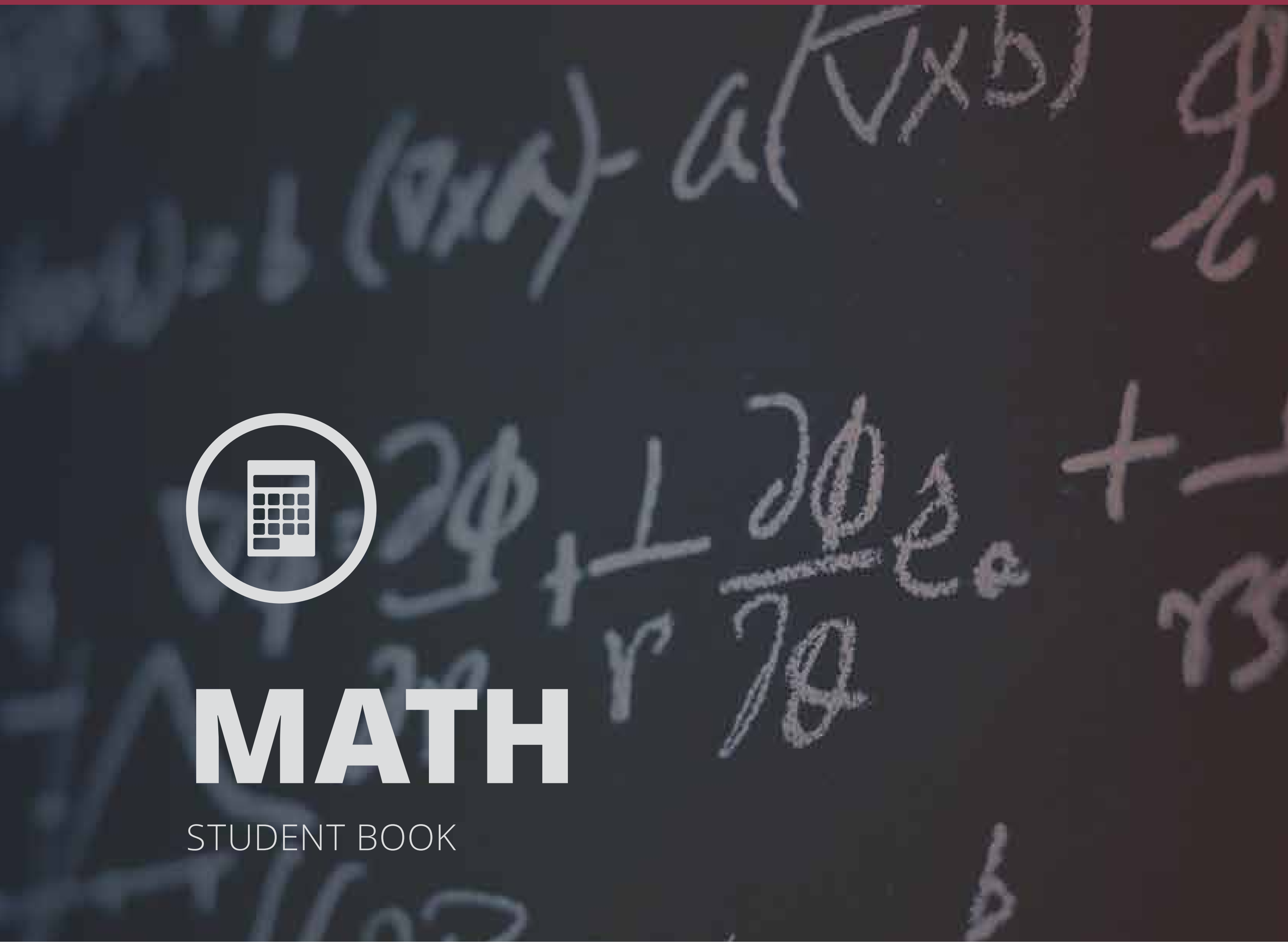
LIFEPAC®



MATH

STUDENT BOOK

11th Grade | Unit 6

Alpha Omega
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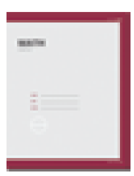
MATH 1106
REAL NUMBERS



LIFEPAC Test is located in the center of the booklet. Please remove before starting the unit.

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Real Numbers

Introduction

In this LIFEPAC® you will study the real-number system and will learn about radicals and imaginary numbers. All of the previously learned properties apply to radicals, because they are real numbers. However, you will learn some special properties of radicals.

In this LIFEPAC you will learn to solve radical equations and quadratic equations. Since much of this LIFEPAC is necessary review, you should also acquire a better understanding of previous material and increased development of your skill-type learning.

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC. When you have finished this LIFEPAC, you should be able to:

1. Identify real numbers as rational or irrational.
2. Perform operations with radicals and simplify the resulting expressions.
3. Solve radical equations.
4. Solve quadratic equations.
5. Write and use the quadratic formula to solve quadratic equations.
6. Define and use imaginary numbers.

Survey the LIFEPAC. Ask yourself some questions about this study and write your questions here.

1. REAL NUMBERS

The simplest and earliest numbers were positive whole numbers. Soon man discovered a need for zero and fractions. Negative numbers also became useful; finally, the concept of irrational numbers was needed. To increase your understanding of our real-number system, you must learn more about radicals and how to solve equations that involve radicals.

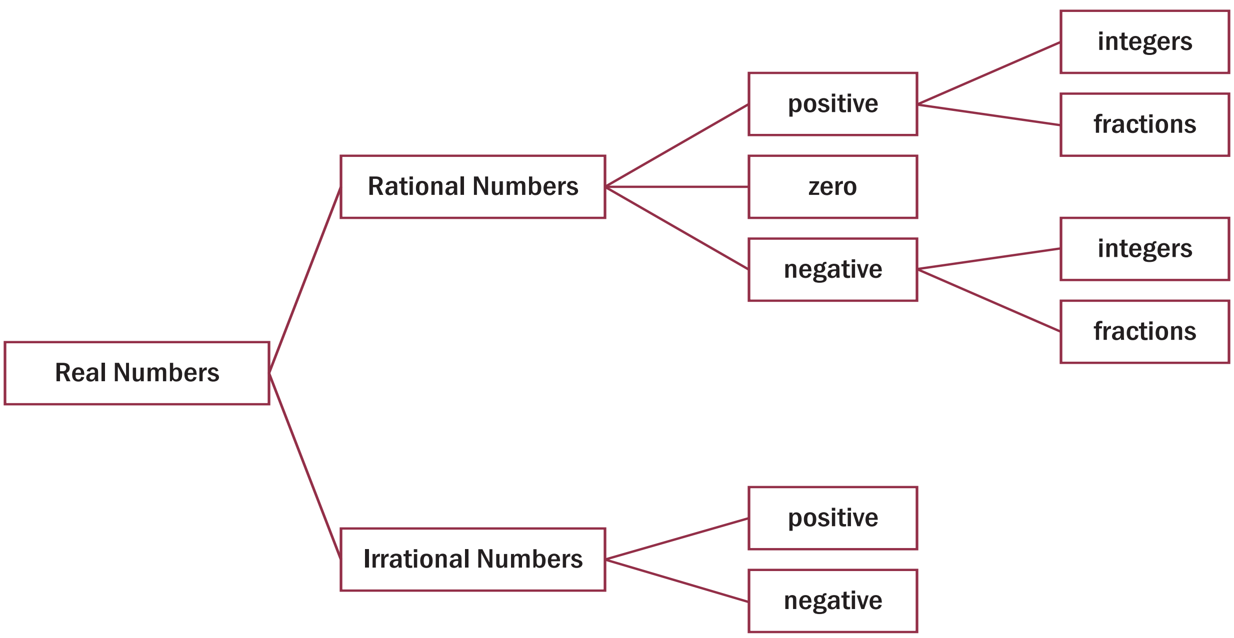
Section Objectives

Review these objectives. When you have completed this section, you should be able to:

1. Identify real numbers as rational or irrational.
2. Perform operations with radicals and simplify the resulting expressions.
3. Solve radical equations.

RATIONAL AND IRRATIONAL NUMBERS

This chart shows the relationship of the real numbers.



Rational numbers are real numbers that can be represented in the form $\frac{a}{b}$ where a and b are integers and $b \neq 0$. When considering rational numbers, remember that we are simply talking about all positive or negative whole numbers, zero, and the fractions you have always used in arithmetic.

Irrational numbers are the real numbers that are not rational numbers. They sometimes occur as special numbers, like π, which is the ratio of the circumference of a circle to its diameter. They may be indicated as nonrepeating, nonterminating decimals.

DEFINITIONS

Rational number: A number that can be expressed as a quotient of two other numbers; a terminating or repeating decimal.

Irrational number: A number that cannot be fully expressed as a quotient of two other numbers; a nonterminating, nonrepeating decimal.

The most commonly occurring type of irrational number is a number that contains a radical that cannot be simplified to a rational number.

Models:

$$\left.\begin{array}{l}\sqrt{16} = 4 \\ \frac{4}{6} = \frac{2}{3}\end{array}\right\} \text{rational numbers}$$

$$\left.\begin{array}{l}\sqrt{3} \\ \frac{\sqrt{11}}{5}\end{array}\right\} \text{irrational numbers}$$

A **radical** is an indicated root of a number. We know that 9 has two square roots, +3 and -3. Any real number has two square roots. When we use the radical symbol, we are referring only to the principal, or positive, square root.

Radicals occur not only as second-degree radicals (square roots) but as third-degree or higher-degree radicals.

DEFINITIONS

Radical: An expression consisting of a number with a radical sign indicating some root of the number beneath the radical sign.

Index: A small number written over the radical sign indicating which root of the number is being sought.

Radicand: The number inside the radical sign.

Radical sign: A symbol indicating that an expression is a radical.

Model: $\sqrt[5]{79}$

5 is the index.

79 is the radicand.

$\sqrt[5]{79}$ is the radical.

$\sqrt{}$ is the radical symbol.

Notice that if the radical has no index number, the root indicated is the square root. Therefore, $\sqrt{} = \sqrt[2]{}$.

Any decimal that does not terminate or repeat is an irrational number. If a decimal does terminate or repeat, it can be written as a common fraction (in the form $\frac{a}{b}$).

Model:

$$0.51 = \frac{51}{100}$$

$$0.25 = \frac{1}{4}$$

$$0.1666\ldots = \frac{1}{6}$$

A repeating decimal can be changed to a common fraction, as shown in the following model, by multiplying by the same power of 10 as the number of digits repeating.

Model: 0.2525...

Let 0.2525... be x. Multiplying by 10^2 or 100, because two digits are repeating, gives $100x = 25.25\ldots$

Subtract.

$$\begin{aligned} 100x &= 25.25\ldots \\ x &= 0.2525\ldots \\ \hline 99x &= 25 \\ x &= \tfrac{25}{99} \end{aligned}$$

Multiplying by a power of 10 that matches the number of digits repeating will allow use of this procedure for a decimal with any number of repeating digits.

Identify each number as *rational* or *irrational*.

1.1 1,384 ____________

1.2 π ____________

1.3 -0.28764... ____________

1.4 $\sqrt{3}$ ____________

1.5 0 ____________

1.6 -19.78 ____________

1.7 $-\frac{\sqrt{3}}{5} + 10.8$ ____________

1.8 0.86258625... ____________

1.9 -7.832198653... ____________

1.10 -0.8662866239... ____________

Change to the form $\frac{a}{b}$ in lowest terms.

1.11 0.225 ____________

1.12 0.625 ____________

1.13 $-0.66\frac{2}{3}$ ____________

1.14 0.888... ____________

1.15 -3.143143... ____________

Complete with the proper word.

1.16 In $\sqrt[3]{18}$, the 3 is called the ______________________________________ .

1.17 In $\sqrt[3]{18}$, the 18 is called the ______________________________________ .

1.18 In $\sqrt[3]{18}$, the $\sqrt{}$ is called the ______________________________________ .

1.19 The name for an expression like $\sqrt[3]{13}$ is a(n) ______________________________________ .

1.20 The square root of 16 is 4; 4 is called the ______________________________________ root of 16.

LAWS OF RADICALS

The laws of radicals are rules about how to combine radicals. Study them carefully. The results of some of the combinations may surprise you!

LAWS OF RADICALS

A. $\sqrt[m]{x}\sqrt[m]{y} = \sqrt[m]{xy}$ $\qquad$ $\sqrt[m]{xy} = \sqrt[m]{x}\sqrt[m]{y}$

B. $\left(\sqrt[m]{x^a}\right)^b = \sqrt[m]{x^{ab}}$ $\qquad$ $\sqrt[m]{x^{ab}} = \left(\sqrt[m]{x^a}\right)^b$

C. $\dfrac{\sqrt[m]{x}}{\sqrt[m]{y}} = \sqrt[m]{\dfrac{x}{y}}$ $\qquad$ $\sqrt[m]{\dfrac{x}{y}} = \dfrac{\sqrt[m]{x}}{\sqrt[m]{y}}$

D. $a\sqrt[n]{x} + b\sqrt[n]{x} = (a + b)\sqrt[n]{x}$

The laws of radicals shown can help you simplify and compute with radicals. Notice that in each law the indices are the same.

A. Model: $\sqrt[3]{9}\sqrt[3]{5} = \sqrt[3]{45}$

B. Model: $\left(\sqrt[4]{a^3}\right)^2 = \sqrt[4]{a^6}$

C. Model: $\dfrac{\sqrt[5]{64}}{\sqrt[5]{2}} = \sqrt[5]{32} = 2$

Radicals can be combined by addition only when radicals (both index and radicand) are the same.

D. Model 1: $6\sqrt[4]{5} + 2\sqrt[4]{5} = 8\sqrt[4]{5}$

Model 2: $3\sqrt[2]{8} + 2\sqrt[3]{4}$ These two radicals cannot be combined.

Model 3: $\left(\sqrt{2} + 3\sqrt{5}\right)\left(2\sqrt{2} - \sqrt{5}\right)$

$2\left(\sqrt{2}\right)^2 + 6\sqrt{2}\sqrt{5} - \sqrt{2}\sqrt{5} - 3\left(\sqrt{5}\right)^2$

$2 \cdot 2 + 5\sqrt{10} - 3 \cdot 5$

$4 + 5\sqrt{10} - 15$

$5\sqrt{10} - 11$

Radical symbols can be eliminated by fractional exponents. In some work with algebraic expressions or equations, however, fractional exponents are useful.

In a **fractional exponent**, the numerator of the fraction represents the power of the number and the denominator represents the degree or index of the radical.

DEFINITION

Fractional exponent: An exponent in the form of a fraction, with the numerator representing the power to which the base is to be raised and the denominator representing the index of the radical.

Models: $x^{\frac{a}{b}} = \sqrt[b]{x^a}$

$7^{\frac{2}{3}} = \sqrt[3]{7^2}$

As fractions must always be simplified to lowest terms, radicals must also be simplified. Three conditions must not exist in the final answer of a radical problem.

A. No perfect *n*th power factors of a radicand can be in an *n*th degree radical.

B. No fractions can be in the radicand.

C. No radical can be in the denominator of a fraction.

Radicals must be simplified until all three of these conditions are fulfilled.

A. Model: $\sqrt[3]{40} = \sqrt[3]{8 \cdot 5} = \sqrt[3]{8} \cdot \sqrt[3]{5} = 2\sqrt[3]{5}$

B. Model: $\sqrt[5]{\frac{3}{4}} = \sqrt[5]{\frac{3 \cdot 8}{4 \cdot 8}} = \sqrt[5]{\frac{24}{32}}$

$\frac{\sqrt[5]{24}}{\sqrt[5]{32}} = \frac{\sqrt[5]{24}}{2}$

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