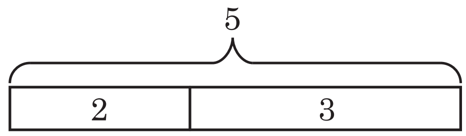
Chapter 9: Equations and Inequalities

Lesson	Objectives	Class Periods	Textbook & Workbook	Teacher's Guide Page	Additional Materials Needed
1	• Determine whether a given number makes an equation true.	1	TB: 28–31 WB: 19–21	38	
2	• Balance simple one-step equations and understand their properties. • Solve simple algebraic equations.	1	TB: 32–39	42	Balance scale, connecting cubes (2 colors), small stickers for labels
3	• Solve word problems using algebraic equations.	1	TB: 39–42 WB: 22–33	50	
4	• Consolidate and extend the material covered thus far.	1	TB: 42–43	54	
5	• Determine whether a given number makes an inequality true.	1	TB: 44–45 WB: 34	58	Balance scale, connecting cubes (2 colors), small stickers for labels
6	• Graph inequalities using a number line.	1	TB: 46–49	60	Graph paper, rulers
7	• Consolidate and extend the material covered thus far.	1	TB: 49–50 WB: 35–39	63	
8	• Summarize and reflect on important ideas learned in this chapter. • Use a spreadsheet to evaluate expressions and solve equations.	1	TB: 51–52	65	
9	• Apply the model method to solve complex word problems involving algebraic equations.	1	TB: 53–56	67	

Chapter 9: Equations and Inequalities

In elementary school, students learned how to solve equations where a $\square$ represents the unknown quantity based on part-whole relationships.

In additive situations:



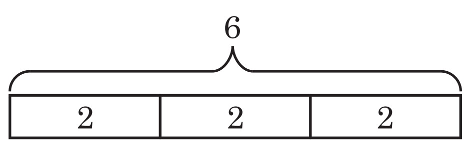
To find an unknown total, we add the parts.

$$2 + 3 = \square \longrightarrow \square = 2 + 3$$
$$\square - 3 = 2 \longrightarrow \square = 2 + 3$$

To find an unknown part, we subtract the known part from the total.

$$2 + \square = 5 \longrightarrow \square = 5 - 2$$
$$5 - \square = 2 \longrightarrow \square = 5 - 2$$

In multiplicative situations:



To find an unknown total, we multiply the factors.

$$3 \times 2 = \square \longrightarrow \square = 3 \times 2$$
$$\square \div 3 = 2 \longrightarrow \square = 2 \times 3$$
$$\square \div 2 = 3 \longrightarrow \square = 3 \times 2$$

To find the value of an unknown factor, we divide the total by the known factor.

$$2 \times \square = 6 \longrightarrow \square = 6 \div 2$$
$$\square \times 3 = 6 \longrightarrow \square = 6 \div 3$$
$$6 \div \square = 3 \longrightarrow \square = 6 \div 3$$
$$6 \div \square = 2 \longrightarrow \square = 6 \div 2$$

In this chapter, these ideas are extended to solving algebraic equations. Instead of $\square$, a variable represents the unknown quantity in the equations.

Chapter 9: Equations and Inequalities

Initially, students can solve algebraic equations by inspection. In the equation $x + 8 = 12$, students can know that $x = 4$ because they know that $4 + 8 = 12$. Students can then substitute 4 for x to see whether the value is the solution to the equation.

$$x + 8 = 12$$
$$4 + 8 = 12$$
$$12 = 12$$

An equation is a statement of equality between two expressions, thus the expressions on both sides of the equal sign are equivalent. When we perform the same operation to both sides of an equation, the expressions on each side of the equal sign remain equivalent.

$$x + 8 = 12 \qquad (x + 8) - 4 = 12 - 4 \qquad 2(x + 8) = 2 \times 12 \qquad (x + 8) \div 2 = 12 \div 2$$

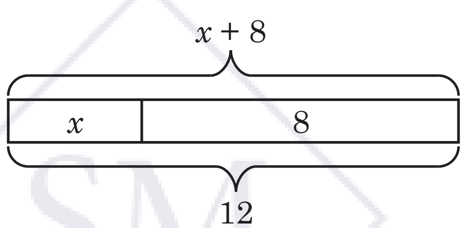
We can use this idea to solve equations. For example,

$$x + 8 = 12$$
$$x + 8 - 8 = 12 - 8$$
$$x = 4$$

The **solution** to an equation is the number that makes the equation true. It is important to encourage students to check their answers by substituting the solution for the variable in the original equation.

$$4 + 8 = 12$$
$$12 = 12$$

Bar models are useful to help students make the connection between arithmetic and algebra. With algebra, they use a letter instead of a question mark to indicate the unknown.



Objective:

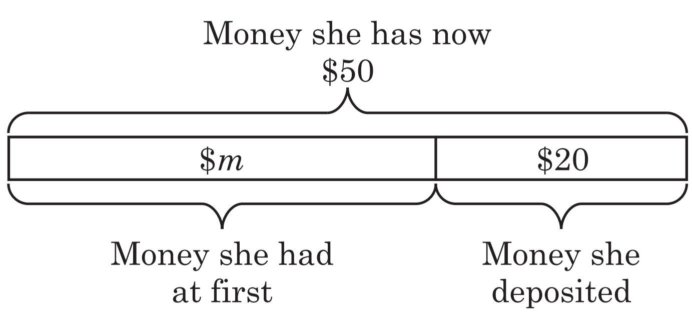
- Solve word problems using algebraic equations.

1. Introduction

Pose the following problem: Brianna had some money in her savings account. She deposited $20 more and now she has $50. How much money did she have at first?

Note: This is the same problem as Example 10, but with simplified numbers.

Tell students to let m = the money she had at first and ask them to draw a bar model to represent the situation.



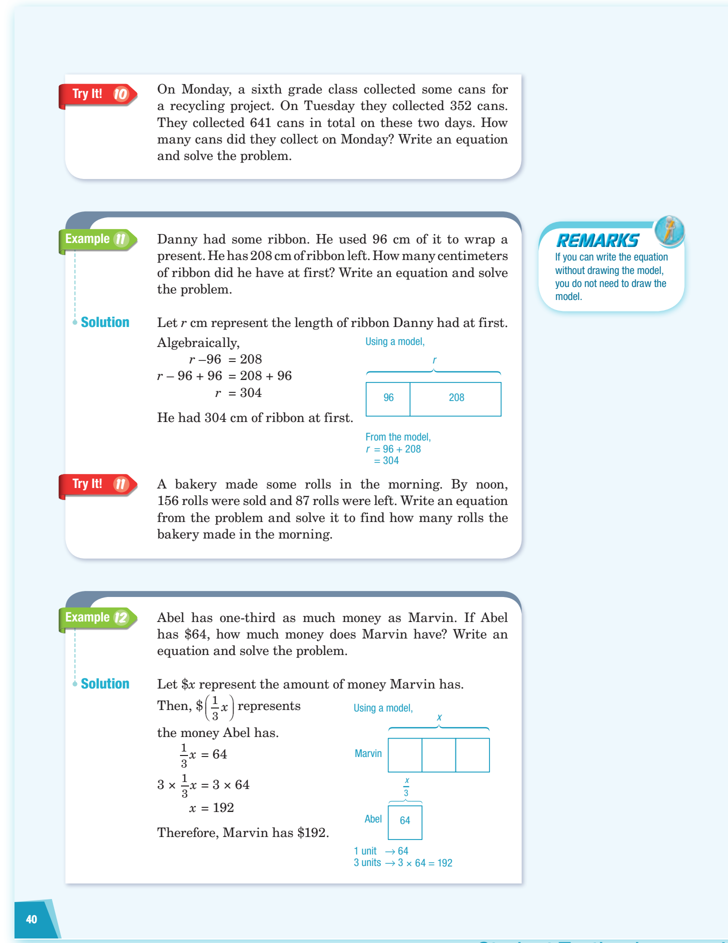
Ask students to write an equation to represent the problem ($m + 20 = 50$).

Pose the problem with the same numbers as Example 10 ($87 and $324), and ask them to represent the problem with an equation, then use the equation to solve the problem.

Have students share their methods for solving the problem, then discuss the method shown for Example 10 on page 39. Discuss REMARKS.

Have students check their answers by substituting $237 for m.

Note: Students who have learned bar models before should be able to see that to find the value of m, we simply subtract 20 from 50.



Try It! 10 On Monday, a sixth grade class collected some cans for a recycling project. On Tuesday they collected 352 cans. They collected 641 cans in total on these two days. How many cans did they collect on Monday? Write an equation and solve the problem.

Example 11 Danny had some ribbon. He used 96 cm of it to wrap a present. He has 208 cm of ribbon left. How many centimeters of ribbon did he have at first? Write an equation and solve the problem.

Solution Let r cm represent the length of ribbon Danny had at first.

Algebraically,

$r - 96 = 208$

$r - 96 + 96 = 208 + 96$

$r = 304$

He had 304 cm of ribbon at first.

Using a model, r: 96 | 208

From the model, $r = 96 + 208 = 304$

REMARKS If you can write the equation without drawing the model, you do not need to draw the model.

Try It! 11 A bakery made some rolls in the morning. By noon, 156 rolls were sold and 87 rolls were left. Write an equation from the problem and solve it to find how many rolls the bakery made in the morning.

Example 12 Abel has one-third as much money as Marvin. If Abel has $64, how much money does Marvin have? Write an equation and solve the problem.

Solution Let $$x$ represent the amount of money Marvin has.

Then, $\$\left(\frac{1}{3}x\right)$ represents the money Abel has.

$\frac{1}{3}x = 64$

$3 \times \frac{1}{3}x = 3 \times 64$

$x = 192$

Therefore, Marvin has $192.

Using a model, Marvin: x; Abel: $\frac{x}{3}$, 64

1 unit → 64

3 units → 3 × 64 = 192

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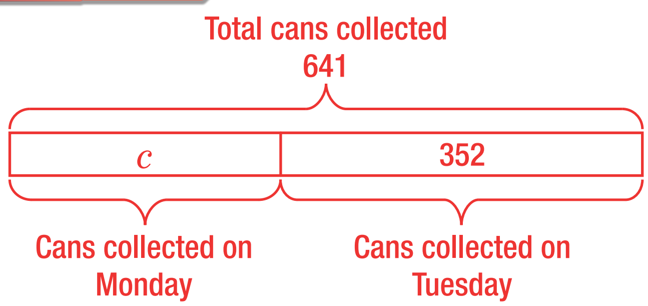
The emphasis here should not be on finding the answer, but on using an equation to model the problem situation. The bar model is helpful to see the equation visually.

Have students solve Try It! 10 by drawing a model and writing an equation. Ask them to share their methods and discuss.

Try It! 10 Answer



$c + 352 = 641$

$c + 352 - 352 = 641 - 352$

$c = 289$

They collected 289 cans on Monday.

Notes:

- Based on the model, students may also write $641 - 352 = c$ or $c = 641 - 352$. These equations are acceptable, but it is usually best for the equation to mirror the order of the problems. I.e., "She started with some cans (c). She collected 352 more cans (+ 352). Then she had a total of 641 cans (= 641)."
- Students can assign any variable for the unknown. Initially, it is helpful to assign the first letter of what we are trying to find. For example, "We are finding cans, so we can use c to represent the cans collected on Monday." Later, students can use x to represent any unknown quantity (see Examples 13 – 14).
- Encourage students to check the answer by substituting 289 for c in the original equation.

2. Application

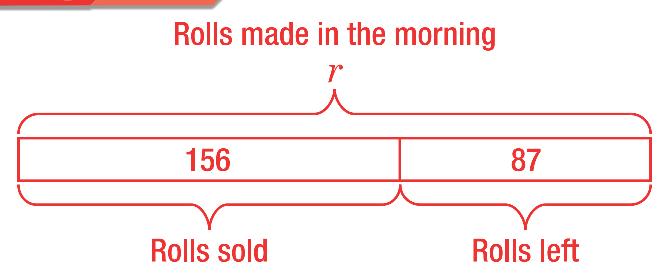
Have students study Examples 11 – 13 and do Try It! 11 – 13.

After students have solved the problems, have them share their equations and models. Relate the quantities in the models to the equations.

Notes:

- Encourage students who are having difficulty to draw a model first and then write an equation.
- Students who do not need to draw a model do not have to (see REMARKS).
- In Example 11, the total is unknown but we know both parts. Remind students that part + part = total. The model shows this clearly.

Try It! 11 Answer



$r - 156 = 87$

$r - 156 + 156 = 87 + 156$

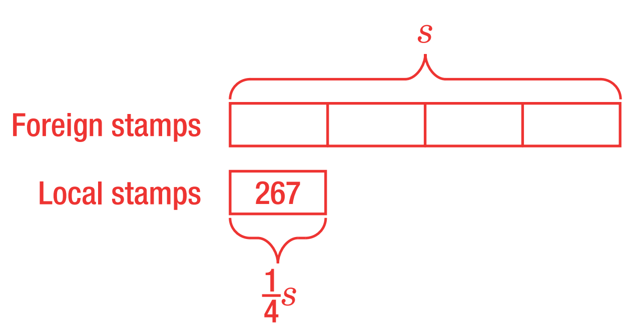
$r = 243$

243 rolls were made in the morning.

Notes:

- In Example 12, we are comparing Marvin's money to Abel's money. The value being compared to is the base, so Marvin's money is $1x = x$. Abel's money is $\frac{1}{3}$ of that or $\frac{1}{3}x$.
- For Try It! 12, students may write the equation $s = 267 \times 4$. This is fine, but help students see that we can express the number of local stamps in terms of the number of foreign stamps. The local stamps become $\frac{1}{4}s$.

Try It! 12 Answer



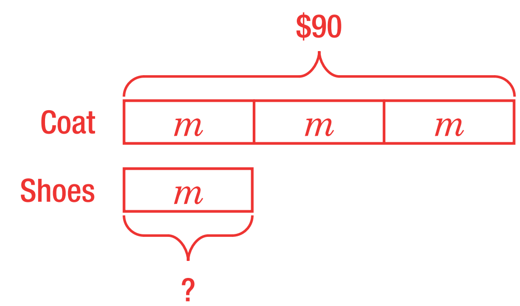
$\frac{1}{4}s = 267$

$\frac{1}{4}s \times 4 = 267 \times 4$

$s = 1{,}068$

He has 1,068 foreign stamps.

Try It! 13 Answer



$3m = 90$

$\frac{3m}{3} = \frac{90}{3}$

$m = 30$

He spent $30 on the shoes.

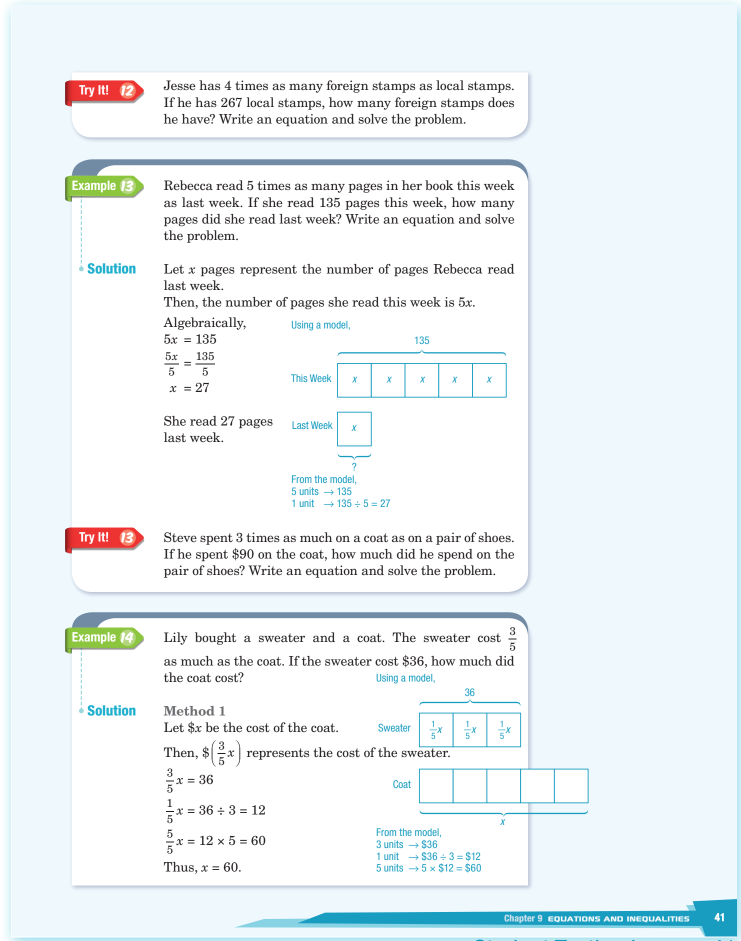
3. Extension

Have students study Example 14 and do Try It! 14. Then, have them share their solutions.

Notes:

- We are comparing the price of the sweater to the price of the coat, so the coat is the base ($1x$). Since the sweater costs $\frac{3}{5}$ as much, we can represent the price of the sweater as $\frac{3}{5}x$.
- From the bar model, we can see that the coat is 5 units. Since all 5 units are x, one unit is $\frac{x}{5}$ or $\frac{1}{5}x$.
- Students learned both methods in Example 9. Method 2 involves fewer steps.

★ **Workbook: Page 22**



Try It! 12 Jesse has 4 times as many foreign stamps as local stamps. If he has 267 local stamps, how many foreign stamps does he have? Write an equation and solve the problem.

Example 13 Rebecca read 5 times as many pages in her book this week as last week. If she read 135 pages this week, how many pages did she read last week? Write an equation and solve the problem.

Solution

Let x pages represent the number of pages Rebecca read last week.
Then, the number of pages she read this week is $5x$.

Algebraically,

$5x = 135$

$\frac{5x}{5} = \frac{135}{5}$

$x = 27$

She read 27 pages last week.

Using a model,

This Week: x x x x x (135)

Last Week: x (?)

From the model,
5 units → 135
1 unit → 135 ÷ 5 = 27

Try It! 13 Steve spent 3 times as much on a coat as on a pair of shoes. If he spent $90 on the coat, how much did he spend on the pair of shoes? Write an equation and solve the problem.

Example 14 Lily bought a sweater and a coat. The sweater cost $\frac{3}{5}$ as much as the coat. If the sweater cost $36, how much did the coat cost?

Solution

Method 1

Let $$x$ be the cost of the coat.

Then, $$\left(\frac{3}{5}x\right)$ represents the cost of the sweater.

$\frac{3}{5}x = 36$

$\frac{1}{5}x = 36 \div 3 = 12$

$\frac{5}{5}x = 12 \times 5 = 60$

Thus, $x = 60$.

Using a model,

Sweater: $\frac{1}{5}x$ $\frac{1}{5}x$ $\frac{1}{5}x$ (36)

Coat: (x)

From the model,
3 units → $36
1 unit → $36 ÷ 3 = $12
5 units → 5 × $12 = $60

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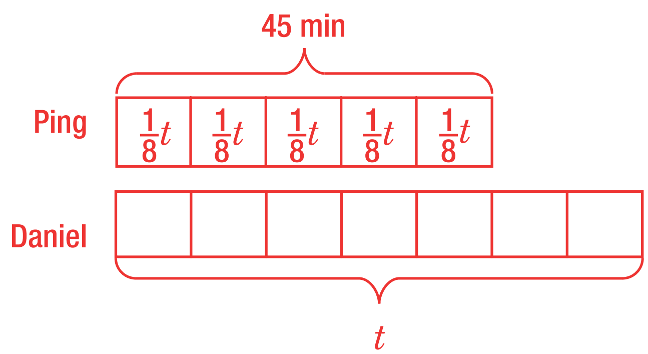
Try It! 14 Answer



$\frac{5}{8}t = 45$

$\frac{5}{8}t \times \frac{8}{5} = 45 \times \frac{8}{5}$

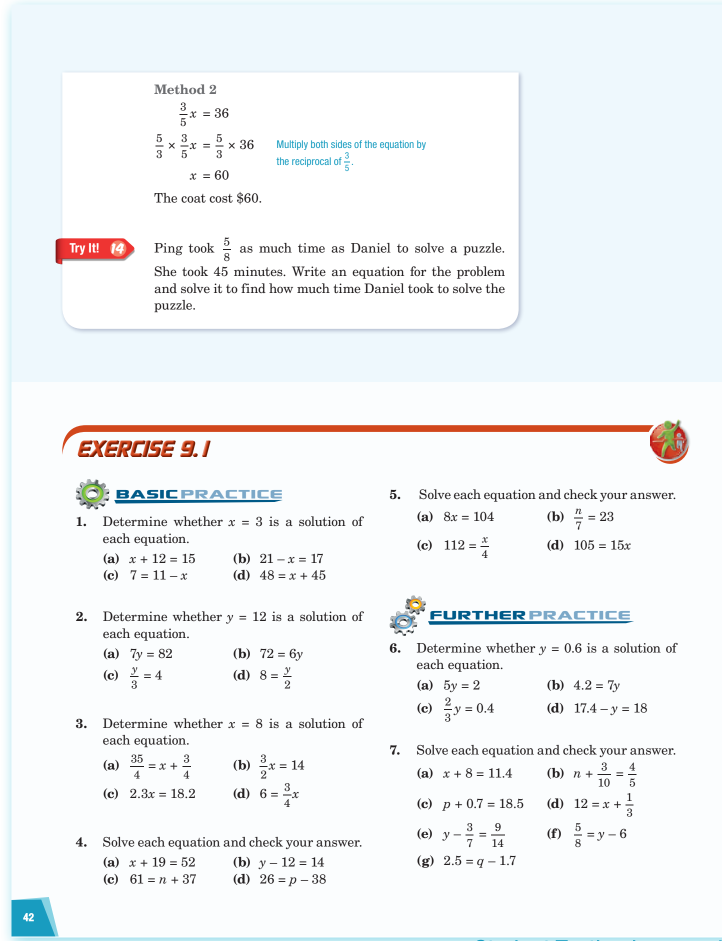
$t = 72$

Daniel took 72 minutes.

4. Conclusion

Summarize the main points of the lesson.

- We can represent and solve problem situations with algebraic equations.
- To think about the equation needed to solve a problem, it is often helpful to draw a bar model. When we learned bar models, we used a question mark to indicate what we were trying to find out (the unknown). With algebra, we use a variable instead of a question mark.



Method 2

$\frac{3}{5}x = 36$

$\frac{5}{3} \times \frac{3}{5}x = \frac{5}{3} \times 36$ Multiply both sides of the equation by the reciprocal of $\frac{3}{5}$.

$x = 60$

The coat cost $60.

Try It! 14 Ping took $\frac{5}{8}$ as much time as Daniel to solve a puzzle. She took 45 minutes. Write an equation for the problem and solve it to find how much time Daniel took to solve the puzzle.

EXERCISE 9.1

BASIC PRACTICE

1. Determine whether $x = 3$ is a solution of each equation.
   (a) $x + 12 = 15$ (b) $21 - x = 17$
   (c) $7 = 11 - x$ (d) $48 = x + 45$

2. Determine whether $y = 12$ is a solution of each equation.
   (a) $7y = 82$ (b) $72 = 6y$
   (c) $\frac{y}{3} = 4$ (d) $8 = \frac{y}{2}$

3. Determine whether $x = 8$ is a solution of each equation.
   (a) $\frac{35}{4} = x + \frac{3}{4}$ (b) $\frac{3}{2}x = 14$
   (c) $2.3x = 18.2$ (d) $6 = \frac{3}{4}x$

4. Solve each equation and check your answer.
   (a) $x + 19 = 52$ (b) $y - 12 = 14$
   (c) $61 = n + 37$ (d) $26 = p - 38$

5. Solve each equation and check your answer.
   (a) $8x = 104$ (b) $\frac{n}{7} = 23$
   (c) $112 = \frac{x}{4}$ (d) $105 = 15x$

FURTHER PRACTICE

6. Determine whether $y = 0.6$ is a solution of each equation.
   (a) $5y = 2$ (b) $4.2 = 7y$
   (c) $\frac{2}{3}y = 0.4$ (d) $17.4 - y = 18$

7. Solve each equation and check your answer.
   (a) $x + 8 = 11.4$ (b) $n + \frac{3}{10} = \frac{4}{5}$
   (c) $p + 0.7 = 18.5$ (d) $12 = x + \frac{1}{3}$
   (e) $y - \frac{3}{7} = \frac{9}{14}$ (f) $\frac{5}{8} = y - 6$
   (g) $2.5 = q - 1.7$

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Objective:

- Consolidate and extend the material covered thus far.

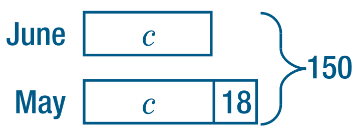
Have students work together with a partner or in groups. Students should try to solve the problems by themselves first, then compare solutions with their partner or group. If they are confused, they can discuss together.

Observe students carefully as they work on the problems. Give help as needed individually or in small groups.

BASIC PRACTICE

1. (a) Yes (b) No
   (c) No (d) Yes

14. Note: This is a two-step problem. Students did not see two-step equations in previous lessons, so it is important to discuss it here. Help them see that first we subtract 18 from each side of the equation, and then divide each side of the equation by 2.

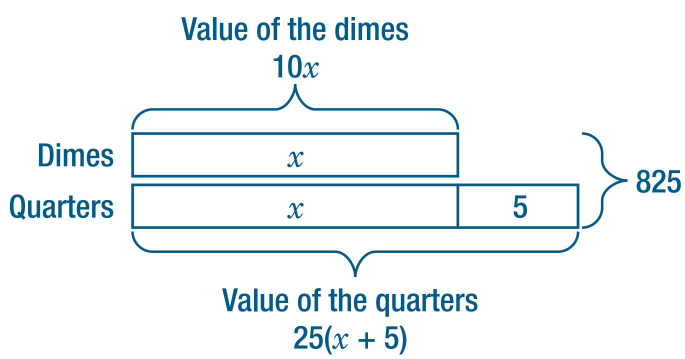


2 units $\longrightarrow$ 150 – 18 = 132
1 unit $\longrightarrow$ 132 ÷ 2 = 66

$c + (c + 18) = 150$
$2c + 18 = 150$
$2c = 132$
$c = 66$

66 cakes

15.



Let x = Number of dimes
$x + 5$ = Number of quarters
$10x$ = Value of the dimes (in cents)
$25(x + 5)$ = Value of quarters (in cents)
825 = Total amount of money (in cents)

$10x + 25(x + 5) = 825$
$10x + 25x + 125 = 825$
$35x + 125 = 825$
$35x = 700$
$x = 20$

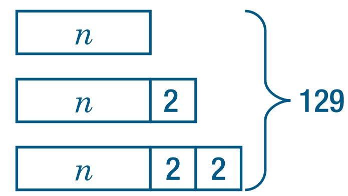
There are 20 dimes.

Note: The equation could be written in dollars as $0.10x + 0.25(x + 5) = 8.25$, but it is easier to eliminate the decimals and just think about it with whole numbers using cents. It is the same as multiplying each side of the equation by 100.

$100(0.10x + 0.25(x + 5)) = 100(8.25)$
$10x + 25(x + 5) = 825$

BRAIN WORKS

16.



Note:
All odd numbers are 2 apart from each other. E.g., 1 + 2 = 3, 3 + 2 = 5, 5 + 2 = 7, etc. so we can represent the numbers as n, $n + 2$ and $n + 4$.

This problem can be solved using the unitary method as:
3 units $\longrightarrow$ 129 – 6 = 123
1 unit $\longrightarrow$ 123 ÷ 3 = 41
So the numbers are 41, 43, and 45.
41 × 43 × 45 = 79,335

$n + (n + 2) + (n + 4) = 129$
$3n + 6 = 129$
$3n = 123$
$n = 41$
$n + 2 = 43$
$n + 4 = 45$
41 × 43 × 45 = 79,335

Be advised that the answer in the back of the textbook on page 249 is listed incorrectly as 79,355.

17. (a) $8x + 3x = 11x$
(b) $11y - 4y = 7y$

Objective: Find the mean absolute deviation of a set of data.

1. Introduction

Ask students to find the mean and make dot plots for each of the following sets of data:

Data Set A: 3, 3, 5, 5

Data Set B: 1, 1, 6, 8

Note: These are the same sets of data given on page 226.

Ask, "For which data set are the data values closer to the mean?" (Data set A)

Ask, "How far is each data value in Set A from the mean?"

- $3 \longrightarrow 1$ unit
  $3 \longrightarrow 1$ unit
  $5 \longrightarrow 1$ unit
  $5 \longrightarrow 1$ unit

Ask, "What is the mean distance of each data value in Set A from the mean?"

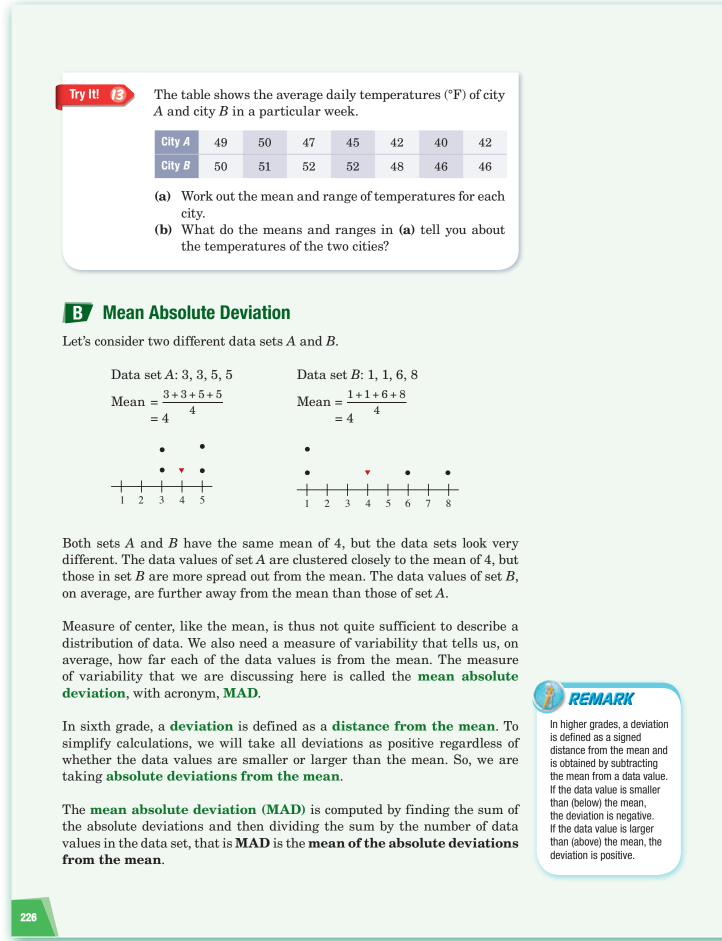
- $\frac{1+1+1+1}{4} = 1$

Ask, "How far is each data value in Set B from the mean?"

- $1 \longrightarrow 3$ units
  $1 \longrightarrow 3$ units
  $6 \longrightarrow 2$ units
  $8 \longrightarrow 4$ units

Ask, "What is the mean distance of each data value in Set A from the mean?"

- $\frac{3+3+2+4}{4} = 3$



Try It! 13 The table shows the average daily temperatures (°F) of city A and city B in a particular week.

City A	49	50	47	45	42	40	42
City B	50	51	52	52	48	46	46

(a) Work out the mean and range of temperatures for each city.

(b) What do the means and ranges in **(a)** tell you about the temperatures of the two cities?

B Mean Absolute Deviation

Let's consider two different data sets A and B.

Data set A: 3, 3, 5, 5

Mean $= \frac{3+3+5+5}{4}$
$= 4$

Data set B: 1, 1, 6, 8

Mean $= \frac{1+1+6+8}{4}$
$= 4$

Both sets A and B have the same mean of 4, but the data sets look very different. The data values of set A are clustered closely to the mean of 4, but those in set B are more spread out from the mean. The data values of set B, on average, are further away from the mean than those of set A.

Measure of center, like the mean, is thus not quite sufficient to describe a distribution of data. We also need a measure of variability that tells us, on average, how far each of the data values is from the mean. The measure of variability that we are discussing here is called the **mean absolute deviation**, with acronym, **MAD**.

In sixth grade, a **deviation** is defined as a **distance from the mean**. To simplify calculations, we will take all deviations as positive regardless of whether the data values are smaller or larger than the mean. So, we are taking **absolute deviations from the mean**.

The **mean absolute deviation (MAD)** is computed by finding the sum of the absolute deviations and then dividing the sum by the number of data values in the data set, that is **MAD** is the **mean of the absolute deviations from the mean**.

REMARK
In higher grades, a deviation is defined as a signed distance from the mean and is obtained by subtracting the mean from a data value. If the data value is smaller than (below) the mean, the deviation is negative. If the data value is larger than (above) the mean, the deviation is positive.

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2. Development

Ask, "What does the average distance of the data values from the mean tell us about the distribution of the data?" Possible responses:

- On average, the values in Set A are 1 away from the mean. On average, the values in Set B are 3 away from the mean.
- In Set A, the values are generally closer to the mean. In Set B the values are generally further away from the mean.
- The values in Set A are spread out more than the values in Set B.
- The greater the average distance from the mean, the more spread out the values are. The smaller the average distance from the mean, the less spread out the values are.

Read and discuss page 226.

3. Application

Use the top of page 227 to show students how to find the sum of the absolute deviations for each set of data using absolute value.

Note: Focus students on the diagrams with the arrows at the top of page 227. The number line can be used to count the absolute deviation of a data value from the mean. This is the same as finding the absolute value of the difference between the data value and the mean.

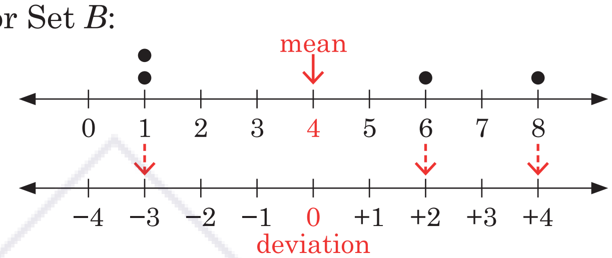
Discuss the procedure for finding Mean Absolute Deviation (MAD).

4. Extension

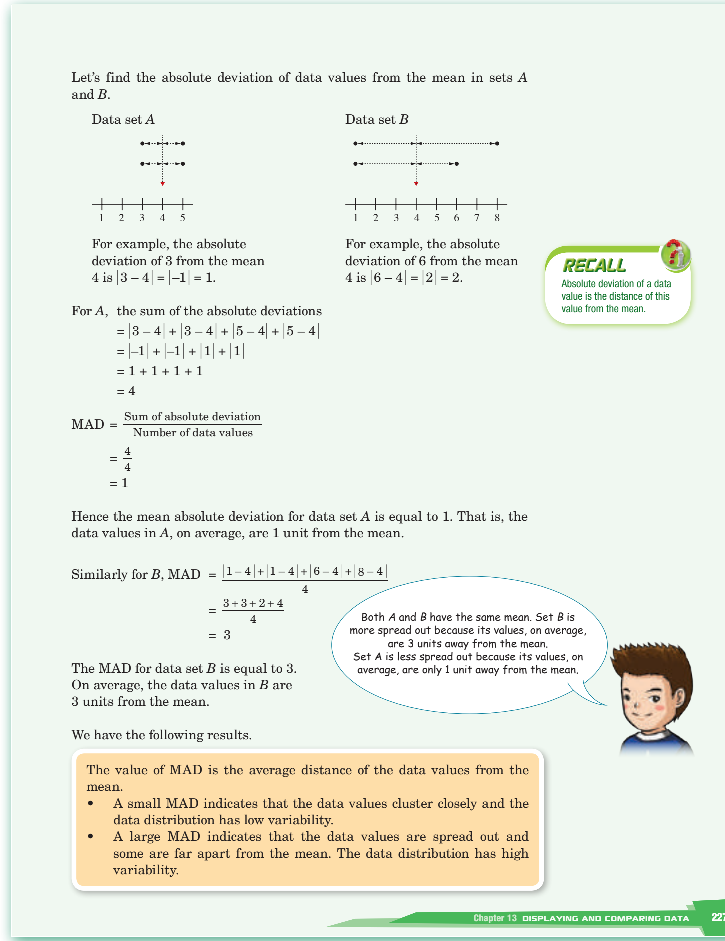
Have students discuss what the boy is saying with a partner or in groups. Read and discuss the summary box at the bottom of page 227.

Notes:

- If we consider the mean as 0, the numbers to the left are considered negative and the numbers to the right positive. For example, for Set *B*:



  The deviation of the data point 1 is −3, which means it deviates 3 units from the mean in the negative direction. Its absolute deviation is $|-3| = 3$. The absolute deviation is the unsigned distance of the data value from the mean. (See RECALL.)
- A small MAD indicates low variability, which means that the values are close to the middle (i.e., close to the mean). A large MAD indicates high variability, which means that the values are more spread out (i.e., farther away from the mean).



Let's find the absolute deviation of data values from the mean in sets *A* and *B*.

Data set *A*

Data set *B*

For example, the absolute deviation of 3 from the mean 4 is $|3 - 4| = |-1| = 1$.

For example, the absolute deviation of 6 from the mean 4 is $|6 - 4| = |2| = 2$.

RECALL
Absolute deviation of a data value is the distance of this value from the mean.

For *A*, the sum of the absolute deviations

$$= |3-4| + |3-4| + |5-4| + |5-4|$$
$$= |-1| + |-1| + |1| + |1|$$
$$= 1 + 1 + 1 + 1$$
$$= 4$$

$$\text{MAD} = \frac{\text{Sum of absolute deviation}}{\text{Number of data values}} = \frac{4}{4} = 1$$

Hence the mean absolute deviation for data set *A* is equal to 1. That is, the data values in *A*, on average, are 1 unit from the mean.

Similarly for *B*, $\text{MAD} = \frac{|1-4|+|1-4|+|6-4|+|8-4|}{4} = \frac{3+3+2+4}{4} = 3$

The MAD for data set *B* is equal to 3. On average, the data values in *B* are 3 units from the mean.

Both *A* and *B* have the same mean. Set *B* is more spread out because its values, on average, are 3 units away from the mean. Set *A* is less spread out because its values, on average, are only 1 unit away from the mean.

We have the following results.

The value of MAD is the average distance of the data values from the mean.
- A small MAD indicates that the data values cluster closely and the data distribution has low variability.
- A large MAD indicates that the data values are spread out and some are far apart from the mean. The data distribution has high variability.

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- Range is a limited measure of variability because it only tells us the difference between the largest and smallest values. If most the data values are clustered around the mean but one data point is farther away from the mean, the data set may have a large range but low variability. Thus, MAD gives us more accurate information about the variability of the data.

5. Conclusion

Summarize the main points of the lesson.

- The Mean Absolute Deviation (MAD) is the mean distance of the data values from the mean.
- MAD tells us about the variability in the data (i.e., how spread out or close together the data values are).

Objectives:

- Solve real-life problems involving Mean Absolute Deviation.
- Determine whether the mean is a good indicator of the typical values in a data set.

1. Introduction

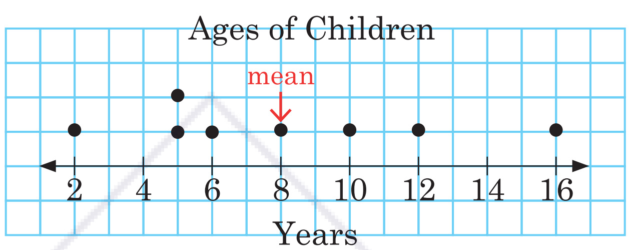
Remind students of the procedure for finding the Mean Absolute Deviation for a set of data.

Have students solve Example 14 (cover the answers).

Notes:
If students are having difficulty finding the absolute deviation of the values from the mean, encourage them to use a number line. (See REMARK.)

Have students share and discuss their solutions. Discuss the solution given on page 228.

Have students make a dot plot for the ages of the children and label the mean.

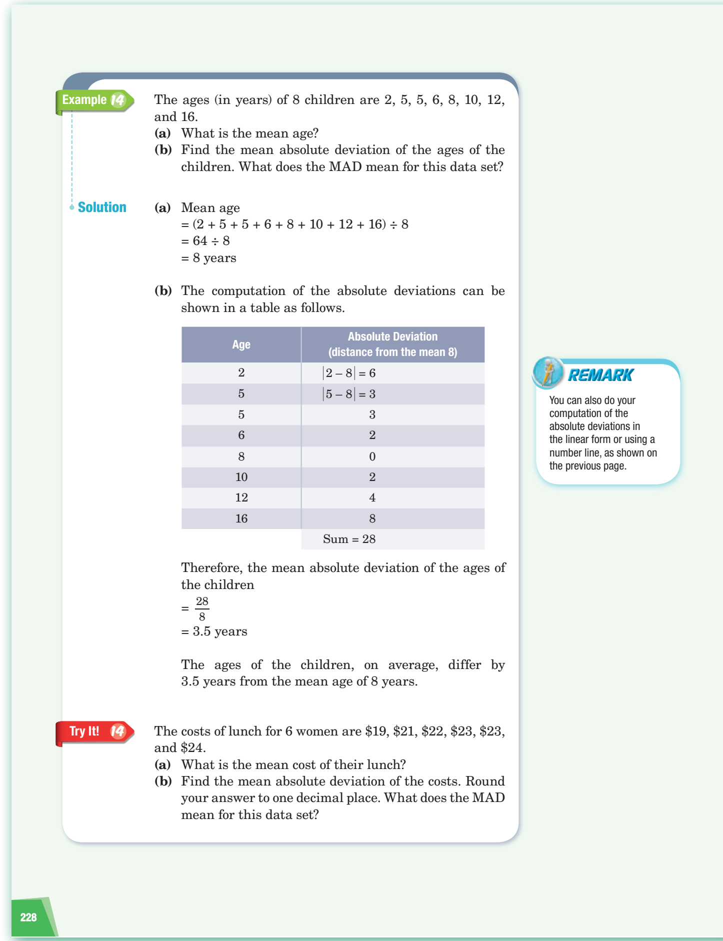


Ask, "What would the dot plot look like if there were a higher Mean Absolute Deviation?"

Possible responses:

- It would be more spread apart.
- There would be fewer values that are close to 8.
- There would more older or younger people.

Ask, "What would the dot plot look like if there were a lower Mean Absolute Deviation?"



Example 14 The ages (in years) of 8 children are 2, 5, 5, 6, 8, 10, 12, and 16.
(a) What is the mean age?
(b) Find the mean absolute deviation of the ages of the children. What does the MAD mean for this data set?

Solution (a) Mean age
= (2 + 5 + 5 + 6 + 8 + 10 + 12 + 16) ÷ 8
= 64 ÷ 8
= 8 years

(b) The computation of the absolute deviations can be shown in a table as follows.

Age	Absolute Deviation (distance from the mean 8)
2	$\lvert 2 - 8 \rvert = 6$
5	$\lvert 5 - 8 \rvert = 3$
5	3
6	2
8	0
10	2
12	4
16	8
	Sum = 28

Therefore, the mean absolute deviation of the ages of the children
$= \frac{28}{8}$
= 3.5 years

The ages of the children, on average, differ by 3.5 years from the mean age of 8 years.

REMARK
You can also do your computation of the absolute deviations in the linear form or using a number line, as shown on the previous page.

Try It! 14 The costs of lunch for 6 women are $19, $21, $22, $23, $23, and $24.
(a) What is the mean cost of their lunch?
(b) Find the mean absolute deviation of the costs. Round your answer to one decimal place. What does the MAD mean for this data set?

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Possible responses:

- The values would be more clustered in the middle.
- There would be more values that are close to 8.
- There would fewer older or younger people.

2. Development

Have students do Try It! 14 and share and discuss their solutions.

Try It! 14 Answers

(a) Mean cost of lunch = $(19 + 21 + 22 + 23 + 23 + 23 + 24) ÷ 6 = $132 ÷ 6 = $22

(b) Sum of absolute deviations

= |19 – 22| + |21 – 22| + |22 – 22| + |23 – 22| + |23 – 22| + |24 – 22|

= 3 + 1 + 0 + 1 + 1 + 2

= 8

MAD = $\frac{8}{6}$ ≈ 1.3 ≈ $1.30

On average, the cost of lunch differs from the mean of $22 by $1.30.

3. Application

Have students draw a dot plot and find the mean for each set of data in Example 15 (cover the answer).

Have students answer questions (a) – (d) and discuss their answers with partners or in groups. Then, ask them to share their conclusions.

Read and discuss the top of page 229.

Discuss the solutions given on page 229 – 230.

How are mean and variability related in a data distribution? How does variability determine if the mean is an accurate indicator of a typical value for a data distribution?

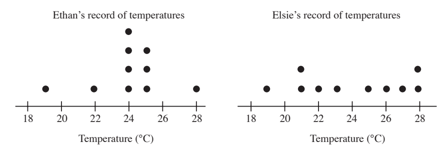
When the mean is used to describe a typical value for a data distribution and the data points are clustered closely around the mean, we say that the mean is a good indicator of a typical value in the data set. This means that if there is little/low variability in a data distribution, the mean is a good indicator of a typical value. The greater the variability, the less precise the mean is as an indicator of a typical value in the data set.

Example 15 Ethan and Elsie recorded the daily temperatures (in °C) over 10 days in May. The distribution of temperatures is shown below.

	Temperatures (in °C)									
Ethan's data	19	22	24	24	24	24	25	25	25	28
Elsie's data	19	21	21	22	23	25	26	27	28	28

(a) What is the mean temperature in Ethan's and Elsie's data?
(b) Without doing any calculations, for which distribution do you think the mean would give a better indicator of a typical value?
(c) Calculate the MAD of Ethan's and Elsie's data.
(d) Justify your answer in **(b)**.

Solution It is very helpful to draw a dot plot to represent a distribution as the dot plot shows a good visual image of the data and their spread.



(a) The mean temperature in Ethan's data
= (19 + 22 + 24 + 24 + 24 + 24 + 25 + 25 + 25 + 28) ÷ 10
= 240 ÷ 10
= 24 °C

The mean temperature in Elsie's data
= (19 + 21 + 21 + 22 + 23 + 25 + 26 + 27 + 28 + 28) ÷ 10
= 240 ÷ 10
= 24 °C

Mean temperature = $\frac{\text{Sum of temperatures}}{\text{Number of days}}$

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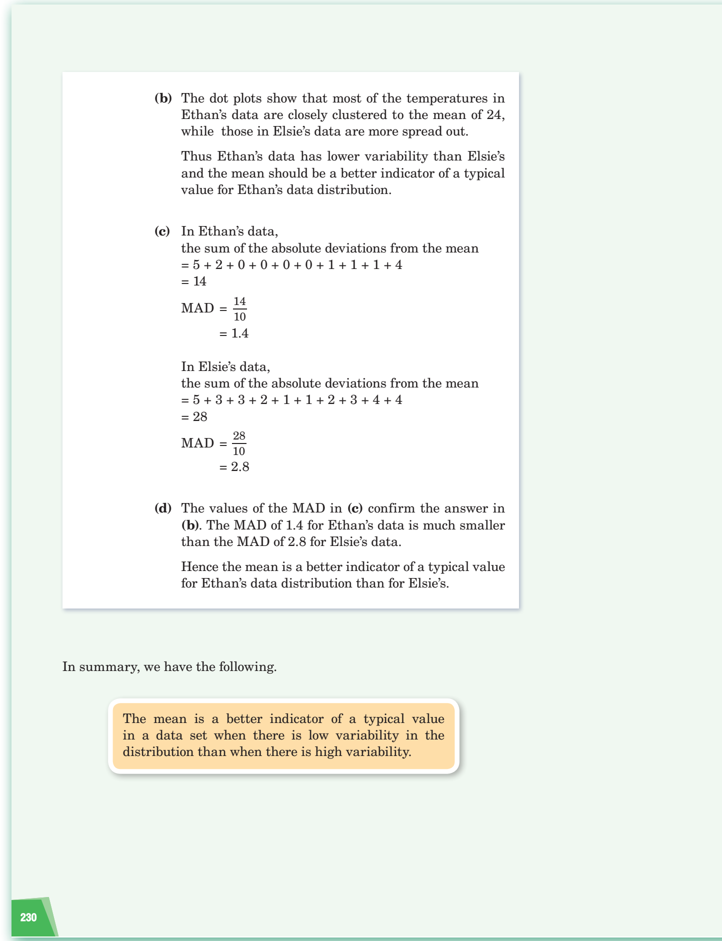
Summarize this part of the lesson by discussing the box at the bottom of page 230.

Note: Just because two sets of data have a similar mean it does not mean that the sets of data are similar. Mean Absolute Deviation helps us to compare two sets of data with similar means by looking at the variability within each data set.

4. Extension

Have students do Try It! 15 and share and discuss their solutions.



(b) The dot plots show that most of the temperatures in Ethan's data are closely clustered to the mean of 24, while those in Elsie's data are more spread out.

Thus Ethan's data has lower variability than Elsie's and the mean should be a better indicator of a typical value for Ethan's data distribution.

(c) In Ethan's data,
the sum of the absolute deviations from the mean
= 5 + 2 + 0 + 0 + 0 + 0 + 1 + 1 + 1 + 4
= 14

$$\text{MAD} = \frac{14}{10} = 1.4$$

In Elsie's data,
the sum of the absolute deviations from the mean
= 5 + 3 + 3 + 2 + 1 + 1 + 2 + 3 + 4 + 4
= 28

$$\text{MAD} = \frac{28}{10} = 2.8$$

(d) The values of the MAD in **(c)** confirm the answer in **(b)**. The MAD of 1.4 for Ethan's data is much smaller than the MAD of 2.8 for Elsie's data.

Hence the mean is a better indicator of a typical value for Ethan's data distribution than for Elsie's.

In summary, we have the following.

The mean is a better indicator of a typical value in a data set when there is low variability in the distribution than when there is high variability.

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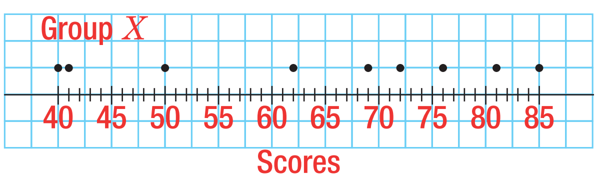
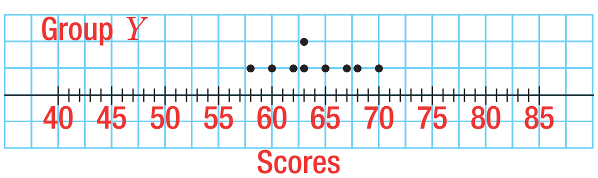
Try It! 15 Answers

(a)





(b) Mean for group $X = (40 + 41 + 50 + 62 + 69 + 72 + 76 + 81 + 85) \div 9 = 576 \div 9 = 64$

MAD for group $X =$

$$= \frac{|40-64| + |41-64| + |50-64| + |62-64| + |69-64| + |72-64| + |76-64| + |81-64| + |85-64|}{9}$$

$$= \frac{24+23+14+2+5+8+12+17+21}{9} = \frac{126}{9} = 14$$

Mean for group Y = (58 + 60 + 62 + 63 + 63 + 65 + 67 + 68 + 70) ÷ 9 = 576 ÷ 9 = 64

MAD for group Y =

$$= \frac{|58-64| + |60-64| + |62-64| + |63-64| + |63-64| + |65-64| + |67-64| + |68-64| + |70-64|}{9}$$

$$= \frac{6+4+2+1+1+1+3+4+6}{9} = \frac{28}{9} \approx 3.1$$

(c) Group Y has a smaller MAD so there is less variability in the data. The data values are clustered around the mean more closely than for Group Y so the mean is a better indicator of a typical value than it is for Group X.