 4x - 2 = 0$

1.011 $\frac{x^2}{3} + \frac{1}{2} = \frac{5}{6}x$

Solve each quadratic equation by factoring (each answer, 3 points).

1.012 $7x^2 + 3x = 0$

1.013 $(x - 2)(x - 3) = 2$

Solve each verbal problem with a quadratic equation using your choice of methods for solving the equation (each answer, 5 points).

1.014 The square of a certain negative number is equal to five more than one-half of that number. Find the number.

1.015 The width and the length of a rectangle are consecutive even integers. If the width is decreased by three inches, then the area of the resulting rectangle is twenty four square inches. Find the dimensions of the original rectangle.

$\frac{53}{66}$ **SCORE** ________ **TEACHER** ________ ________
initials date

MAT0910 – May '14 Printing

ISBN 978-0-86717-630-8

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Alpha Omega
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