Chapter 5: Ratios

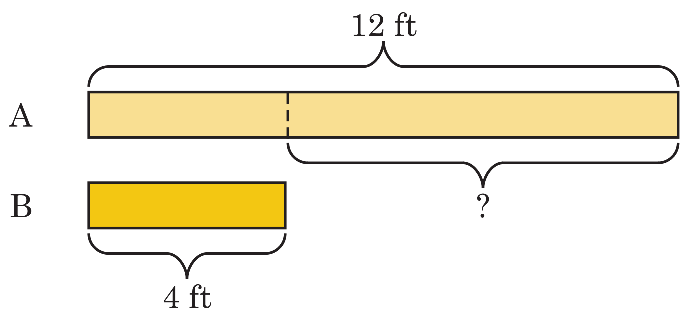
Lesson	Objectives	Class Periods	Textbook & Workbook	Teacher's Guide Page	Additional Materials Needed
1	• Compare quantities using ratios. • Use ratio language to describe a relationship between two quantities.	1	TB: 131–136 WB:110–113	192	
2	• Express ratios using terms and fractions. • Find equivalent ratios and explore their properties. • Simplify ratios.	1	TB: 137–141	198	Cubes of two different colors
3	• Find the missing term in a ratio. • Apply ratio to real-world situations.	1	TB: 141–143 WB:114–120	202	
4	• Consolidate and extend the material from the previous section.	1	TB: 144–145	207	
5	• Relate ratios and fractions. • Apply ratio relationships to solve real world problems.	1	TB: 145–152 WB:121–127	209	Rectangular paper strips of equal length
6	• Consolidate and extend the material from the previous section.	1	TB: 153–154	217	
7	• Summarize, reflect on, and extend important ideas learned in this chapter.	1	TB: 155–156	222	
8	• Apply the model method to solve complex problems involving ratios.	1	TB: 157–163	223	

In **Dimensions Math® 5**, students learned how to use ratio to compare quantities with the same units. This chapter extends this learning with more complex situations and problems.

Ratio and Its Value

Suppose we have two ribbons. Ribbon A is 12 ft long and Ribbon B is 4 ft long. There are two ways that we can compare the lengths of the ribbons.

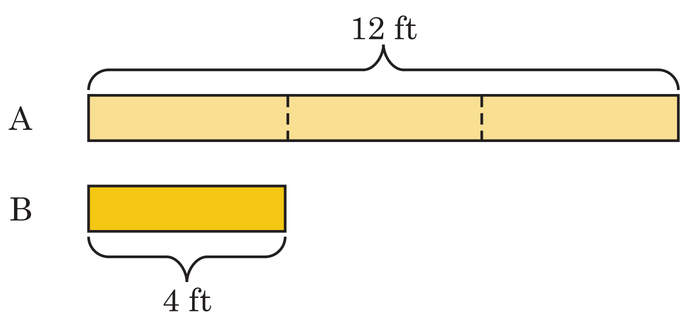
We can use subtraction (additive comparison) to find the difference between the lengths of the ribbons.



$12 - 4 = 8$

Ribbon A is 8 ft longer than Ribbon B.

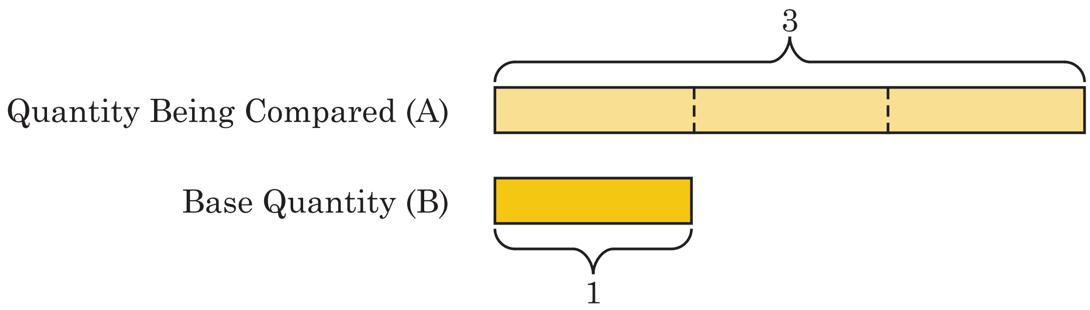
We can also use division (multiplicative comparison) to find how many times the length of one ribbon is to the other ribbon. **Ratio** is a multiplicative comparison (i.e., a comparison of two quantities using division).



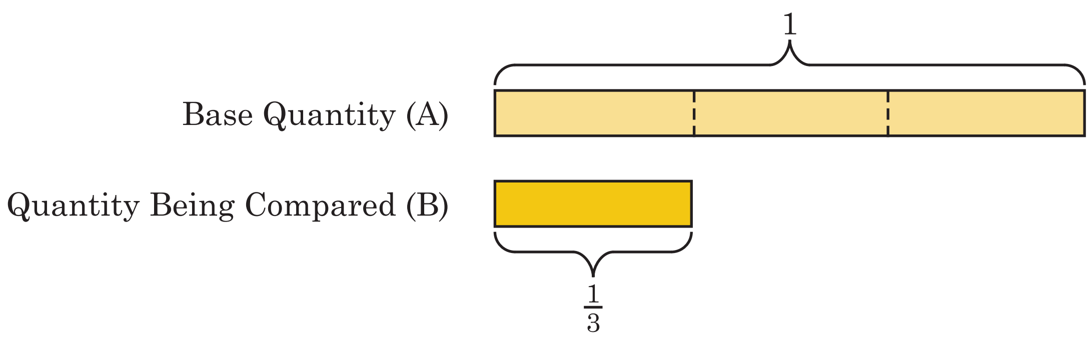
The ratio of the length of A compared to the length of B is 12 : 4. $12 \div 4 = 3$, so Ribbon A is 3 times as long as Ribbon B.

The ratio of the length of B compared to the length of A is 4 : 12. $4 \div 12 = \frac{4}{12} = \frac{1}{3}$, so Ribbon B is $\frac{1}{3}$ as long as Ribbon A.

In the ratio A : B, A is the **first term** and B is the **second term**. The first term is the quantity being compared and the second term is the quantity we are comparing it to, or the **base quantity**. The quotient of the terms, A ÷ B, is the **value of the ratio**. The value of the ratio shows the **relative value** of the quantity being compared to the base quantity when the base quantity is considered as 1. If we consider the length of Ribbon B as 1, then the length of A relative to B is $3 \div 1 = \frac{3}{1} = 3$.



If we compare B to A, the ratio is now 1 : 3 and its value is $1 \div 3 = \frac{1}{3}$. This means that the length of B relative to A is $\frac{1}{3}$.



The value of the ratio can be expressed as a fraction. Since we are dividing the quantity being compared by the base quantity, the base quantity cannot be zero, for the same reason a fraction cannot have a denominator of zero (i.e., division by zero is undefined).

We use ratio to compare similar types of quantities so we do not write the units. To compare the length of the ribbons using division, the units (feet) are the same so they cancel each other out when we divide.

$12 \text{ ft} \div 3 \text{ ft} = \frac{12 \text{ ft}}{3 \text{ ft}} = \frac{12}{3}$

We say the ratio is 12 : 3, not 12 ft : 3 ft.

When we use ratio to compare quantities involving measurement units, the units must be the same. For example, we do not express 12 in to 3 ft as a ratio, but instead 1 ft to 3 ft or 12 in to 36 in. Students will learn how to use division to compare quantities with different units by using compound units such as mi/h (miles per hour) in **Chapter 6: Rate**.

Objectives:

- Express ratios using terms and fractions.
- Find equivalent ratios and explore their properties.
- Simplify ratios.

1. Introduction

Give groups of students 30 cubes (or counters) of one color and 18 cubes (or counters) of another color. Tell them that the cubes represent 30 apples and 18 oranges. Ask, “What is the ratio of the number of apples to the number of oranges?” (30 : 18)

Pose the following problem:

- A store owner wants to put the apples and oranges in boxes without mixing the fruits together (i.e., you cannot have apples and oranges in the same box). All the boxes have to have the same number of fruits in them.
  - In what different ways could the store owner box the apples?
  - Write the ratio of boxes of apples to boxes of oranges for each different way.

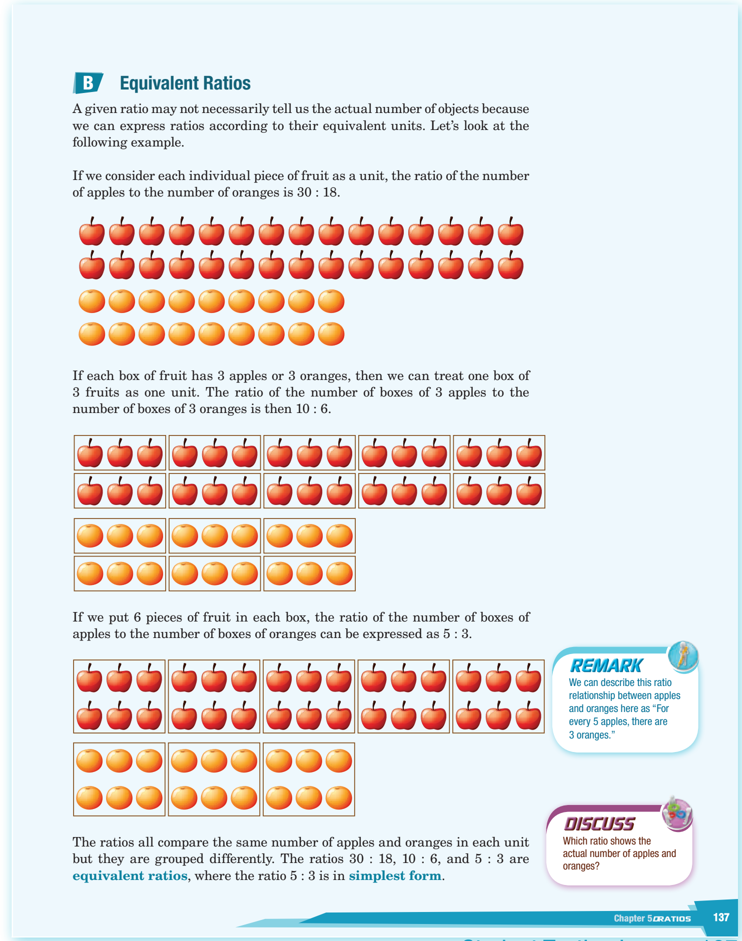
Have students work together in groups to solve the problem and share their solutions. Possible solutions:

- We can put them in groups of 2. The ratio of boxes of apples to boxes of oranges is 15 : 9.
- We can put them in groups of 3. The ratio of boxes of apples to boxes of oranges is 10 : 6.
- We can put them in groups of 6. The ratio of boxes of apples to boxes of oranges is 5 : 3.

Ask, “What do you notice about these ratios, 30 : 18, 15 : 9, 10 : 6 and 5 : 3?”

Ask students to rewrite the ratios as fractions and ask them what they notice. Possible responses:

- They are all equivalent.
- They all simplify to $\frac{5}{3}$.



B Equivalent Ratios

A given ratio may not necessarily tell us the actual number of objects because we can express ratios according to their equivalent units. Let’s look at the following example.

If we consider each individual piece of fruit as a unit, the ratio of the number of apples to the number of oranges is 30 : 18.

If each box of fruit has 3 apples or 3 oranges, then we can treat one box of 3 fruits as one unit. The ratio of the number of boxes of 3 apples to the number of boxes of 3 oranges is then 10 : 6.

If we put 6 pieces of fruit in each box, the ratio of the number of boxes of apples to the number of boxes of oranges can be expressed as 5 : 3.

REMARK
We can describe this ratio relationship between apples and oranges here as “For every 5 apples, there are 3 oranges.”

DISCUSS
Which ratio shows the actual number of apples and oranges?

The ratios all compare the same number of apples and oranges in each unit but they are grouped differently. The ratios 30 : 18, 10 : 6, and 5 : 3 are **equivalent ratios**, where the ratio 5 : 3 is in **simplest form**.

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Read and discuss page 137, including REMARK.

Notes:

- Depending on the size of the unit, we can write the ratio differently. If we consider 1 fruit as the unit, the ratio of boxes of apples to boxes of oranges is 30 : 18. If we consider 3 fruits as the unit, the ratio becomes 10 : 6. If we consider 6 fruits as the unit, the ratio becomes 5 : 3.
- If we write equivalent ratios as fractions we can see that the fractions are equivalent. If we divide the terms of equivalent ratios the quotients are equal. Have students rewrite the ratios as fractions and simplify them to see this.

$$\frac{30}{18} = \frac{5}{3}$$

$$\frac{15}{9} = \frac{5}{3}$$

$$\frac{10}{6} = \frac{5}{3}$$

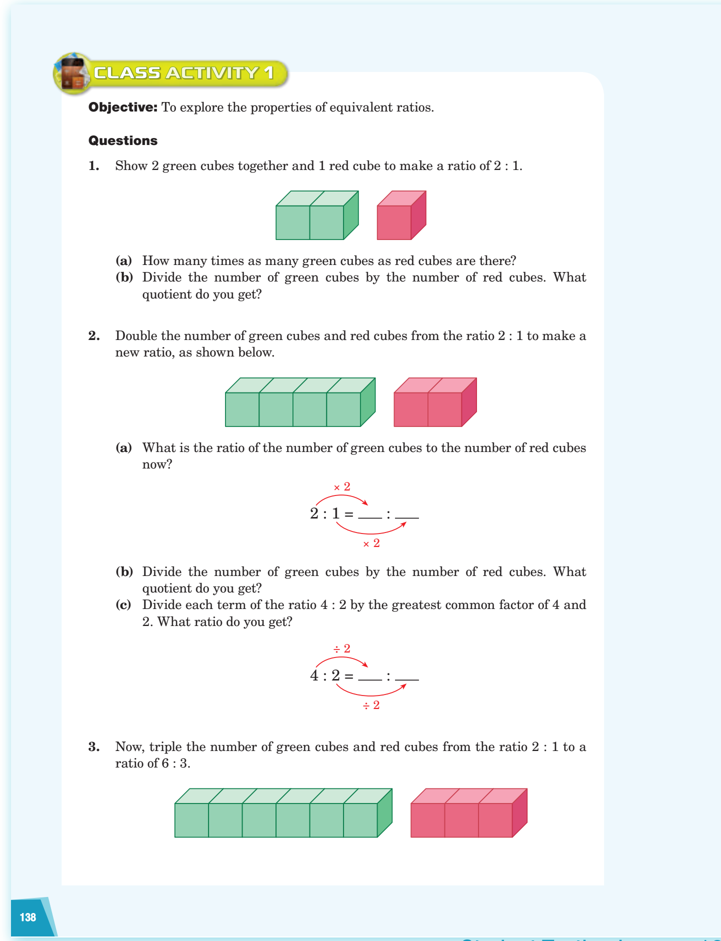
2. Development

Give students 6 cubes of one color and 3 cubes of another color.

Have students work in pairs or groups to complete Class Activity 1 and share their conclusions.

Answers

1. (a) There are two times as many green as red cubes.
   (b) The quotient of 2 and 1 is 2.
2. (a) The ratio of green to red cubes now is 4 : 2.
   (b) The quotient of 4 and 2 is 2.
   (c) 4 : 2 = 2 : 1 when you divide each term of the ratio by the greatest common factor of 4 and 2, which is 2.
3. (a) The ratio of the number of green cubes to the number of red cubes now is 6 : 3.
   (b) The quotient of 6 and 3 is 2.
   (c) There are two times as many green cubes as red cubes.
   (d) The greatest common factor of 6 and 3 is 3. $6 \div 3 = 2$ $3 \div 3 = 1$
4. $2 : 1 = \frac{2}{1}$ $4 : 2 = \frac{4}{2}$ $6 : 3 = \frac{6}{3}$


CLASS ACTIVITY 1

Objective: To explore the properties of equivalent ratios.

Questions

1. Show 2 green cubes together and 1 red cube to make a ratio of 2 : 1.
   (a) How many times as many green cubes as red cubes are there?
   (b) Divide the number of green cubes by the number of red cubes. What quotient do you get?
2. Double the number of green cubes and red cubes from the ratio 2 : 1 to make a new ratio, as shown below.
   (a) What is the ratio of the number of green cubes to the number of red cubes now?
   $2 : 1 =$ ___ : ___ (× 2, × 2)
   (b) Divide the number of green cubes by the number of red cubes. What quotient do you get?
   (c) Divide each term of the ratio 4 : 2 by the greatest common factor of 4 and 2. What ratio do you get?
   $4 : 2 =$ ___ : ___ (÷ 2, ÷ 2)
3. Now, triple the number of green cubes and red cubes from the ratio 2 : 1 to a ratio of 6 : 3.

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5. Possible answers:
- The fractions are equivalent.
  $\frac{2}{1} = \frac{4}{2} = \frac{6}{3}$
- If we divide the numerator by the denominator, the values of the quotients are equal.

  $\frac{2}{1} = 2$

  $\frac{4}{2} = 2$

  $\frac{6}{3} = 2$

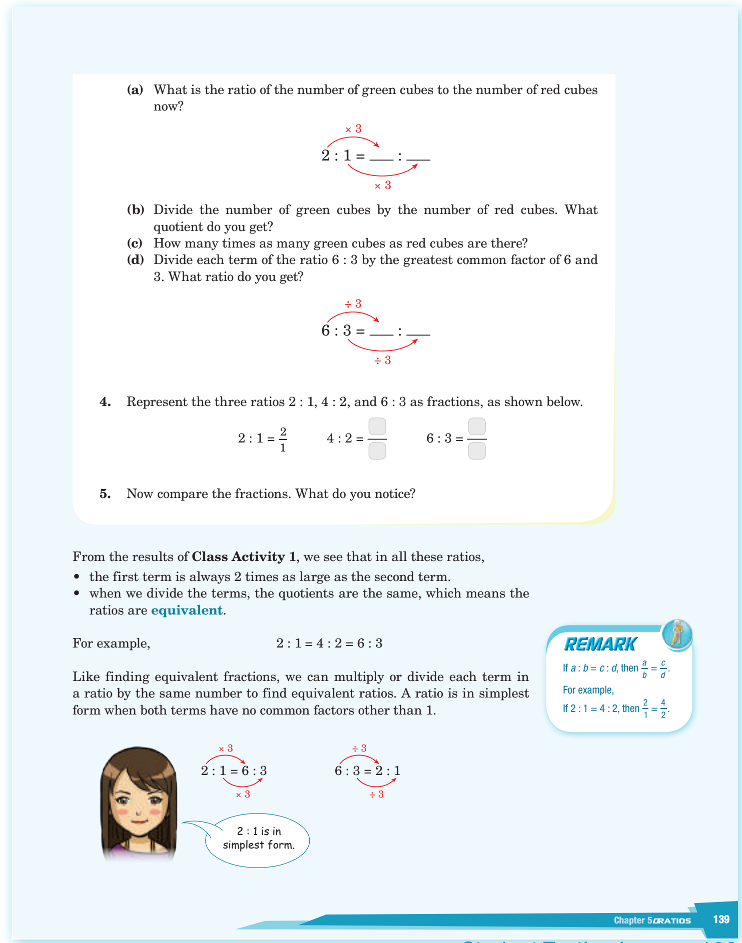
Read and discuss the bottom of page 139, and the REMARK to summarize the activity.

Notes:
- When we compare two quantities using ratio, the quotient of the terms is called the value of the ratio. The value of a ratio $a : b$ is the relative value of a (the quantity being compared) when b (the base quantity) is considered as 1. The value of the ratio $(a \div b)$ can be written as a fraction $(\frac{a}{b})$.
- Since equivalent ratios have the same value, we can determine whether two ratios are equivalent by comparing their values (i.e., the quotient of the terms). For example, $4 : 2 = 6 : 3$ are equivalent because $\frac{4}{2} = \frac{6}{3}$.
- Help students make the connection between equivalent ratios and equivalent fractions. Write the ratio as a fraction to illustrate why this works. For example, to find ratios that are equivalent to 4 : 6, we can write the ratio as a fraction and multiply or divide the numerator and denominator by the same number.

  $4 : 6 = \frac{4}{6} = \frac{4 \times 2}{6 \times 2} = \frac{8}{12} = 8 : 12$

  $4 : 6 = \frac{4}{6} = \frac{4 \div 2}{6 \div 2} = \frac{2}{3} = 2 : 3$

  Thus, $4 : 6 = 8 : 12 = 2 : 3$.



(a) What is the ratio of the number of green cubes to the number of red cubes now?

$2 : 1 = \_\_ : \_\_$ (× 3, × 3)

(b) Divide the number of green cubes by the number of red cubes. What quotient do you get?

(c) How many times as many green cubes as red cubes are there?

(d) Divide each term of the ratio 6 : 3 by the greatest common factor of 6 and 3. What ratio do you get?

$6 : 3 = \_\_ : \_\_$ (÷ 3, ÷ 3)

4. Represent the three ratios 2 : 1, 4 : 2, and 6 : 3 as fractions, as shown below.

$2 : 1 = \frac{2}{1}$ $\quad 4 : 2 = \frac{\square}{\square}$ $\quad 6 : 3 = \frac{\square}{\square}$

5. Now compare the fractions. What do you notice?

From the results of **Class Activity 1**, we see that in all these ratios,
- the first term is always 2 times as large as the second term.
- when we divide the terms, the quotients are the same, which means the ratios are **equivalent**.

For example, $2 : 1 = 4 : 2 = 6 : 3$

Like finding equivalent fractions, we can multiply or divide each term in a ratio by the same number to find equivalent ratios. A ratio is in simplest form when both terms have no common factors other than 1.

REMARK
If $a : b = c : d$, then $\frac{a}{b} = \frac{c}{d}$.
For example,
If $2 : 1 = 4 : 2$, then $\frac{2}{1} = \frac{4}{2}$.

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- We can also think about it using the properties of division.
  $4 : 6 = 4 \div 6 = (4 \times 2) \div (6 \times 2) = 8 \div 12 = 8 : 12$
  $4 : 6 = 4 \div 6 = (4 \div 2) \div (6 \div 2) = 2 \div 3 = 2 : 3$
  Thus, $4 : 6 = 8 : 12 = 2 : 3$.
- To determine whether two ratios are equivalent it is often easier to simplify them first and see if the simplified ratios are the same. For example, to determine if the ratios 4 : 6 and 6 : 9 are equivalent...
  $4 : 6 = 2 : 3$
  $6 : 9 = 2 : 3$
  Therefore, $4 : 6 = 6 : 9$.

3. Application

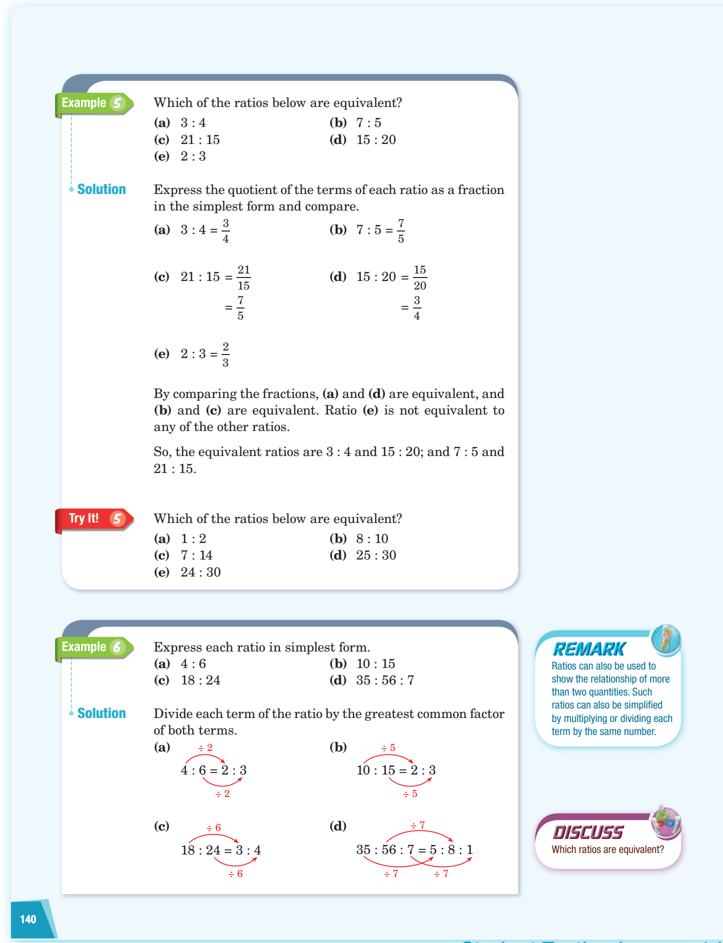
Have students study Example 5, do Try It! 5 and discuss the answers.

Note: In this problem, there are several ratios. We can compare them easily by writing them as fractions in simplest form and then comparing their values. Alternatively, we could divide the terms and express the ratio as a decimal to compare. For example, 3 : 4 = 0.75, 15 : 20 = 0.75, thus 3 : 4 = 15 : 20.

Try It! 5 Answers

(a) and (c) are equivalent, as are (b) and (e).

(a) 1 : 2 is in simplest form.
(b) 8 : 10 = 4 : 5
(c) 7 : 14 = 1 : 2
(d) 25 : 30 = 5 : 6
(e) 24 : 30 = 4 : 5



Example 5 Which of the ratios below are equivalent?
(a) 3 : 4 (b) 7 : 5
(c) 21 : 15 (d) 15 : 20
(e) 2 : 3

Solution Express the quotient of the terms of each ratio as a fraction in the simplest form and compare.
(a) $3 : 4 = \frac{3}{4}$ (b) $7 : 5 = \frac{7}{5}$
(c) $21 : 15 = \frac{21}{15} = \frac{7}{5}$ (d) $15 : 20 = \frac{15}{20} = \frac{3}{4}$
(e) $2 : 3 = \frac{2}{3}$

By comparing the fractions, (a) and (d) are equivalent, and (b) and (c) are equivalent. Ratio (e) is not equivalent to any of the other ratios.

So, the equivalent ratios are 3 : 4 and 15 : 20; and 7 : 5 and 21 : 15.

Try It! 5 Which of the ratios below are equivalent?
(a) 1 : 2 (b) 8 : 10
(c) 7 : 14 (d) 25 : 30
(e) 24 : 30

Example 6 Express each ratio in simplest form.
(a) 4 : 6 (b) 10 : 15
(c) 18 : 24 (d) 35 : 56 : 7

Solution Divide each term of the ratio by the greatest common factor of both terms.
(a) 4 : 6 = 2 : 3 (÷ 2) (b) 10 : 15 = 2 : 3 (÷ 5)
(c) 18 : 24 = 3 : 4 (÷ 6) (d) 35 : 56 : 7 = 5 : 8 : 1 (÷ 7)

REMARK Ratios can also be used to show the relationship of more than two quantities. Such ratios can also be simplified by multiplying or dividing each term by the same number.

DISCUSS Which ratios are equivalent?

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4. Extension

Have students study Example 6, do Try It! 6, and discuss the answers.

Notes:

- Unlike fractions, with ratios we can compare more than two quantities (see REMARK). To help students understand what this means it might be helpful to give them an example such as a cookie recipe that calls for 3 cups of flour, 2 cups of butter, and 1 cup of sugar, the ratio of flour to butter to sugar is 3 : 2 : 1. Within this ratio we can make different comparisons; e.g., cups of flour to cups of butter, cups of butter to cups of sugar, cups of flour to total number of cups of all the ingredients, etc.
- Problem (d) is a ratio of three quantities. It can be simplified the same way as a ratio of two quantities by finding common factors of the terms. 35, 56, and 7 all have a common factor of 7.

- If we find the greatest common factor (GCF), we can simplify in one step, but we could also simplify in several steps, just as we do with fractions. (See Note on page 141).
- Have students talk about the DISCUSS on page 141 with partners or group.

Try It! 6 Answers

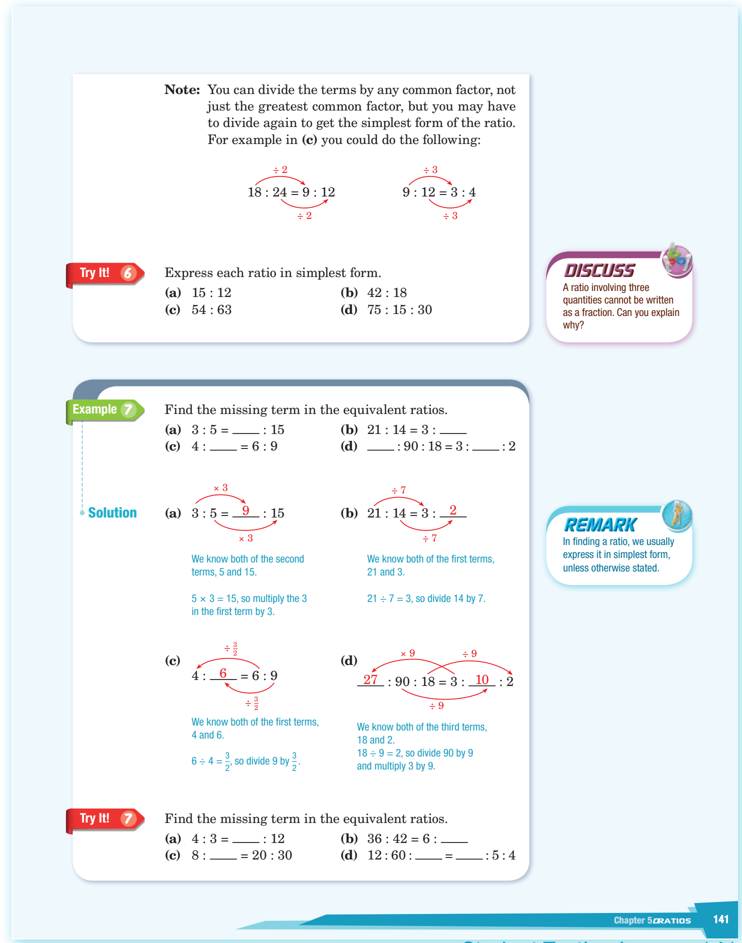
(a) 15 : 12 = 5 : 4 (b) 42 : 18 = 7 : 3

(c) 54 : 63 = 6 : 7 (d) 75 : 15 : 30 = 5 : 1 : 2

5. Conclusion

Summarize the main points of the lesson.

- Equivalent ratios are ratios that have the same value. This value can be expressed as a fraction.
- We simplify ratios by finding a common factor of the terms, similar to the way we simplify fractions.
- We can have ratios of more than two quantities.



Note: You can divide the terms by any common factor, not just the greatest common factor, but you may have to divide again to get the simplest form of the ratio. For example in **(c)** you could do the following:

$18 : 24 = 9 : 12$ (÷ 2) $9 : 12 = 3 : 4$ (÷ 3)

Try It! 6 Express each ratio in simplest form.

(a) 15 : 12 **(b)** 42 : 18

(c) 54 : 63 **(d)** 75 : 15 : 30

DISCUSS A ratio involving three quantities cannot be written as a fraction. Can you explain why?

Example 7 Find the missing term in the equivalent ratios.

(a) 3 : 5 = ____ : 15 **(b)** 21 : 14 = 3 : ____

(c) 4 : ____ = 6 : 9 **(d)** ____ : 90 : 18 = 3 : ____ : 2

Solution

(a) 3 : 5 = 9 : 15 (× 3)

We know both of the second terms, 5 and 15.

5 × 3 = 15, so multiply the 3 in the first term by 3.

(b) 21 : 14 = 3 : 2 (÷ 7)

We know both of the first terms, 21 and 3.

21 ÷ 7 = 3, so divide 14 by 7.

(c) 4 : 6 = 6 : 9 ($\div \frac{3}{2}$)

We know both of the first terms, 4 and 6.

$6 \div 4 = \frac{3}{2}$, so divide 9 by $\frac{3}{2}$.

(d) 27 : 90 : 18 = 3 : 10 : 2 (× 9, ÷ 9)

We know both of the third terms, 18 and 2.

18 ÷ 9 = 2, so divide 90 by 9 and multiply 3 by 9.

REMARK In finding a ratio, we usually express it in simplest form, unless otherwise stated.

Try It! 7 Find the missing term in the equivalent ratios.

(a) 4 : 3 = ____ : 12 **(b)** 36 : 42 = 6 : ____

(c) 8 : ____ = 20 : 30 **(d)** 12 : 60 : ____ = ____ : 5 : 4

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Objectives:

- Find the missing term in a ratio.
- Apply ratio to real-world situations.

1. Introduction

Have students think of Example 7 (a) and (b) without looking at the answers (they can cover the answers with an index card or the teacher can write the questions on the board).

(a) 3 : 5 = ____ : 15

(b) 21 : 14 = 3 : ____

Ask students to talk with a partner or group to think about how we can find the missing term in each problem and have them share their ideas.

Possible responses:

- Look for which term is given on both sides of the equation and use that relationship to find what number to multiply by.

For (a), we know the second term of each ratio.

× 3

3 : 5 = ____ : 15

× 3

For (b), we know the first term of each ratio.

÷ 7

21 : 14 = 3 : ____

÷ 7

- Change the ratios to fractions and think about what we need to multiply or divide the numerator and denominator by.

× 3 ÷ 7

$\frac{3}{5} = \frac{\quad}{15}$ $\frac{21}{14} = \frac{3}{\quad}$

× 3 ÷ 7

Notes:

- Help students see that when we go from a smaller term to a larger term we multiply, and when we go from a larger term to a smaller we divide. To know what to multiply or divide by we can divide the larger term by the smaller term.
    - For (a), we are going from a smaller term (5) to a larger term (15). 15 ÷ 5 = 3, so we multiply 3 by 3.
    - For (b), we are going from a larger term (21) to a smaller term (3). 21 ÷ 3 = 7, so we divide 14 by 7.

2. Development

Have students study Example 7 (c) and (d) and discuss.

Notes:

- For problem (c), we are dividing the 6 on the right side of the equation by the quotient of 6 and 4, which is $\frac{6}{4} = \frac{3}{2}$. It may be easier to divide by $\frac{6}{4}$ instead of $\frac{3}{2}$ because we can just cancel out the sixes.

$6 \div \frac{6}{4} = \cancel{6}^{1} \times \frac{4}{\cancel{6}_{1}} = 4$

We could also think of what number we need to multiply 6 by to get 4 and then multiply the 9 in the second term by the same number.

$6 \times ? = 4$

$\cancel{6}^{1} \times \frac{4}{\cancel{6}_{1}} = 4$

If we multiply 6 by a fraction that has a denominator of 6, the sixes will cancel each other out. To be left with a product of 4, the numerator needs to be 4.

$\times \frac{4}{6}$

4 : ____ = 6 : 9

$\times \frac{4}{6}$

Problem (d) is a ratio of three quantities. We need to look for which term is known on both sides of the equation. Going to the right to get from 18 to 2 we divide by 9. We need to do the same thing with 90 to find the missing second term since we are going to the right. For the first term, we are going to the left so we multiply by 9.

Have students do Try It! 7, and discuss their answers.

- For students who are having difficulty, it may be helpful to write the ratios as equivalent fractions with a missing numerator or denominator.
- For Try It! 7 (c), $20 \div \frac{20}{8} = 8$, we divide 30 by $\frac{20}{8}$ (or $\frac{5}{2}$) to find the missing term. Alternatively, students could simplify the ratio 20 : 30 to 2 : 3, which makes it is easier to think about (8 : ___= 2 : 3).
- For Try It! 7 (d), we know the second term on both sides of the equation. $60 \div 5 = 12$, so to find the first term on the right side we divide 12 by 12. To find the third term on the left side we multiply 4 by 12.

Try It! 7 Answers

(a) 4 : 3 = 16 : 12

(b) 36 : 42 = 6 : 7

(c) 8 : 12 = 20 : 30

(d) 12 : 60 : 48 = 1 : 5 : 4