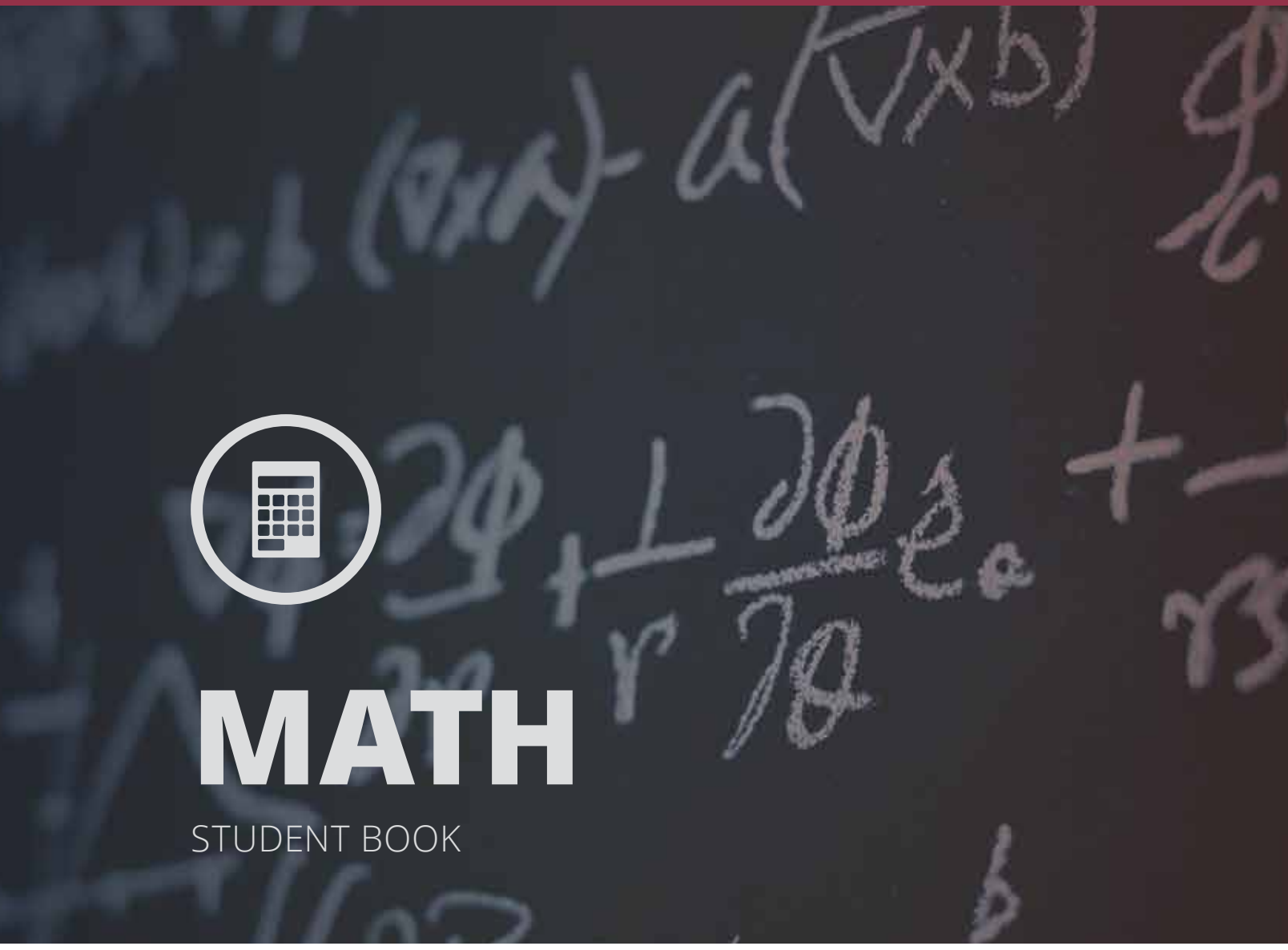
LIFEPAC®



MATH

STUDENT BOOK

10th Grade | Unit 7

Alpha Omega
PUBLICATIONS

MATH 1007

Construction and Locus

LIFEPAC Test is located in the center of the booklet. Please remove before starting the unit.

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Construction and Locus

Introduction

In your previous LIFEPACs you have represented geometric figures by sketching them on your paper. You had no particular concern for the accuracy of the figures, as long as they did not give misleading information.

Some of you may have made accurate drawings using a ruler, compass, and protractor. You may be surprised to learn that in many situations you can make these figures with the same or greater accuracy using only the compass and an unmarked straightedge.

In this LIFEPAC® you will learn how to construct a variety of figures using only the compass and straightedge. You will also learn a technique to locate and describe sets of points that satisfy one or more given conditions.

Objectives

Read these objectives. The objectives tell you what you will be able to do when you have successfully completed this LIFEPAC. When you have finished this LIFEPAC, you should be able to:

1. Copy segments and angles.
2. Bisect angles, segments, and arcs.
3. Construct perpendiculars to a line.
4. Construct tangents to a circle.
5. Construct parallel lines.
6. Divide a segment into a given number of equal segments.
7. Construct a fourth proportional.
8. Construct a geometric mean.
9. Divide a segment into a given ratio.
10. Construct triangles.
11. Construct special circles.
12. Construct certain polygons.
13. Define locus.
14. Solve locus problems.
15. Construct figures using locus concepts.

1. BASIC CONSTRUCTION

You will learn five basic constructions in this section: copying figures, bisecting figures, constructing perpendiculars, constructing parallels, and constructing with measurements. Each of these constructions will be done using only the compass and the straightedge. With these five basic constructions you can then construct other geometric figures such as triangles, rectangles, and parallelograms.

Section Objectives

Review these objectives. When you have finished this section, you should be able to:

1. Copy segments and angles.
2. Bisect angles, segments, and arcs.
3. Construct perpendiculars to a line.
4. Construct tangents to a circle.
5. Construct parallel lines.
6. Divide a segment into a given number of equal segments.
7. Construct a fourth proportional.
8. Construct a geometric mean.
9. Divide a segment into a given ratio.

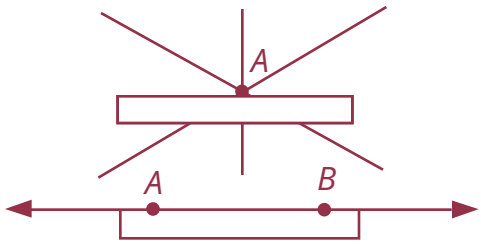
COPYING FIGURES

To copy a figure means to duplicate it on your paper so that the segments and angles of the copy are exactly the same size as the original. Copying is done with just two tools: a compass and an unmarked straightedge.

DEFINITION

Straightedge: an instrument used only to draw lines through one given point and draw the unique line through two given points.

We can use many items to draw lines, if they are straight. The edge of a book, the edge of a piece of cardboard, a piece of plastic, or a ruler will all serve. We can use the ruler so long as we do not use the numbers on it for measuring.

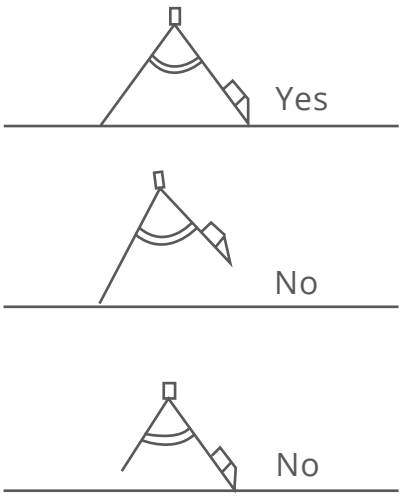


DEFINITION

Compass: an instrument used only to draw circles or arcs of a circle having a given center and to draw the unique circle having a given center and a given radius.

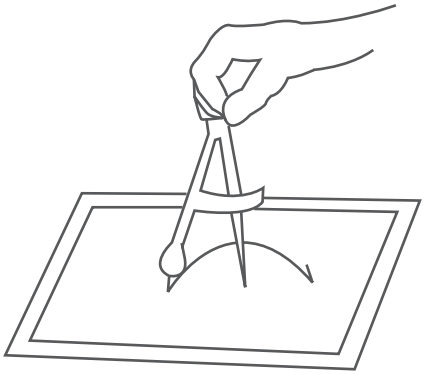
A compass can take many forms, but the two main parts of any compass are its points. One point is a fixed sharp metal point that you stick into your paper at the center of the circle. The

other point contains the writing tool, usually a pencil. The pencil point is adjustable up and down. The compass should be adjusted so that the tip of the pencil is the same length as the tip of the center point. The distance between the two points will be the radius of the circle you are drawing.



When using a compass, you may wish to back your paper with some thin cardboard so that the center point of the compass will stick in and not slide off your paper.

When drawing a circle, hold the compass at the top. Do not hold the legs of the compass, or you may change the radius of your circle.



Tip the compass slightly in the direction you are drawing so that it is not perpendicular to the paper and pull the compass through your circle. Never push the compass, because pushing may tear your paper.

Use your straightedge to construct lines through the given points. Use your compass to construct circles.

1.1 $\overline{AB}$

1.2 $\overrightarrow{BC}$

1.3 $\overleftrightarrow{DC}$

1.4 $\overleftrightarrow{BD}$

1.5 $\overline{AD}$

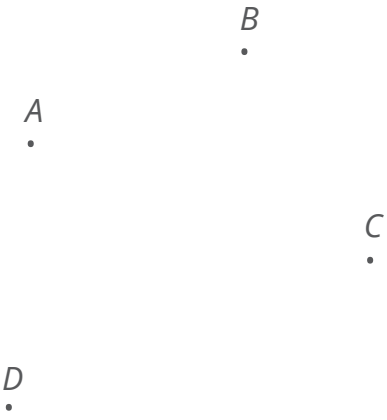
1.6 $\overrightarrow{AC}$

1.7 Circle *A*, any radius

1.8 Circle *B*, any radius

1.9 Circle *C*, same radius as circle *B*

1.10 Circle *D*, same radius as circle *A*



Construct circles with the given segment as radius and the given point as center.

1.11 circle *A*

A

1.12 circle *B*

B

C

1.13 circle *C*

1.14 circle *D*

D

E

1.15 circle *E*

F

1.16 circle *F*

To copy any segment, use Construction 1.

CONSTRUCTION 1

Given a segment, construct another segment with a length equal to the given segment.

Given: $\overline{AB}$

To construct: segment $\overline{RS}$ with length equal to AB

Steps: Draw a work line t. Pick a point on t; call it R. Put point of compass on A and pencil on B. This distance will be the radius of an arc. Place point of compass at R and, using radius from previous step, make an arc that intersects t at S. $\overline{RS}$ is the required segment.

To copy any angle, use Construction 2.

CONSTRUCTION 2

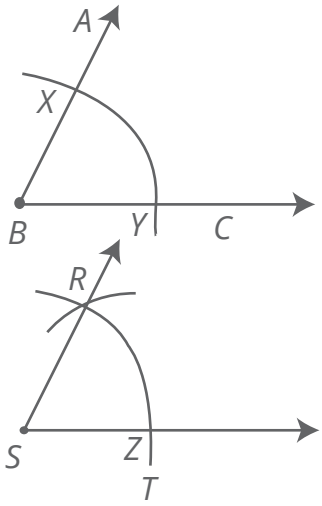
Given a segment, construct another angle with measure equal to the given angle.

Given: $\angle ABC$

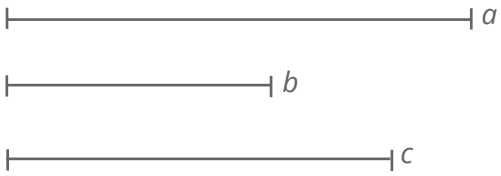
To construct: $\angle RST$ with measure equal to $\angle ABC$

Steps: Construct a ray $\overrightarrow{ST}$. With B as center and a convenient radius, construct an arc that intersects $\overrightarrow{BA}$ at X and $\overrightarrow{BC}$ at Y. With the same radius and center at S, construct an arc intersecting $\overrightarrow{ST}$ at Z. With center at Z and radius YX, construct an arc that intersects the arc from S. Call this point R.

$\angle RST$ is the required angle.



Use segments a, b, and c to complete the following constructions of segments with length equal to the indicated segments.



1.17 $a + b$

1.18 $a - b$

1.19 $2b + a$

1.20 $a + b + c$

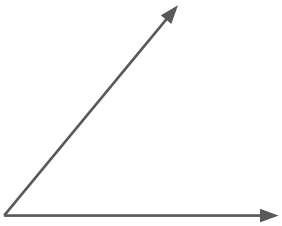
1.21 $a + 2b - c$

1.22 $a - b + c$

1.23 $a + c - b$

By construction, copy the angles given. Leave all construction arcs in your final figure.

1.24



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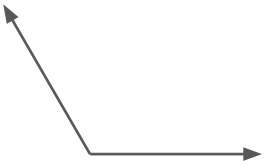

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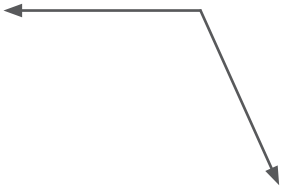

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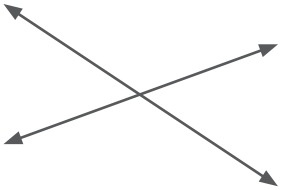

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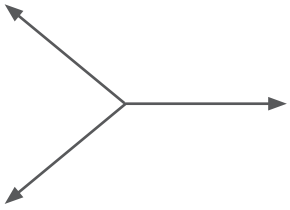

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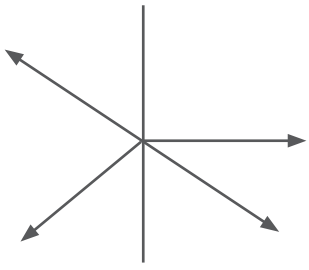
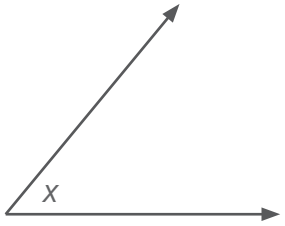
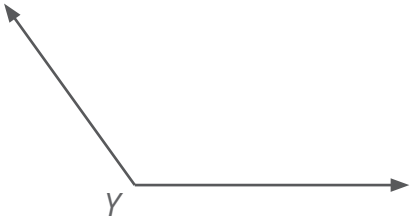


1.31 Construct an angle equal to 2*x*.



1.32 Construct a supplement to to $\angle Y$.



BISECTING

In many of our constructions, we want to divide a segment, an angle, or an arc into two equal parts. When this division has been accomplished, we have bisected the segment, angle, or arc.

To bisect a segment, follow Construction 3.

CONSTRUCTION 3

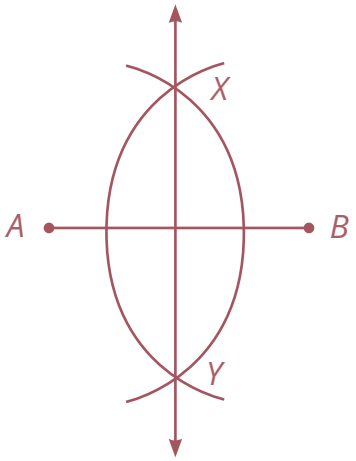
Given a segment, construct a bisector of the segment.

Given: $\overline{AB}$

To construct: a bisector of $\overline{AB}$

Steps: Take a radius greater than half of AB and construct intersecting arcs using A as center and then B as center.

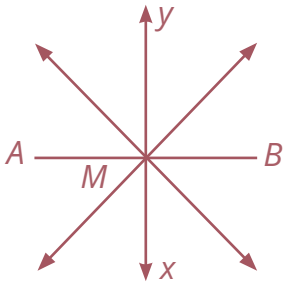
Let X and Y be the two points of intersection of the arcs. Draw $\overleftrightarrow{XY}$. $\overleftrightarrow{XY}$ is the required bisector of $\overline{AB}$.



This bisector of the segment is also perpendicular to the segment at its midpoint. It is sometimes called the *perpendicular bisector* of the segment.

The point where $\overleftrightarrow{XY}$ intersects $\overline{AB}$ is the *midpoint* of $\overline{AB}$.

Therefore, any segment, line, or ray passing through this point would also be a bisector of the segment.

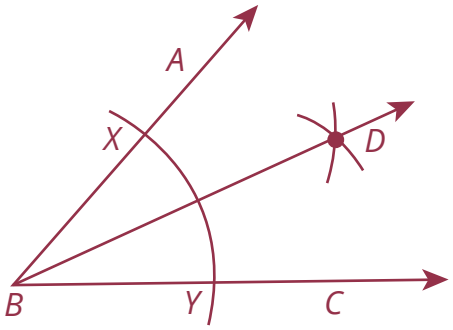


Bisecting an angle is a bit more involved. Follow Construction 4 to bisect an angle.

CONSTRUCTION 4

Given an angle, construct a ray that bisects the angle.

To construct: $\overrightarrow{BD}$ that bisects $\angle ABC$



Steps: Using a convenient radius and the vertex of the angle B as center, construct an arc that intersects both sides of the angle. Call these points X and Y as centers and a radius greater than half of XY, construct two arcs that intersect in the interior of $\angle ABC$. Call this point D.

Draw $\overrightarrow{BD}$. $\overrightarrow{BD}$ is the ray that bisects $\angle ABC$.

Make sure that the radius you use to make the arcs from X and Y is long enough. If it is not long enough, then the arcs will not intersect. Remember that we need two points to determine a line, segment, or ray.

Constructing the bisector of an arc is very similar to bisecting a segment. Construction 5 shows you how.

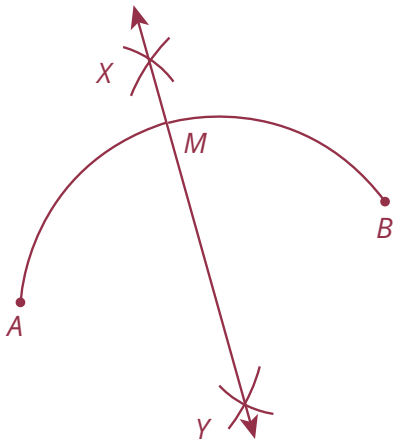
CONSTRUCTION 5

Given an arc, construct a line that bisects the given arc.

Given: $\overset{\frown}{AB}$

To construct: $\overleftrightarrow{XY}$ that bisects $\overset{\frown}{AB}$

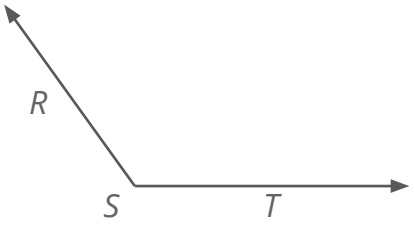
Steps: Using A and B as centers and a radius greater than half of $\overset{\frown}{AB}$, construct two arcs that intersect.Let X and Y be the points of intersection of the arcs. Draw $\overleftrightarrow{XY}$. $\overleftrightarrow{XY}$ is the desired line that bisects $\overset{\frown}{AB}$.



The point M where the bisector intersects the arc is called the *midpoint of the arc*. Any line, segment, or ray that passes through this midpoint is also a bisector of the arc.

Construct the bisector of the following figures.

1.33



1.34

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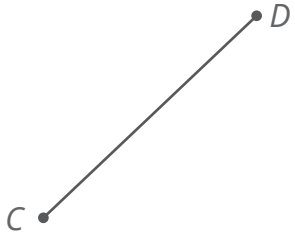

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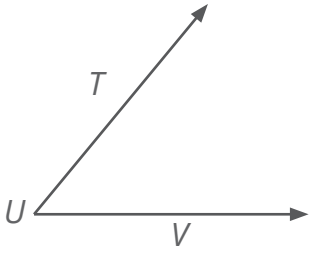


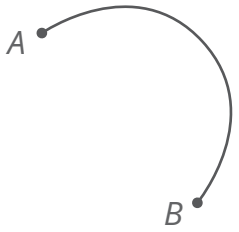
1.37

1.38

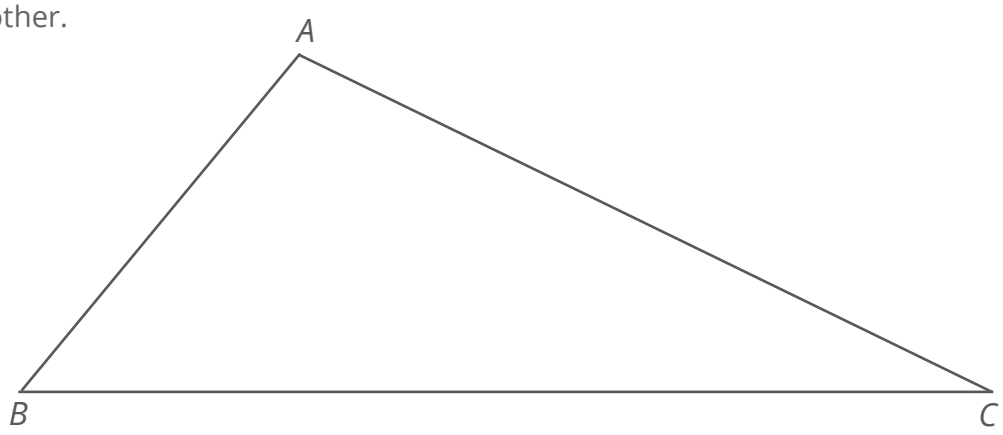
Complete the following activities.

1.39 Find the midpoint of $\overline{AB}$. Name it *R*.

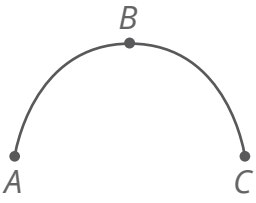
1.40 Find the midpoint of $\overset{\frown}{AB}$. Name it *S*.



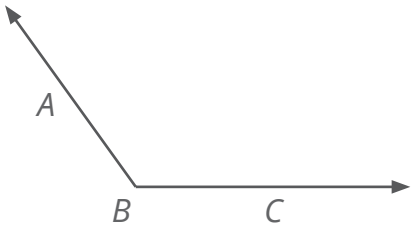
1.41 Find the angle bisector of each angle in this triangle. Extend the bisectors until they intersect each other.



1.42 Find the bisector of $\overset{\frown}{AB}$ and of $\overset{\frown}{BC}$. Extend the bisectors until they intersect. What do you think the point of intersection is in relation to the arc?

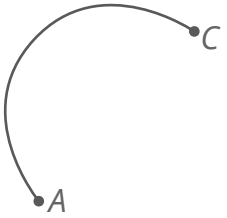


1.43 Divide $\angle ABC$ into four equal angles. (Hint: Use bisectors)



1.44 Divide $\overline{AB}$ into four equal segments.

1.45 Find the center of the circle that *AC* is a part of.

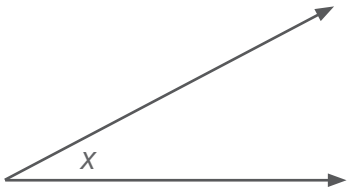


SELF TEST 1

Complete the following constructions. Leave all construction arcs on your paper (each construction, 5 points)**.**

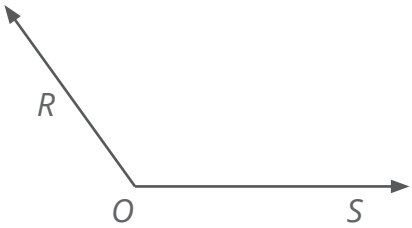
1.01 Construct segment $MN = 3a$.

1.02 Construct an angle, $\angle ABC = 2x$.

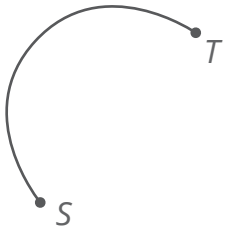


1.03 Bisect $\overline{AB}$ with $\overleftrightarrow{CD}$.

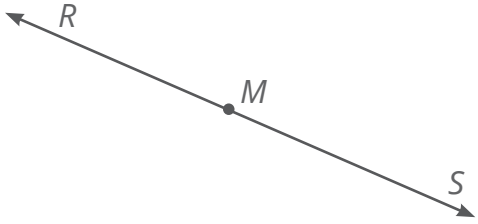
1.04 Bisect $\angle ROS$ with $\overrightarrow{OT}$.



1.05 Find the midpoint, M, of $\overset{\frown}{ST}$.

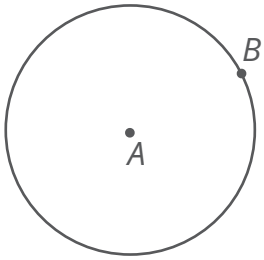


1.06 Construct $\overleftrightarrow{CT}$ perpendicular to $\overleftrightarrow{RS}$ at M.

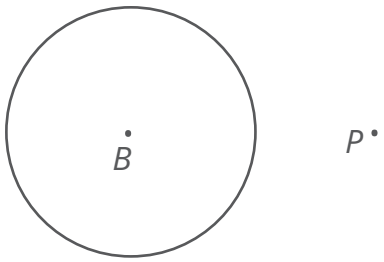


1.07 Construct $\overleftrightarrow{CT}$ perpendicular to $\overleftrightarrow{RS}$ at N.

1.08 Construct a tangent, t, to circle A at B.



1.09 Construct a tangent, $\overrightarrow{PX}$, to circle B.



1.010 Construct l parallel to m through P.

1.011 Divide $\overline{AB}$ into five equal parts.

1.012 Find x, the fourth proportional between r, s, and t.

_____ r

_____ s

_____ t

1.013 Find x, the geometric mean between c and d.

_____ c

_____ d

52/65 **SCORE** _____ **TEACHER** _____ _____
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MAT1007 – Apr '15 Printing

ISBN 978-0-86717-637-7