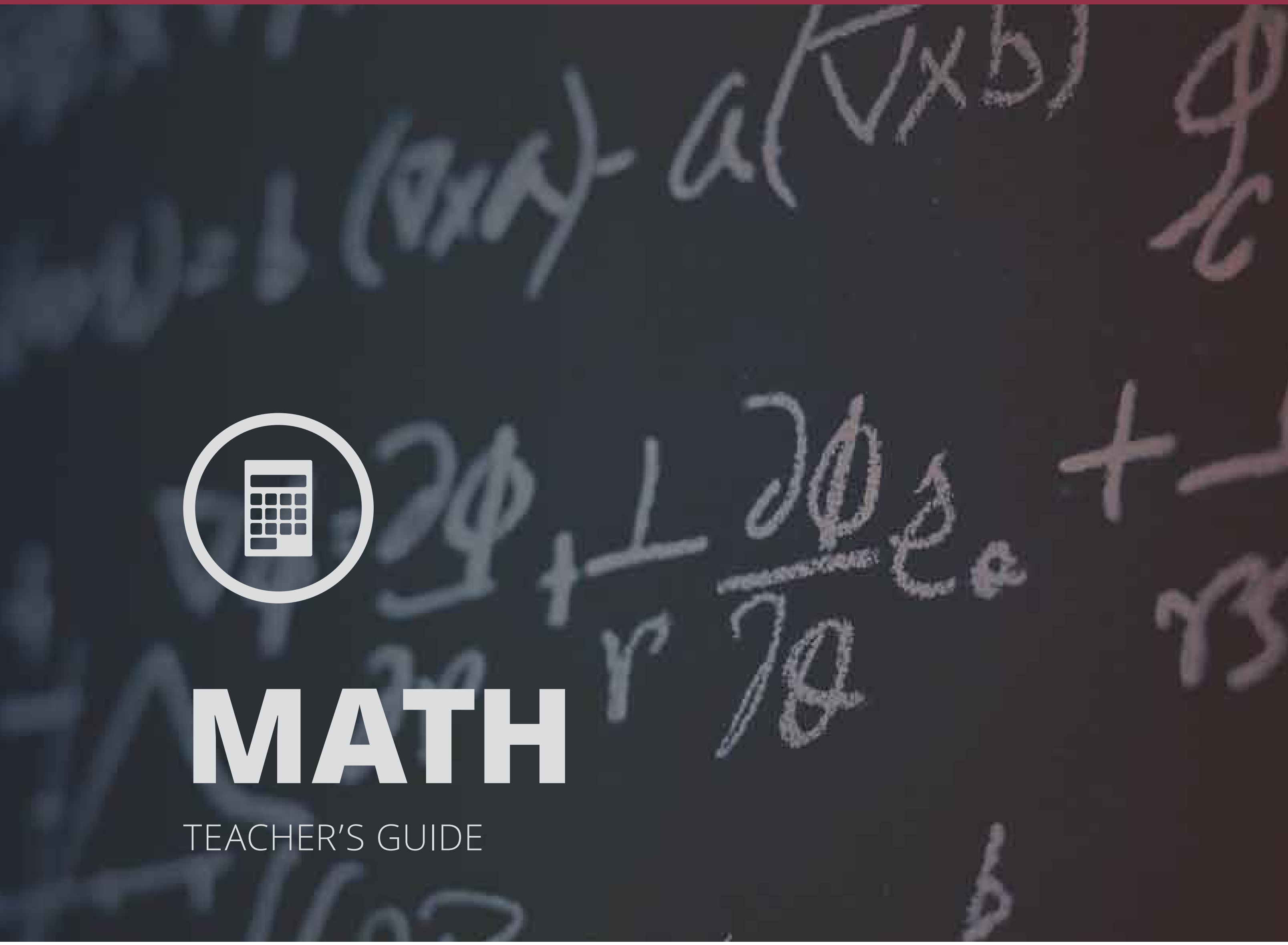
LIFEPAC®



MATH

TEACHER'S GUIDE

12th Grade

Alpha Omega
PUBLICATIONS

MATH 1200

Teacher's Guide

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INSTRUCTIONS FOR MATH

The LIFEPAC curriculum from grades 2 through 12 is structured so that the daily instructional material is written directly into the LIFEPACs. The student is encouraged to read and follow this instructional material in order to develop independent study habits. The teacher should introduce the LIFEPAC to the student, set a required completion schedule, complete teacher checks, be available for questions regarding both content and procedures, administer and grade tests, and develop additional learning activities as desired. Teachers working with several students may schedule their time so that students are assigned to a quiet work activity when it is necessary to spend instructional time with one particular student.

Math is a subject that requires skill mastery. But skill mastery needs to be applied toward active student involvement. Measurements require measuring cups, rulers, and empty containers. Boxes and other similar items help the study of solid shapes. Construction paper, beads, buttons, and beans are readily available and can be used for counting, base ten, fractions, sets, grouping, and sequencing. Students should be presented with problem situations and be given the opportunity to find their solutions.

Any workbook assignment that can be supported by a real world experience will enhance the student's ability for problem solving. There is an infinite challenge for the teacher to provide a meaningful environment for the study of math. It is a subject that requires constant assessment of student progress. Do not leave the study of math in the classroom.

This section of the Math Teacher's Guide includes the following teacher aids: Answer Keys and reproducible Alternate LIFEPAC Tests.

MATH 1201

Unit 1: Relations and Functions

ANSWER KEY

SECTION 1: Ordered-Pair Numbers

Ordered-Pair Numbers: Relations

1.1 h. domain
1.2 f. element
1.3 e. function
1.4 d. ordered pair
1.5 a. range
1.6 c. relation
1.7 b. $R \times R$
1.8 g. set
1.9 i. subset

1.10 b. {5, 6, 7}
The domain is the set of all first numbers of each ordered pair in a relation. Therefore, the domain of this relation is {5, 6, 7}.

1.11 a. {0, 1, 2}
The range is the set of all second numbers of each ordered pair in a relation. Therefore, the range of this relation is {0, 1, 2}.

1.12 a. {6, 7, 8, 9}
The domain is the set of all first numbers of each ordered pair in a relation. Therefore, the domain of this relation is {6, 7, 8, 9}.

1.13 b. $\{\sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}\}$
The range is the set of all second numbers of each ordered pair in a relation. Therefore, the range of this relation is $\{\sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}\}$.

1.14 b. 1
The domain is the set of all first numbers of each ordered pair in a relation. Since the first number of each ordered pair in this relation is the same, the number is only listed once. The domain of this relation is $\{\frac{1}{2}\}$.

1.15 c. 4
The range is the set of all second numbers of each ordered pair in a relation. The range of this relation is $\{\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}\}$.

1.16 c. {6.2, 7.3, 8.4, 9.5}
The domain is the set of all first numbers of each ordered pair in a relation. Therefore, the domain of this relation is {6.2, 7.3, 8.4, 9.5}.

1.17 a. {0.3}
The range is the set of all second numbers of each ordered pair in a relation. Since the second number of each ordered pair in this relation is the same, the number is only listed once. The range of this relation is {0.3}.

1.18 a. {(10, 2), (15, 3), (20, 4), (30, 6), (60, 8), (90, 10)}
The domain is the first number of the ordered pair and the range is the second. Therefore, combining the elements of the domain, *D*, with the elements of the range, *R*, will give the set of the ordered pairs, *Q*.
Q = {(10, 2), (15, 3), (20, 4), (30, 6), (60, 8), (90, 10)}.

1.19 b. {(1, 16), (2, 64), (3, 144), (4, 256), (5, 400)}
The domain is the first number of the ordered pair and the range is the second. In this case, *D* = {1, 2, 3, 4, 5}, and *R* = {16, 64, 144, 256, 400}. Therefore, combining the elements of the domain, *D*, with the elements of the range, *R*, will give the set of the ordered pairs, *F*. *F* = {(1, 16), (2, 64), (3, 144), (4, 256), (5, 400)}.

1.20 b. Domain $\{x: x \in R, x \geq 0\}$
Since the relation contains a square root radical, domain values must be excluded that would make the radicand negative. Otherwise, you have imaginary numbers instead of real numbers. Therefore, $y \geq 0$ and Domain $\{x: x \in R, x \geq 0\}$.

1.21 d. Domain $\{r: r \in R, r \neq 0\}$
Since the relation contains a fraction, domain values must be excluded that would make the denominator zero. Therefore, $13r \neq 0$ or $r \neq 0$. Domain $\{r: r \in R, r \neq 0\}$.

1.22 b. Domain $\{a: a \in R, a \neq 0\}$
The relation is $ab = 12$, or $b = \frac{12}{a}$. Therefore, $a \neq 0$. Domain $\{a: a \in R, a \neq 0\}$.

1.23 b. odd integers
If x = an even integer, then $x + 1$ must be an odd integer = y. Therefore, Domain {*x*: *x* is an even integer} and Range {*y*: *y* is an odd integer}.

1.24 c. Domain $\{x: x \in R\}$, Range $\{y: y \in R\}$

Ordered-Pair Numbers: Functions

1.25 function

1.26 a. Yes, no two ordered pairs in this list have the same first element.

1.27 d. No, each ordered pair in this list has the same first element.

1.28 a. Yes, no two ordered pairs in this list have the same first element.

1.29 c. Yes, no two ordered pairs in this list have the same first element.

1.30 c and d

Graphs *c* and *d* are not functions because a vertical line would intersect each graph at more than one point.

1.31 b. No, some of the ordered pairs on this graph have the same first element.

1.32 d. No, some of the ordered pairs on this graph have the same first element.

1.33 a. No, some of the ordered pairs on this graph have the same first element.

1.34 c. Yes, no two ordered pairs on this graph have the same first element.

1.35 b. Yes, no two ordered pairs on this graph have the same first element.

Ordered-Pair Numbers: Rules of Correspondence

1.36 c. linear function

1.37 b. quadratic function

1.38 a. slope

1.39 $y = 3x - 1$

Check for a linear function. First, determine the ratios of the difference in the *y*-coordinates compared to the *x*-coordinates among the ordered pairs.

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{5-2}{2-1} = \frac{3}{1} = 3$

$\frac{y_3 - y_1}{x_3 - x_1} = \frac{8-2}{3-1} = \frac{6}{2} = 3$

$\frac{y_3 - y_2}{x_3 - x_2} = \frac{8-5}{3-2} = \frac{3}{1} = 3$

Since the ratios are the same, it is a linear function and the slope is 3. Now use the point-slope formula and the first of the ordered pairs, (1, 2), to determine the equation.

$y - y_1 = m(x - x_1)$

$y - 2 = 3(x - 1)$

$y - 2 = 3x - 3$

$y = 3x - 1$

1.40 $x - 2y = -4$

Check for a linear function. First, determine the ratios of the difference in the *y*-coordinates compared to the *x*-coordinates among the ordered pairs.

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{4-3}{4-2} = \frac{1}{2}$

$\frac{y_3 - y_1}{x_3 - x_1} = \frac{5-3}{6-2} = \frac{2}{4} = \frac{1}{2}$

$\frac{y_3 - y_2}{x_3 - x_2} = \frac{5-4}{6-4} = \frac{1}{2}$

Since the ratios are the same, it is a linear function and the slope is $\frac{1}{2}$.

Now use the point-slope formula and the first of the ordered pairs, (2, 3), to determine the equation.

$y - y_1 = m(x - x_1)$

$y - 3 = \frac{1}{2}(x - 2)$

$y - 3 = \frac{1}{2}x - 1$

$2(y = \frac{1}{2}x + 2)$

$2y = x + 4$

$y = \frac{x}{2} + 2$

In general form: $x - 2y = -4$

1.41 $y = 3x - 3$

Check for a linear function. First, determine the ratios of the difference in the *y*-coordinates compared to the *x*-coordinates among the ordered pairs.

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{21-15}{8-6} = \frac{6}{2} = 3$

$\frac{y_3 - y_1}{x_3 - x_1} = \frac{27-15}{10-6} = \frac{12}{4} = 3$

$\frac{y_3 - y_2}{x_3 - x_2} = \frac{27-21}{10-8} = \frac{6}{2} = 3$

Since the ratios are the same, it is a linear function and the slope is 3.

Now use the point-slope formula and the first of the ordered pairs, (6, 15), to determine the equation.

$y - y_1 = m(x - x_1)$

$y - 15 = 3(x - 6)$

$y - 15 = 3x - 18$

$y = 3x - 3$

1.42 $y = 5x + 5$

Check for a linear function. First, determine the ratios of the difference in the *y*-coordinates compared to the *x*-coordinates among the ordered pairs.

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{30-15}{5-2} = \frac{15}{3} = 5$

$\frac{y_3 - y_1}{x_3 - x_1} = \frac{45-15}{8-2} = \frac{30}{6} = 5$

$\frac{y_3 - y_2}{x_3 - x_2} = \frac{45-30}{8-5} = \frac{15}{3} = 5$

Since the ratios are the same, it is a linear function and the slope is 5. Now use the point-slope formula and the first of the ordered pairs, (2, 15), to determine the equation.

$y - y_1 = m(x - x_1)$

$y - 15 = 5(x - 2)$

$y - 15 = 5x - 10$

$y = 5x + 5$

1.43 $y = x^2$

Check for a linear function. First, determine the ratios of the difference in the *y*-coordinates compared to the *x*-coordinates among the ordered pairs.

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{25 - 9}{5 - 3} = \frac{16}{2} = 8$

$\frac{y_3 - y_1}{x_3 - x_1} = \frac{4 - 9}{-2 - 3} = \frac{-5}{-5} = 1$

Since the ratios are not the same, it is not a linear function. Check the ordered pairs to see if it is a quadratic function. Note that the second term in the ordered pair, *y*, is the square of the first term, *x*. Therefore, it is a quadratic function and can be written as $y = x^2$.

1.44 b. $50t$

Note that D is 50 times t. Hence, this is a linear function and $D = 50t$.

1.45 c. $\frac{n(n - 3)}{2}$

1.46 c. $S = 16t^2$

1.47 a. 0

$D = \{(n, S)\text{: } (0, 0), (1, 1), (2, 3), (3, 6), (100, 5{,}050)\}$

$S = \frac{n}{2}(n + 1)$

For $n = 0$, $S = \frac{0}{2}(0 + 1) = 0$

For $n = 1$, $S = \frac{1}{2}(1 + 1) = \frac{1}{2}(2) = 1$

For $n = 2$, $S = \frac{2}{2}(2 + 1) = 1(3) = 3$

For $n = 100$, $S = \frac{100}{2}(100 + 1) = 50(101) = 5{,}050$

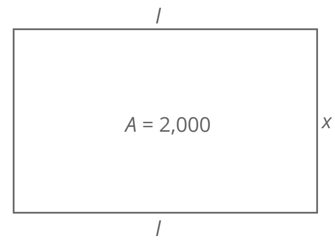
For $S = 6$,

$$6 = \frac{n}{2}(n + 1)$$
$$12 = n(n + 1)$$
$$12 = n^2 + n$$
$$n^2 + n - 12 = 0$$
$$(n + 4)(n - 3) = 0$$
$$n + 4 = 0 \qquad n - 3 = 0$$
$$n = -4 \qquad n = 3$$

Therefore,

$D = \{(n, S)\text{: } (0, 0), (1, 1), (2, 3), (3, 6), (100, 5{,}050)\}$

1.48 b. $x > 0$



$xl = 2{,}000$

$l = \frac{2{,}000}{x}$

$C(x) = 0.50x + 0.30(x + 2l)$

Substitute for l:

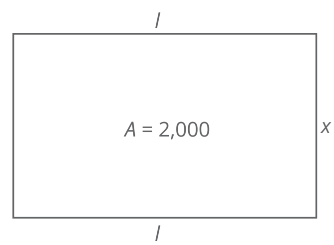
$C(x) = 0.50x + 0.30(x + 2[\frac{2{,}000}{x}])$

$C(x) = 0.50x + 0.30x + \frac{0.30(2)(2{,}000)}{x}$

$C(x) = 0.80x + \frac{1{,}200}{x}$

The domain is $x > 0$.

1.49 b. $0.8x + \frac{1{,}200}{x}$



$xl = 2{,}000$

$l = \frac{2{,}000}{x}$

$C(x) = 0.50x + 0.30(x + 2l)$

Substitute for l:

$C(x) = 0.50x + 0.30(x + 2[\frac{2{,}000}{x}])$

$C(x) = 0.50x + 0.30x + \frac{0.30(2)(2{,}000)}{x}$

$C(x) = 0.80x + \frac{1{,}200}{x}$

The domain is $x > 0$.

SELF TEST 1: Ordered-Pair Numbers

1.01 a. No, each ordered pair in this list has the same first element.

1.02 c. Yes, no two ordered pairs in this list have the same first element.

1.03 d. No, the elements of the set of y are not all ordered pairs.

1.04 b. {2}
The domain of $x = \{2\}$. It is the first number of each ordered pair.

1.05 b. {4}
The range of $x = \{4\}$. It is the second number of each ordered pair.

1.06 b. Yes, no two ordered pairs on this graph have the same first element.

1.07 c. Yes, no two ordered pairs on this graph have the same first element.

1.08 b. No, some of the ordered pairs on this graph have the same first element.

1.09 a. Yes, no two ordered pairs on this graph have the same first element.

1.010 $y = 4x - 4$

Check for a linear function. First, determine the ratios of the difference in the y-coordinates compared to the x-coordinates among the ordered pairs.

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 0}{2 - 1} = \frac{4}{1} = 4$

$\frac{y_3 - y_1}{x_3 - x_1} = \frac{8 - 0}{3 - 1} = \frac{8}{2} = 4$

$\frac{y_4 - y_1}{x_4 - x_1} = \frac{12 - 0}{4 - 1} = \frac{12}{3} = 4$

Since the ratios are the same, this is a line with a slope = 4. To find the equation of the relation, use the point-slope method.

$y - y_1 = m(x - x_1)$
$y - 0 = 4(x - 1)$
$y = 4x - 4$

1.011 $y = x^3$

Since the ratios between ordered pairs are different, this is not a linear relation. By trial and error observation, note that the second element of each ordered pair is the cube of the first element of the ordered pair. Therefore, the equation of the relation is:
$y = x^3$

1.012 $y = \frac{x + 4}{2}$

Check for a linear function. First, determine the ratios of the difference in the y-coordinates compared to the x-coordinates among ordered pairs.

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 3}{4 - 2} = \frac{1}{2}$

$\frac{y_3 - y_1}{x_3 - x_1} = \frac{5 - 3}{6 - 2} = \frac{2}{4} = \frac{1}{2}$

$\frac{y_4 - y_1}{x_4 - x_1} = \frac{6 - 3}{8 - 2} = \frac{3}{6} = \frac{1}{2}$

Since the ratios are the same, this is a line with a slope of $\frac{1}{2}$. To find the equation of the relation, use the point-slope method.

$y - y_1 = m(x - x_1)$

$y - 3 = \frac{1}{2}(x - 2)$

$y - 3 = \frac{1}{2}x - 1$

$y = \frac{1}{2}x + 2$

$2y = x + 4$ or $y = \frac{x + 4}{2}$

1.013 $y = \frac{x + 6}{2}$

Check for a linear function. First, determine the ratios of the difference in the y-coordinates compared to the x-coordinates among ordered pairs.

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 5}{8 - 4} = \frac{2}{4} = \frac{1}{2}$

$\frac{y_3 - y_1}{x_3 - x_1} = \frac{9 - 5}{12 - 4} = \frac{4}{8} = \frac{1}{2}$

$\frac{y_4 - y_1}{x_4 - x_1} = \frac{11 - 5}{16 - 4} = \frac{6}{12} = \frac{1}{2}$

Since the ratios are the same, this is a line with a slope of $\frac{1}{2}$. To find the equation of the relation, use the point-slope method.

$y - y_1 = m(x - x_1)$

$y - 5 = \frac{1}{2}(x - 4)$

$y - 5 = \frac{1}{2}x - 2$

$y = \frac{1}{2}x + 3$

$2y = x + 6$ or $y = \frac{x + 6}{2}$

SECTION 2: Algebra of Functions

Algebra of Functions: Notation

2.1 b. dependent variable

2.2 a. independent variable

2.3 a. 2

$F(x) = x^2 + 3x - 2$
$F(1) = (1)^2 + 3(1) - 2$
$= 1 + 3 - 2$
$= 2$

2.4 b. 16

$F(x) = x^2 + 3x - 2$
$F(3) = (3)^2 + 3(3) - 2$
$= 9 + 9 - 2$
$= 16$

2.5 a. -4

$F(x) = x^2 + 3x - 2$
$F(-1) = (-1)^2 + 3(-1) - 2$
$= 1 - 3 - 2$
$= -4$

2.6 b. 95

$G(x) = 3x^2 - 2x - 1$
$G(6) = 3(6)^2 - 2(6) - 1$
$= 3(36) - 12 - 1$
$= 108 - 13$
$= 95$

2.7 $x^4 + 2$

$F(x) = x^2 + 2$
$F(x^2) = (x^2)^2 + 2$
$= x^4 + 2$

2.8 $a^2 + 3a - 2$

$F(x) = x^2 + 3x - 2$
$F(a) = (a)^2 + 3(a) - 2$
$= a^2 + 3a - 2$

2.9 $x^2 + x - 4$

$F(x) = x^2 + 3x - 2$
$F(x - 1) = (x - 1)^2 + 3(x - 1) - 2$
$= x^2 - 2x + 1 + 3x - 3 - 2$
$= x^2 + x - 4$

2.10 $3a^2 + 6ab + 3b^2 - 2a - 2b - 1$

$G(x) = 3x^2 - 2x - 1$
$G(a + b) = 3(a + b)^2 - 2(a + b) - 1$
$= 3(a^2 + 2ab + b^2) - 2a - 2b - 1$
$= 3a^2 + 6ab + 3b^2 - 2a - 2b - 1$

2.11 $x^2 + 2xh + h^2 + 2$

$F(x) = x^2 + 2$
$F(x + h) = (x + h)^2 + 2$
$= x^2 + 2xh + h^2 + 2$

2.12 $x^2 - 2xh + h^2 + 2$

$F(x) = x^2 + 2$
$F(x - h) = (x - h)^2 + 2$
$= x^2 - 2xh + h^2 + 2$

Algebra of Functions: Arithmetic

2.13 d. same, equal

2.14 b. {2, 4, 6}

The common domain is the set of all the first elements of the ordered pairs that are the same in *F* and *G*. It is {2, 4, 6}.

2.15 c. 9

First, find the common domain of *F* and *G*. Then, add the range elements of the common domain. The answer is
$(F + G)(x) = \{(2, 4 + 5), (4, 6 + 7), (6, 8 + 9)\}$
$= \{(2, 9),(4, 13), (6, 17)\}$.

2.16 b. -1

First, find the common domain of *F* and *G*. Then, subtract the range elements of the common domain. The answer is
$(F - G)(x) = \{(2, 4 - 5), (4, 6 - 7), (6, 8 - 9)\}$
$= \{(2, -1), (4, -1), (6, -1)\}$.

2.17 b. $4x + 7$

$(F + G)(x) = (x + 2) + (3x + 5)$
$= 4x + 7$

2.18 c. $-2x - 3$

$(F - G)(x) = (x + 2) - (3x + 5)$
$= -2x - 3$

2.19 $2x + 4 - \frac{1}{x - 1}$

$f(x) = x + 2$
$h(x) = \frac{1}{x - 1}$
$2f(x) - h(x) = 2(x + 2) - \frac{1}{x - 1}$
$= 2x + 4 - \frac{1}{x - 1}$

2.20 b. 20

First, find the common domain of *F* and *G*. Then, multiply the range elements of the common domain. The answer is
$(F \cdot G)(x) = \{(2, 4 \cdot 5), (4, 6 \cdot 7), (6, 8 \cdot 9)\}$
$= \{(2, 20), (4, 42), (6, 72)\}$

2.21 a. $\frac{6}{7}$

First, find the common domain of *f* and *g*. Then, divide the range elements of the common domain. The answer is

$\frac{f(x)}{g(x)} = \{(2, \frac{4}{5}),(4, \frac{6}{7}),(6, \frac{8}{9})\}$

2.22 $3x^2 + 11x + 10$

$(F \bullet G)(x) = (x + 2)(3x + 5)$
$= 3x^2 + 5x + 6x + 10$
$= 3x^2 + 11x + 10$

2.23 b. $\frac{f(x)}{g(x)} = \frac{x + 2}{3x + 5}$

2.24 $x^2 + 4x + 4$

$f(x) = x + 2$
$[f(x)]^2 = (x + 2)^2$
$= x^2 + 4x + 4$

2.25 a. $\frac{1}{4}$

$f(x) = x + 2$

$h(x) = \frac{1}{x - 1}$

$\frac{[h(x)]^2}{f(x)} = \frac{[\frac{1}{x-1}]^2}{(x + 2)} = \frac{1}{(x - 1)^2(x + 2)}$

2.26 $x^2 + 2xh + h^2 + 2x + 2h + 1$

$f(x) = x^2 + 2x + 1$
$f(x + h) = (x + h)^2 + 2(x + h) + 1$
$= x^2 + 2xh + h^2 + 2x + 2h + 1$

2.27 c. $2x + h$

$F(x) = x^2$

$\frac{F(x + h) - F(x)}{h} = \frac{(x + h)^2 - x^2}{h}$

$= \frac{x^2 + 2xh + h^2 - x^2}{h}$

$= \frac{2xh + h^2}{h} = \frac{h(2x + h)}{h}$

$= 2x + h$

2.28 b. 2

$F(x) = 2x + 3$

$\frac{F(x + h) - F(x)}{h} = \frac{[2(x + h) + 3] - (2x + 3)}{h}$

$= \frac{2x + 2h + 3 - 2x - 3}{h} = \frac{2h}{h} = 2$

Algebra of Functions: Composition

2.29 d. composition of functions
2.30 b. constant function
2.31 a. identity function
2.32 c. zero function
2.33 d. $F[G(x)]$
2.34 a. $3x + 7$

$g[f(x)] = (3x + 2) + 5$
$= 3x + 7$

2.35 b. $2x^2 + 11$

$g[f(x)] = 2(x^2 + 6) - 1$
$= 2x^2 + 12 - 1$
$= 2x^2 + 11$

2.36 a. $x^2 + 8$

$g[h(x)] = (x^2 + 1) + 7$
$= x^2 + 8$

2.37 c. $\frac{1}{3x - 2}$

$h[k(x)] = \frac{1}{(3x - 4) + 2} = \frac{1}{3x - 2}$

2.38 a. $+ 16x + 16$

$f[g(x)] = \frac{1}{(\frac{1}{2x + 4})^2} = \frac{1}{\frac{(1)^2}{(2x + 4)^2}} = \frac{1}{\frac{1}{4x^2 + 16x + 16}}$

$= \frac{1}{1} \bullet \frac{4x^2 + 16x + 16}{1} = 4x^2 + 16x + 16$

2.39 c. 169

$f(x) = x^2$
$g(x) = x + 6$
$h(x) = 7$

$f\{g[h(x)]\} = \{[(7) + 6]\}^2$
$= \{13\}^2$
$= 169$

2.40 b. 55

$f(x) = x^2$
$g(x) = x + 6$
$h(x) = 7$

$g\{f[h(x)]\} = [(7)]^2 + 6$
$= 49 + 6$
$= 55$

2.41 a. 7

$h(x)$ is constant and equal to 7 for any value of x.

2.42 c. $4x^2 + 4x + 1$

$f[g(x)] = f(2x + 1)$
$= (2x + 1)^2$
$= 4x^2 + 4x + 1$

2.43 c. 5

$g[f(-2)] = g[(x^2)]$
$= (-2)^2 + 1$
$= 5$

2.44 c. zero
2.45 b. constant
2.46 a. identity

Algebra of Functions: Inverse

2.47 c. the range and the domain are interchanged
2.48 b. Yes, each element in the domain has only one range value.

2.49 c. No, each element in the domain does not have one range value.

2.50 a. Yes, each element in the domain has only one range value.

2.51 c. No, each element in the domain does not have one range value.

2.52 b. $\frac{x-6}{3}$, Yes

2.53 c. $\pm\sqrt{x-5}$, No

2.54 a. $\frac{1}{x} + 1$, Yes

2.55 c. $\pm\sqrt{\frac{x-2}{2}}$, No

2.56 We must check that $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$. If both of these statements are true, then f and g are inverses.

$$(f \circ g)(x) = \frac{5 - 3(\frac{5-2x}{3})}{2} = \frac{5 - \frac{15}{3} + \frac{6x}{3}}{2}$$
$$= \frac{5 - 5 + 2x}{2}$$
$$= \frac{2x}{2}$$
$$= x$$

and

$$(g \circ f)(x) = \frac{5 - 2(\frac{5-3x}{2})}{3} = \frac{5 - \frac{10}{2} + \frac{6x}{2}}{3}$$
$$= \frac{5 - 5 + 3x}{3}$$
$$= \frac{3x}{3}$$
$$= x$$

Since $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$, f and g are inverses.

2.57 We must check that $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$. If both statements are true, then f and g are inverses.

$$(f \circ g)(x) = \frac{(2x-3)+3}{2}$$
$$= \frac{2x-3+3}{2}$$
$$= \frac{2x}{2}$$
$$= x$$

and

$$(g \circ f)(x) = 2(\frac{x+3}{2}) - 3$$
$$= \frac{2x}{2} + \frac{6}{2} - 3$$
$$= x + 3 - 3$$
$$= x$$

Since $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$, f and g are inverses.

SELF TEST 2: Algebra of Functions

2.01 b. {-3, 0, 2, 5}

The domain is the set of all first numbers of each ordered pair in a relation. Therefore, the domain of this relation, *A*, from lowest to highest, is {-3, 0, 2, 5}.

2.02 a. {-2, 1, 3}

The range is the set of all second numbers of each ordered pair in a relation. The range number is only listed once in the range set. Therefore, the range of this relation, from lowest to highest, is {-2, 1, 3}.

2.03 d. $R \times R$

2.04 c. *r* is not a set of ordered pairs

2.05 d. $x \in R$

The domain of *R* is all real numbers. Therefore, $x \in R$.

2.06 a. $y \leq 0$

Since x^2 is positive or zero, then $-x^2$ will always be negative or zero. Therefore, the range of *R* is $y \leq 0$.

2.07 a. $2x - 6$

$f(x) = 3 - x$
$g(x) = -2x$
$g[f(x)] = -2(3 - x)$
$= -6 + 2x$
or
$= 2x - 6$

2.08 b. 7

$f(x) = 3 - x$
$g(x) = -2x$
$f[g(2)] = 3 - [-2(2)]$
$= 3 - [-4]$
$= 3 + 4$
$= 7$

2.09 a. -8

$f(x) = 3 - x$
$g(x) = -2x$
$g[f(-1)] = -2[3 - (-1)$
$= -2[3 + 1]$
$= -2(4)$
$= -8$

2.010 a. $4x + 4$

$F(x) = 2x - 1$
$G(x) = 3x + 2$
$H(x) = x^2$
$F[G(x)] - F(x) = [2(3x + 2) - 1] - (2x - 1)$
$= [6x + 4 - 1] - 2x + 1$
$= 4x + 4$

2.011 a. 5

$F(x) = 2x - 1$
$G(x) = 3x + 2$
$H(x) = x^2$
$G[H(1)] = [3(1)^2] + 2$
$= 3 + 2$
$= 5$

2.012 a. 27

$F(x) = 2x - 1$
$G(x) = 3x + 2$
$H(x) = x^2$
$F\{G[H(2)]\} = 2\{3[(2)^2] + 2\} - 1$
$= 2\{3(4) + 2\} - 1$
$= 2(12 + 2) - 1$
$= 2(14) - 1$
$= 28 - 1$
$= 27$

2.013 b. $a^2 + 2ax$

$F(x) = 2x - 1$
$G(x) = 3x + 2$
$H(x) = x^2$
$H(x + a) - H(x) = (x + a)^2 - x^2$
$= x^2 + 2ax + a^2 - x^2$
$= a^2 + 2ax$

2.014 a. $2x + a$

$F(x) = 2x - 1$
$G(x) = 3x + 2$
$H(x) = x^2$

$$\frac{H(x + a) - H(x)}{a} = \frac{(x + a)^2 - x^2}{a}$$
$$= \frac{x^2 + 2ax + a^2 - x^2}{a}$$
$$= \frac{a(a + 2x)}{a}$$
$$= a + 2x$$

2.015 b. $x^2 + 5x + 1$

$F(x) = 2x - 1$
$G(x) = 3x + 2$
$H(x) = x^2$
$F(x) + G(x) + H(x) = (2x - 1) + (3x + 2) + (x^2)$
$= x^2 + 5x + 1$

2.016 c. $y = \frac{x}{2}$

Note that the second number of the ordered pair is one-half the first number:

$y = \frac{1}{2}x$ or $y = \frac{x}{2}$

2.017 b. 7

$f(x) = 2x^2 - 3x + 1$
$f(3) = 2(3)^2 - 3(3) + 1$
$= 2(9) - 9 + 1$
$= 18 - 8 = 10$

$f(2) = 2(2)^2 - 3(2) + 1$
$= 2(4) - 6 + 1$
$= 8 - 5 = 3$

$f(3) - f(2) = 10 - 3$
$= 7$

2.018 a. Yes, no two ordered pairs in this list have a repeat of the domain element.

2.019 b. $y^2 = x$

Note that the first number of each ordered pair is the square of the second number. Therefore, $y^2 = x$.

2.020 c. No, two ordered pairs in this list have a repeat of the domain element.

2.021 a. $\frac{n(n + 1)}{2}$

2.022 b. $\frac{x + 2}{5}$

To find the inverse, let $y = G(x)$ and interchange x and y:

$G(x) = 5x - 2 \quad \rightarrow \quad y = 5x - 2$
$x = 5y - 2$
$x + 2 = 5y$
$\frac{x + 2}{5} = y \quad \rightarrow \quad G^{-1}(x) = \frac{x + 2}{5}$

2.023 If $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$, then $f(x)$ and $g(x)$ are inverses of each other.

$$(f \circ g)(x) = \frac{2}{(\frac{6x + 2}{x}) - 6} = \frac{2}{6x + 2 - 6x} = \frac{2x}{2} = x$$

and

$$(g \circ f)(x) = \frac{6(\frac{2}{x - 6}) + 2}{\frac{2}{x - 6}} = \frac{\frac{12}{x - 6} + \frac{2(x - 6)}{x - 6}}{\frac{2}{x - 6}}$$

$$= \frac{\frac{12 + 2x - 12}{x - 6}}{\frac{2}{x - 6}} = \frac{\frac{2x}{x - 6}}{\frac{2}{x - 6}} = \frac{2x}{x - 6} \bullet \frac{x - 6}{2} = x$$

Since $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$, $f(x)$ and $g(x)$ are inverses of each other.

2.024 If $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$, then $f(x)$ and $g(x)$ are inverses of each other.

$$(f \circ g)(x) = \tfrac{4}{5}(\frac{5x - 5}{4}) + 1$$

$$= \frac{20x - 20}{20} + 1$$

$$= \frac{20x}{20} - \frac{20}{20} + 1$$

$$= x - 1 + 1$$

$$= x$$

and

$$(g \circ f)(x) = \frac{5(\frac{4}{5}x + 1) - 5}{4}$$

$$= \frac{\frac{20}{5}x + 5 - 5}{4}$$

$$= \frac{4x + 5 - 5}{4}$$

$$= \frac{4x}{4}$$

$$= x$$

Since $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$, $f(x)$ and $g(x)$ are inverses of each other.

SECTION 3: Review Relations and Functions

3.1 b. $\{\sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}\}$

The range is the set of all second numbers of each ordered pair in a relation. Therefore, the range of this relation is $\{\sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}\}$.

3.2 c. {6.2, 7.3, 8.4, 9.5}

The domain is the set of all first numbers of each ordered pair in a relation. Therefore, the domain of this relation is {6.2, 7.3, 8.4, 9.5}.

3.3 c. Yes, no two ordered pairs on this graph have the same first element.

3.4 $y = 3x - 1$

Check for a linear function. First, determine the ratios of the difference in the y-coordinates compared to the x-coordinates among the ordered pairs.

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{5-2}{2-1} = \frac{3}{1} = 3$$
$$\frac{y_3 - y_1}{x_3 - x_1} = \frac{8-2}{3-1} = \frac{6}{2} = 3$$
$$\frac{y_3 - y_2}{x_3 - x_2} = \frac{8-5}{3-2} = \frac{3}{1} = 3$$

Since the ratios are the same, it is a linear function and the slope is 3.

Now use the point-slope formula and the first of the ordered pairs, (1, 2), to determine the equation.

$$y - y_1 = m(x - x_1)$$
$$y - 2 = 3(x - 1)$$
$$y - 2 = 3x - 3$$
$$y = 3x - 1$$

3.5 b. 95

$$G(x) = 3x^2 - 2x - 1$$
$$\begin{aligned} G(6) &= 3(6)^2 - 2(6) - 1 \\ &= 3(36) - 12 - 1 \\ &= 108 - 13 \\ &= 95 \end{aligned}$$

3.6 b. -1

First, find the common domain of F and G. Then, subtract the range elements of the common domain. The answer is

$$\begin{aligned} (F - G)(x) &= \{(2, 4 - 5), (4, 6 - 7), (6, 8 - 9)\} \\ &= \{(2, -1), (4, -1), (6, -1)\} \end{aligned}$$

3.7 b. $\frac{x+2}{3x+5}$

3.8 c. $\frac{1}{3x-2}$

$$h(x) = \frac{1}{x+2}$$
$$k(x) = 3x - 4$$
$$\begin{aligned} h[k(x)] &= \frac{1}{(3x-4)+2} \\ &= \frac{1}{3x-2} \end{aligned}$$

LIFEPAC TEST

1. c. $\{x: x \in R, x \neq -4, x \neq 7\}$

$\{(x, y): y = \frac{x(x-3)}{(x+4)(x-7)}\}$

A rational expression has a domain of all real numbers with the exception of values that make the denominator zero:

$x + 4 \neq 0$ $\quad$ $x - 7 \neq 0$
$x \neq -4$ $\quad$ $x \neq 7$

2. b. $(\sqrt{3}, -4)$ and c. $(-\sqrt{3}, 4)$
3. c. 2
4. c. $\{x: x \geq 3\}$
5. b. all real numbers
6. b. $x^2 + 2$
7. b. 13

$F(-2) = 3(-2)^2 + 1$
$= 3(4) + 1$
$= 12 + 1$
$= 13$

8. a. $3x^2 + 2x - 2$

$F(x) + G(x) = (3x^2 + 1) + (2x - 3)$
$= 3x^2 + 2x - 2$

9. c. $12x^2 - 36x + 28$

$F(x) = 3x^2 + 1$
$G(x) = 2x - 3$
$H(x) = x$
$F \circ G = F[G(x)]$
$= 3(2x - 3)^2 + 1$
$= 3(4x^2 - 12x + 9) + 1$
$= 12x^2 - 36x + 27 + 1$
$= 12x^2 - 36x + 28$

10. b. $\frac{x+3}{2}$

$F(x) = 3x^2 + 1$
$G(x) = 2x - 3$
$H(x) = x$
$G(x) = y = 2x - 3$

Interchange x and y:
$x = 2y - 3$
$x + 3 = 2y$
$y = \frac{x+3}{2}$
$G^{-1}(x) = \frac{x+3}{2}$

11. a. x

$H(x) = y = x$

Interchange x and y:
$x = y$
$H^{-1}(x) = x$

12. b. 23

$F(3) + G(4) - 2H(5) = 3[(3)^2 + 1 + [2(4) - 3] - 2(5)$
$= [3(9) + 1] + 8 - 3 - 10$
$= 27 + 1 - 5$
$= 23$

13. c. $\frac{x+26}{5}$

$F(x) = 5x - 6$
$G(x) = x - 4$

First find the composition $F \circ G$ and then find the inverse of that composition.

$F \circ G = F[G(x)]$
$= 5(x - 4) - 6$
$= 5x - 20 - 6$
$= 5x - 26$

Find $(F \circ G)^{-1}$:
$y = 5x - 26$

Interchange x and y:
$x = 5y - 26$
$x + 26 = 5y$
$y = \frac{x+26}{5}$
$(F \circ G)^{-1} = \frac{x+26}{5}$

14. a. $\frac{x+26}{5}$

$F(x) = 5x - 6$
$G(x) = x - 4$
$F^{-1}(x) = y = 5x - 6$

Interchange x and y:
$x = 5y - 6$
$x + 6 = 5y$
$y = \frac{x+6}{5}$
$F^{-1}(x) = \frac{x+6}{5}$

$G(x) = y = x - 4$

Interchange x and y:
$x = y - 4$
$y = x + 4$
$G^{-1}(x) = x + 4$
$G^{-1} \circ F^{-1} = G^{-1}[F^{-1}(x)]$
$= (\frac{x+6}{5}) + 4$
$= \frac{x+6+20}{5}$
$= \frac{x+26}{5}$

15. b. $x^2 + 2x + 2$

$F(x) = 2x$

$G(x) = x^2 + 2$

$(F + G)(x) = 2x + x^2 + 2$

$= x^2 + 2x + 2$

16. c. $x^2 - 2x + 2$

$F(x) = 2x$

$G(x) = x^2 + 2$

$(G - F)(x) = (x^2 + 2) - 2x$

$= x^2 - 2x + 2$

17. a. $-\frac{2}{3}$

$\frac{F}{G}(x) = \frac{2x}{x^2 + 2}$

$\frac{F}{G}(-1) = \frac{(2)(-1)}{(-1^2 + 2)}$

$-\frac{2}{3}$

18. We must check that $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$. If both statements are true, then f and g are inverses.

$(f \circ g)(x) = \frac{1}{2}(2x + 4) - 2$

$= x + 2 - 2$

$= x$

and

$(g \circ f)(x) = 2(\frac{1}{2}x - 2) + 4$

$= x - 4 + 4$

$= x$

Since $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$, $f(x)$ and $g(x)$ are inverses of each other.

ALTERNATE LIFEPAC TEST

1. d. $\{x: x \in R, x \neq -4, x \neq 7\}$

A rational expression has a domain of all real numbers with the exception of values that make the denominator zero:

$x + 4 \neq 0$ $\quad$ $x - 7 \neq 0$

$x \neq -4$ $\quad$ $x \neq 7$

2. b. (2, 3) and c. (-2, 3)

3. b. 0

4. b. $\{x: x \geq 4\}$

5. b. all real numbers

6. b. $x^2 + 1$

7. b. 9

$F(-2) = 2(-2)^2 + 1$

$= 2(4) + 1$

$= 9$

8. a. $2x^2 + 2x$

$F(x) + G(x) = [2x^2 + 1] + [2x - 1]$

$= 2x^2 + 1 + 2x - 1$

$= 2x^2 + 2x$

9. c. $12x^2 - 12x + 4$

$3(2x - 1)^2 + 1$

$3(4x^2 - 4x + 1) + 1$

$12x^2 - 12x + 4$

10. b. $\frac{x + 1}{2}$

To find the inverse, interchange x and y and solve for y.

$x = 2y - 1$

$x + 1 = 2y$

$\frac{x + 1}{2} = y$

$G^{-1}(x) = \frac{x + 1}{2}$

11. a. x

$H(x) = y = x$

Interchange x and y:

$x = y$

$H^{-1}(x) = x$

12. a. 16

$F(3) + G(4) - 2H(5) = [2(3)^2 + 1] + [2(4) - 1] - 2[5]$

$= [2(9) + 1] + [8 - 1] - 10$

$= 19 + 7 - 10$

$= 16$

13. a. $\frac{x+6}{5}$

$$\begin{aligned} F(x) &= 5x - 6 \\ y &= 5x - 6 \\ x &= 5y - 6 \\ x + 6 &= 5y \\ y &= \frac{x+6}{5} \\ F^{-1}(x) &= \frac{x+6}{5} \end{aligned}$$

14. a. $\frac{x+26}{5}$

$F(x) = 5x - 6$
$G(x) = x - 4$

Find G^{-1}:

$$\begin{aligned} x &= y - 4 \\ x + 4 &= y \\ G^{-1} &= x + 4 \end{aligned}$$

Find F^{-1}:

$$\begin{aligned} x &= 5y - 6 \\ x + 6 &= 5y \\ \frac{x+6}{5} &= y \\ F^{-1} &= \frac{x+6}{5} \end{aligned}$$

Now do the composition:

$$\begin{aligned} G^{-1} \circ F^{-1} &= \frac{x+6}{5} + 4 \\ &= \frac{x+6}{5} + \frac{20}{5} \\ &= \frac{x+26}{5} \end{aligned}$$

15. b. $x^2 + 3x + 1$

$$\begin{aligned} (F + G)(x) &= (3x) + (x^2 + 1) \\ &= x^2 + 3x + 1 \end{aligned}$$

16. c. $x^2 - 3x + 1$

$$\begin{aligned} (G - F)(x) &= (x^2 + 1) - (3x) \\ &= x^2 - 3x + 1 \end{aligned}$$

17. c. $-\frac{3}{2}$

$$\begin{aligned} \frac{F}{G}(x) &= \frac{F(x)}{G(x)} \\ &= \frac{3x}{x^2 + 1} \end{aligned}$$

18. We must check that $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$. If both statements are true, then f and g are inverses.

$$\begin{aligned} (f \circ g)(x) &= \frac{5x - 3 - 3}{5} \\ &= \frac{5x - 6}{5} \\ &= x - \frac{6}{5} \end{aligned}$$

Since $(f \circ g)(x) \neq x$, f and g are NOT inverses.

MATH 1201

ALTERNATE LIFEPAC TEST

NAME ________________________________

DATE ________________________________

SCORE ________________________________

$\frac{43}{54}$

Work the following problems (each answer, 3 points).

1. What is the domain of the relation $\{(x, y): y = \frac{x(x-3)}{(x+4)(x-7)}\}$? ______

a. $\{x: x \approx R, x \leq -4, x \neq 7\}$
b. $\{x: x \notin R, x = -4, x \neq 7\}$
c. $\{x: x = R, x \neq -4, x \geq 7\}$
d. $\{x: x \in R, x \neq -4, x \neq 7\}$

2. Choose the ordered pairs that belong to the given relation. Select all that apply.

$\{(x, y): x^2 < |y|^3 - 5\}$ ____________

a. (0, 1)
b. (2, 3)
c. (-2, 3)
d. (0, 0)

3. Complete the ordered pair for the relation $\{(x, y): y = 3|x + 2|$ and $x \in \{-2, -1, 0, 1, 2\}\}$.

(-2, ______)

a. 12
b. 0
c. 3

4. $\{(x, y): y = \sqrt{x - 4}\}$
The domain of the set is represented by ______ .

a. $\{x: x \in R\}$
b. $\{x: x \geq 4\}$
c. $\{x: x \leq 4\}$
d. $\{x: x \geq 0\}$

5. The domain of $\{(x, y): y = 2x^2 + 1\}$ is _______ .

a. $x < 0$
b. all real numbers
c. $x > 0$

6. $D = \{[x, f(x)]: (-1, 2), (0, 1), (1, 2), (2, 5), (3, 10)\}$

Write the rule for $f(x)$. _______

a. $x + 1$
b. $x^2 + 1$
c. $2x$

7. Given: $F(x) = 2x^2 + 1$, $G(x) = 2x - 1$, $H(x) = x$ $F(-2) =$ _______

a. -7
b. 9
c. 17

8. Given: $F(x) = 2x^2 + 1$, $G(x) = 2x - 1$, $H(x) = x$ $F(x) + G(x) =$ _______

a. $2x^2 + 2x$
b. $4x^3$
c. $2x^2 + 2x + 2$

9. Given: $F(x) = 3x^2 + 1$, $G(x) = 2x - 1$, $H(x) = x$ $F \circ G =$ _______

a. $4x^2$
b. $12x^2 + 4$
c. $12x^2 - 12x + 4$

10. Given: $F(x) = 2x^2 + 1$, $G(x) = 2x - 1$, $H(x) = x$ $G^{-1}(x) =$ _______

a. $-2x + 1$
b. $\frac{x + 1}{2}$
c. $2(x + 1)$

11. Given: $F(x) = 2x^2 + 1$, $G(x) = 2x - 1$, $H(x) = x$ $H^{-1}(x) =$ _______

a. x
b. $\frac{1}{x}$
c. $-x$

12. Given: $F(x) = 2x^2 + 1$, $G(x) = 2x - 1$, $H(x) = x$ $F(3) + G(4) - 2H(5) =$ _______

a. 16
b. 26
c. 34

13. Given: $F(x) = 5x - 6$ and $G(x) = x - 4$ $\quad F^{-1} =$ _______

a. $\frac{x+6}{5}$

b. $-6x + 10$

c. $\frac{x+10}{5}$

14. Given: $F(x) = 5x - 6$ and $G(x) = x - 4$ $\quad G^{-1} \circ F^{-1} =$ _______

a. $\frac{x+26}{5}$

b. $-6x + 10$

c. $\frac{x+10}{5}$

15. Given: $F(x) = 3x$ and $G(x) = x^2 + 1$ $\quad$ Find $(F + G)(x)$. _______

a. $3x^3 + 1$
b. $x^2 + 3x + 1$
c. $3x^2 + 1$

16. Given: $F(x) = 3x$ and $G(x) = x^2 + 1$ $\quad$ Find $(G - F)(x)$. _______

a. $x^2 - 3x - 1$
b. $-2x + 1$
c. $x^2 - 3x + 1$

17. Given: $F(x) = 3x$ and $G(x) = x^2 + 1$ $\quad$ Find $\frac{F}{G}(-1)$. _______

a. $-\frac{2}{3}$

b. 1

c. $-\frac{3}{2}$

18. Verify if $f(x) = \frac{x-3}{5}$ and $g(x) = 5x - 3$ are inverses.

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