Chapter 4 Algebraic Manipulation

Suggested Approach

Algebraic manipulation is crucial for students in learning mathematics and science. Careful elaboration of the concepts is necessary in order to build a strong foundation. Students should be encouraged not to skip steps when presenting their solutions.

An algebraic expression can be considered as a machine, with the terms as its parts. Analogously, like terms and unlike terms are like parts and unlike parts respectively of the machine. We may use an activity to ask students to identify coefficients of given terms, like terms and unlike terms. Models can be used to introduce the idea of simplification of like terms.

Numerical expressions and geometrical interpretation can be used to introduce distributive property. Teachers may present various cases and help students discuss how to handle brackets in algebraic expressions.

Teachers may draw the analogy between prime factorization of a whole number and the factorization of an algebraic expression. For factorization by grouping terms, teachers may provide some guided practice to students and let them discover the skill by themselves.

4.1 Like Terms and Unlike Terms

It should be emphasized that the sign of a term is attached to its coefficient and not the variable. Terms with the same variables may not be like terms. For instance, (x and x^3) and (a^2b and ab^2) are two pairs of unlike terms.

4.2 Distributive Law, Addition, and Subtraction of Linear Algebraic Expressions

Students should be careful when removing brackets. The vertical form of addition and subtraction will be used in long multiplication and division. Thus, it is helpful to teach this method as well.

4.3 Simplification of Linear Algebraic Expressions

This section is confined to the expansion of linear algebraic expressions. Students should learn the skill of both distribution from the left and distribution from the right.

4.4 Factorization by Extracting Common Factors

Factorization by extracting the common factor may be considered as the reverse process of expansion. After sufficient practice, students should be able to locate the appropriate factor. They should develop the habit of expanding the factorization result and see whether the original expression can be obtained. Instead of factoring $ax + ay + a$ as $a(x + y + 1)$, some students drop the 1 and get the wrong answer $a(x + y)$.

4.5 Factorization by Grouping Terms

Students should be encouraged to try different combinations of grouping terms. They should be made aware that some algebraic expressions cannot be factorized.

Chapter 3 Introduction To Algebra

Extend Your Learning Curve

Hand-Shaking Problem

There are n persons attending a party. Each person shakes hands with every other person just once. Let T be the total number of handshakes.

(a) Complete the following table.

n	2	3	4	5	6
T					

(b) Establish a formula connecting n and T.

Solution

(a)

n	2	3	4	5	6
T	**1**	**3**	**6**	**10**	**15**

(b) Suppose the n persons in the party are arranged in order.

The 1st person will shake hands with $n - 1$ persons.
The 2nd person will shake hands with the remaining $n - 2$ persons, without repeating.
The 3rd person will shake hands with the remaining $n - 3$ persons, without repeating.

The $(n - 1)$th person will just shake hands with the nth person, without repeating.
$\therefore$ the total number of handshakes is given by
$T = (n - 1) + (n - 2) + (n - 3) + ... + 3 + 2 + 1$ ....(1)

By reversing the above expression, we have

$T = 1 + 2 + 3 + ... + (n - 3) + (n - 2) + (n - 1)$..........................(2)

(1) + (2), $\quad 2T = n + n + n + ...$ for $(n - 1)$ terms
$$2T = n(n - 1)$$
$$\therefore\ T = \frac{1}{2}n(n - 1)$$

17. The price $P for a birthday cake of radius r centimeters and height h centimeters is given by the formula

$$P = \frac{1}{25}r^2h.$$

(a) Find the price for a cake of radius 10 cm and height 8 cm.

(b) If the height of the cake in **(a)** is increased to 10 cm, what is the increase in price?

Solution

(a) When $r = 10$ and $h = 8$,

$$\begin{aligned} P &= \frac{1}{25}r^2h \\ &= \frac{1}{25} \times 10^2 \times 8 \\ &= 32 \end{aligned}$$

The price of the cake is $32.

(b) When $r = 10$ and $h = 10$,

$$\begin{aligned} P &= \frac{1}{25}r^2h \\ &= \frac{1}{25} \times 10^2 \times 10 \\ &= 40 \end{aligned}$$

Increase of price = $(40 – 32)
= $8

The increase in price is $8.

Brainworks

18. **(a)** When two resistors of resistance a ohms and b ohms are connected to two points, X and Y, by using different wires in a circuit as shown in Diagram 1, the equivalent resistance R ohms is given by the formula

$$R = \frac{ab}{a+b}.$$

Find the value of R when $a = 20$ and $b = 30$.

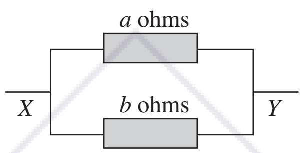


Diagram 1

(b) Suppose 3 resistors of 20 ohms, 30 ohms, and 15 ohms are connected to the points X and Y in the circuit as shown in Diagram 2. Using the formula in **(a)**, find their equivalent resistance.

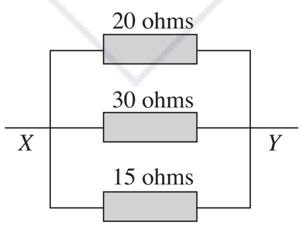


Diagram 2

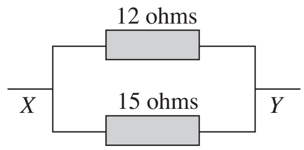
Solution

(a) When $a = 20$ and $b = 30$,

$$\begin{aligned} R &= \frac{ab}{a+b} \\ &= \frac{20 \times 30}{20 + 30} \\ &= 12 \end{aligned}$$

The value of R is 12.

(b) Combining the 20 ohms and 30 ohms resistors first, we have an equivalent circuit as shown.
Take $a = 12$ and $b = 15$,



$$\begin{aligned} R &= \frac{ab}{a+b} \\ &= \frac{12 \times 15}{12 + 15} \\ &= \frac{180}{27} \\ &= \frac{20}{3} \\ &= 6\frac{2}{3} \end{aligned}$$

The equivalent resistance is $6\frac{2}{3}$ ohms.

Exercise 3.3

Basic Practice

1. The length of a rectangular room is 10 feet more than its width. Suppose the width of the room is w feet, what is the length of the room in terms of w?

Solution

Length of the room = $(w + 10)$ ft

2. The price of a burger is $6 less than that of a pizza. Given that the price of the pizza is $$p$, express the price of the burger in terms of p.

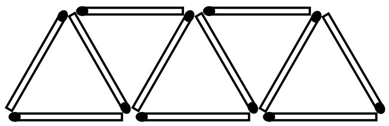
Solution

Price of burger = $\$(p – 6)$

Extend Your Learning Curve

Matchstick Triangle Patterns

Johnny uses matchsticks to form a pattern of triangles as shown below.



Suppose m matchsticks are required to form n triangles.

(a) Copy and complete the following table.

n	1	2	3	4	5	6
m	3					

(b) Find a formula connecting m and n.
(c) How many matchsticks are required to form 100 triangles?
(d) How many triangles can be formed with 2,005 matchsticks?
(e) Suppose the area of a triangle is $\sqrt{3}$ cm^2. Find the total area of the triangles formed in **(d)**. Give your answer correct to the closest whole number.

Suggested Answers

(a)

n	1	2	3	4	5	6
m	3	**5**	**7**	**9**	**11**	**13**

(b) Except for the first triangle, each triangle in a pattern of n triangles is formed by adding 2 matchsticks.
$\therefore\ m = 3 + 2(n - 1)$ or $m = 2n + 1$

(c) When $n = 100$,
$m = 2(100) + 1$
$= 201$
$\therefore$ 201 matchsticks are required to form 100 triangles.

(d) When $m = 2{,}005$,
$2{,}005 = 2n + 1$
$2{,}004 = 2n$
$\therefore\ n = 1{,}002$
1,002 triangles can be formed with 2,005 matchsticks.

(e) Total area of 1,002 triangles $= 1{,}002 \times \sqrt{3}$
$= 1{,}736$ cm^2 (correct to the closest whole number)

Exercise 6.1

Basic Practice

1. Express each ratio in the simplest form.
   (a) 18 : 27 (b) 144 : 132
   (c) $1\frac{1}{2} : 4\frac{1}{2}$ (d) $2\frac{2}{3} : 1\frac{1}{5}$
   (e) 0.250 : 0.375 (f) $0.48 : 2\frac{2}{15}$
   (g) 1.6 feet : 36 inches
   (h) 850 grams : 3.4 kilograms
   (i) $1\frac{1}{3}$ hours : 20 minutes
   (j) 80¢ : $2

Solution

(a) $18 : 27 = \frac{18}{9} : \frac{27}{9}$
$= 2 : 3$

(b) $144 : 132 = \frac{144}{12} : \frac{132}{12}$
$= 12 : 11$

(c) $1\frac{1}{2} : 4\frac{1}{2} = \frac{3}{2} : \frac{9}{2}$
$= 1 : 3$

(d) $2\frac{2}{3} : 1\frac{1}{5} = \frac{8}{3} : \frac{6}{5}$
$= 8 \times 5 : 6 \times 3$
$= 40 : 18$
$= 20 : 9$

(e) $0.250 : 0.375 = 250 : 375$
$= \frac{250}{125} : \frac{375}{125}$
$= 2 : 3$

(f) $0.48 : 2\frac{2}{15} = \frac{48}{100} : \frac{32}{15}$
$= \frac{48}{100} \times \frac{300}{16} : \frac{32}{15} \times \frac{300}{16}$
$= 9 : 40$

(g) 1.6 feet : 36 inches = 19.2 inches : 36 inches
$= \frac{19.2}{2.4} : \frac{36}{2.4}$
$= 8 : 15$

(h) 850 grams : 3.4 kilograms = 850 g : 3,400 g
= 1 : 4

(i) $1\frac{1}{3}$ hours : 20 minutes = 80 minutes : 20 minutes
= 4 : 1

(j) 80¢ : $2 = 80¢ : 200¢
$= \frac{80}{40} : \frac{200}{40}$
$= 2 : 5$

2. Given that $a : b : c = 20 : 35 : 15$,
   (a) simplify $a : b : c$,
   (b) find $a : b$,
   (c) find $c : b$.

Solution

(a) $a : b : c = 20 : 35 : 15$
$= \frac{20}{5} : \frac{35}{5} : \frac{15}{5}$
$= 4 : 7 : 3$

(b) $a : b = 4 : 7$

(c) $c : b = 3 : 7$

3. Given that $x : y : z = 5\frac{1}{2} : 4.62 : 33$,
   (a) simplify $x : y : z$, (b) find $y : x$,
   (c) find $x : z$.

Solution

(a) $x : y : z = 5\frac{1}{2} : 4.62 : 33$
$= \frac{11}{2} : \frac{462}{100} : 33$
$= \frac{1}{2} : \frac{42}{100} : 3$
$= \frac{1}{2} \times 50 : \frac{42}{100} \times 50 : 3 \times 50$
$= 25 : 21 : 150$

(b) $y : x = 21 : 25$

(c) $x : z = 25 : 150 = 1 : 6$

4. Find the ratio of $a : b : c$.
   (a) $a : b = 3 : 4$, $b : c = 4 : 9$
   (b) $a : b = 5 : 3$, $b : c = 4 : 1$
   (c) $a : b = \frac{1}{2} : 1$, $b : c = 1 : \frac{1}{3}$
   (d) $a : b = 3 : 7$, $b : c = 3 : 7$

Solution

(a) $a : b = 3 : 4$
$b : c = 4 : 9$
$\therefore\ a : b : c = 3 : 4 : 9$

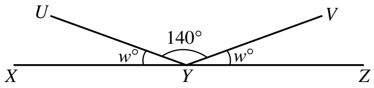
(b) $a : b = 5 : 3$
$= 20 : 12$
$b : c = 4 : 1$
$= 12 : 3$
$\therefore\ a : b : c = 20 : 12 : 3$

(c) $a : b = \frac{1}{2} : 1$
$b : c = 1 : \frac{1}{3}$
$\therefore\ a : b : c = \frac{1}{2} : 1 : \frac{1}{3}$
$= \frac{1}{2} \times 6 : 1 \times 6 : \frac{1}{3} \times 6$
$= 3 : 6 : 2$

Try It!

Section 8.2

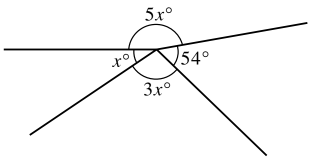
1. In the diagram, XYZ is a straight line. Find the value of w.



Solution

$w° + 140° + w° = 180°$ (adj. $\angle$s on a st. line)
$2w = 40$
$w = 20$

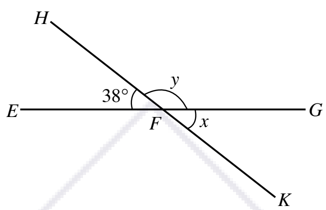
2. Find the value of x in the diagram.



Solution

$5x° + x° + 3x° + 54° = 360°$ ($\angle$s at a point)
$9x + 54 = 360$
$9x = 306$
$x = 34$

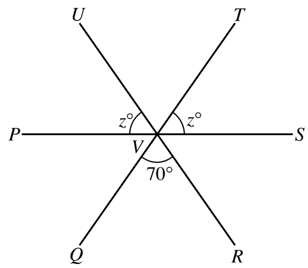
3. In the diagram, EFG and HFK are straight lines. Find $m\angle x$ and $m\angle y$.



Solution

$m\angle x = m\angle EFH$ (vert. opp. $\angle$s)
$= 38°$
$m\angle y + 38° = 180°$ (adj. $\angle$s on a st. line)
$m\angle y = 142°$

4. In the diagram, PS, QT and RU are straight lines, intersecting at V. Find the value of z.



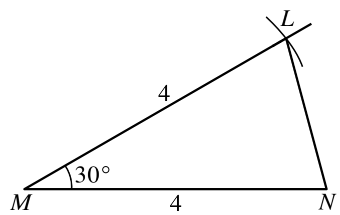
Solution

$m\angle UVT = 70°$ (vert. opp. $\angle$s)
$z° + m\angle UVT + z° = 180°$ (adj. $\angle$s on a st. line)
$z° + 70° + z° = 180°$
$2z = 110$
$z = 55$

Section 8.4

5. Construct $\triangle LMN$ with $LM = 4$ cm, $MN = 4$ cm, and $\angle LMN = 30°$. Measure and write down the length of LN.

Solution



Construction Steps:

1. Construct a line segment MN 4 cm long.
2. Draw a ray with the end point M and making an angle of 30° with MN using a protractor.
3. With M as centre and 4 cm as radius, draw an arc to cut the ray at L.
4. Join L and N. $\triangle LMN$ is the required triangle.
   $LN = 2$ cm

Chapter 14 Proportions

Suggested Approach

Teachers can show why it is relevant to learn how to calculate distance and area based on a map using some daily life examples. Students can integrate the knowledge learned in this topic with what they learn on map reading.

Teachers can also introduce direct and indirect proportions by using daily life examples. Students are expected to recognize these relations from tables, graphs, equations, and verbal descriptions of proportional relationships. Further real-life problems that can be modeled by these relations could be explored. Students should learn to differentiate cases such as "x and y are in direct (or inverse) proportion" and "x^2 and y are in direct (or inverse) proportion".

14.1 Scale Drawings

Students should understand that the scale factor is the ratio of a side of the image to the corresponding side of the object. They are expected to draw simple scale drawings of models and floor plans, and get actual dimensions from scale drawings.

14.2 Map Scale and Calculation of Area

Students should note that a map scale can be represented as a ratio or a fraction. They should learn to calculate the actual distance from a given distance on a map and vice versa. Teachers, when teaching the concept of ratio of actual area to the area on a map, can use squares and rectangles to illustrate the relationship with the scale of the map.

14.3 Direct Proportion

When learning direct proportion, students are expected to observe patterns of the variation between two quantities and express the generalization in algebraic form. Students should be able to relate linear graphs learned earlier with direct relation of two quantities. Real-life examples will make learning more effective and interesting.

14.4 Inverse Proportion

Teachers need to emphasize to their students that they have to draw a linear graph of y against $\frac{1}{x}$ to show that x and y are in inverse proportion. This is because some students may have a misconception that a straight line with a negative slope in x-y plane indicates an inverse proportion between x and y.

Chapter 14 Proportions

Class Activity 1

Objective: To understand the idea of direct proportion.

Questions

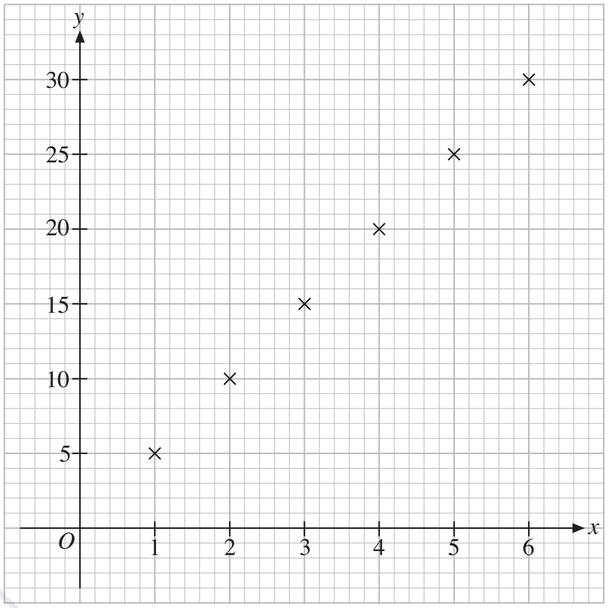
The following table shows the relationship between the number of books bought (x) and the total cost of the books (\$$y$).

Number of books (x)	1	2	3	4	5	6
Total cost (\$$y$)	5	10	15	20	25	30

(a) Copy and complete the following table.

x	1	2	3	4	5	6
y	5	10	15	20	25	30
$\frac{y}{x}$	5	5	5	5	5	5

(b) On a sheet of graph paper, plot the corresponding points (x, y) in **(a)** using the scale for both axes as shown.



(c) What can you say about the points you have plotted in **(b)**?

The points are on a straight line.

(d) Write down an equation connecting x and y.

$y = 5x$

(e) What is the value of y when $x = 8$?

When $x = 8$, $y = 5 \times 8 = 40$.

(f) Does the graph of the equation in **(d)** pass through the origin (0, 0)?

Yes.

Solution

(a) Map scale = 1 : 1,500,000
= 1 cm : 1,500,000 cm
= 1 cm : 15 km

Actual north-south dimension of California
= 84 × 15 km
= 1,260 km

(b) Actual east-west dimension of California
= 480 km
= 480 × 1,000 × 100 cm
= 48,000,000 cm

Actual north-south dimension of California
= 1,260 km
= 1,260 × 1,000 × 100 cm
= 126,000,000 cm

Let the scale of the required map be 1 : n.
We should have
$(21 - 2 \times 1)n > 48,000,000$
and $(28 - 2 \times 1)n > 126,000,000$,
i.e., $n > 2,526,315.79$ and $n > 4,846,153.85$.

Hence, one appropriate scale would be 1 : 5,000,000.

Exercise 14.3

Basic Practice

1. In each of the following tables, determine whether x and y are in direct proportion.

(a)

x	1	2	3	4
y	3	6	9	12

(b)

x	2	4	6	8
y	10	20	25	40

(c)

x	3	6	7	10
y	21	42	49	70

(d)

x	8	12	15	20
y	30	48	60	80

Solution

(a) $\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12}$
$\therefore$ x and y are in direct proportion.

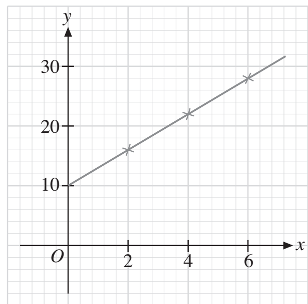
(b) $\frac{2}{10} \neq \frac{6}{25}$
$\therefore$ x and y are NOT in direct proportion.

(c) $\frac{3}{21} = \frac{6}{42} = \frac{7}{49} = \frac{10}{70}$
$\therefore$ x and y are in direct proportion.

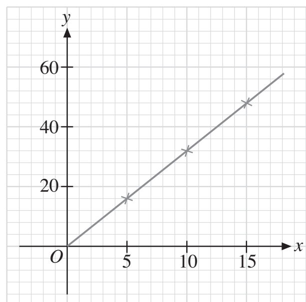
(d) $\frac{8}{30} \neq \frac{12}{48}$
$\therefore$ x and y are NOT in direct proportion.

2. In each of the following graphs, determine whether x and y are in direct proportion.

(a)



(b)



Solution

(a) The straight line graph does not pass through the origin.
$\therefore$ x and y are NOT in direct proportion.

(b) The straight line graph passes through the origin.
$\therefore$ x and y are in direct proportion.

3. In each of the following equations, determine whether x and y are in direct proportion.

(a) $y = 4x$ **(b)** $y = x + 2$

(c) $y = x^2$ **(d)** $y = \frac{1}{2}x$

Solution

(a) $y = 4x$ is in the form of $y = kx$ with $k = 4$.
$\therefore$ x and y are in direct proportion.

(b) $y = x + 2$ cannot be written as $y = kx$.
$\therefore$ x and y are NOT in direct proportion.

(c) $y = x^2$ cannot be written as $y = kx$.
$\therefore$ x and y are NOT in direct proportion.

(d) $y = \frac{1}{2}x$ is in the form of $y = kx$ with $k = \frac{1}{2}$.
$\therefore$ x and y are in direct proportion.

4. If two quantities, x and y, are in direct proportion, find the values of p and q in the following table.

x	12	18	q
y	8	p	24

Solution

$$\frac{12}{8} = \frac{18}{p}$$
$$\therefore\ p = 18 \times \frac{8}{12}$$
$$= 12$$
$$\frac{12}{8} = \frac{n}{24}$$
$$q = 24 \times \frac{12}{8}$$
$$= 36$$

5. It is given that w is directly proportional to t. When $t = 4$, $w = 20$. Find
(a) the value of w when $t = 6$,
(b) the value of t when $w = 45$.

Solution

(a) Let $w = kt$, where k is a constant.
When $t = 4$, $w = 20$.
$$20 = k \times 4$$
$$k = 5$$
$$w = 5t$$
When $t = 6$,
$$w = 5 \times 6$$
$$= 30$$

(b) When $w = 45$,
$$45 = 5t$$
$$t = 9$$

6. It is given that A is directly proportional to r^2 and $r > 0$. When $r = 5$, $A = 75$. Find
(a) the value of A when $r = 4$,
(b) the value of r when $A = 147$.

Solution

(a) Let $A = kr^2$, where k is a constant.
When $r = 5$, $A = 75$.
$$75 = k \times 5^2$$
$$k = 3$$
$$\therefore\ A = 3r^2$$
When $r = 4$,
$$A = 3 \times 4^2$$
$$= 48$$

(b) When $A = 147$,
$$147 = 3r^2$$
$$r^2 = 49$$
$$r = \sqrt{49}$$
$$= 7$$

Further Practice

7. The following table shows the total price ($\$P$) for x copies of books.

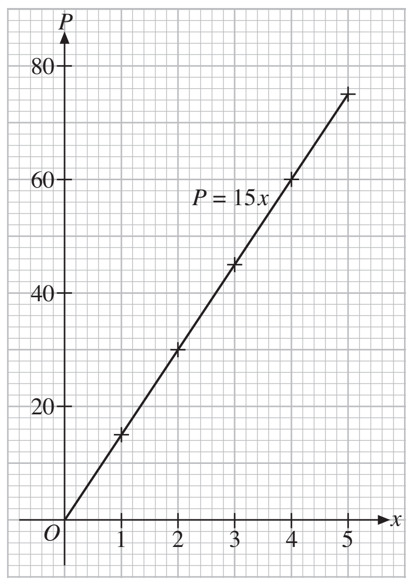
Copies of books (x)	1	2	3	4	5
Total price ($\$P$)	15	30	45	60	75

(a) Show that x and P are in direct proportion.
(b) Draw the graph of P against x.
(c) Describe the graph in **(b)**.
(d) Find the equation connecting x and P.
(e) Hence, find the total price for 8 copies of books.

Solution

(a) $\frac{1}{15} = \frac{2}{30} = \frac{3}{45} = \frac{4}{60} = \frac{5}{75}$
$\therefore$ x and P are in direct proportion.

(b) The diagram below shows the graph of P against x.



(c) The graph in **(b)** is a straight line that passes through the origin and its slope is positive.

(d) As $\frac{x}{P} = \frac{1}{15}$,

$P = 15x$.

(e) When $x = 8$,

$P = 15 \times 8$

$= 120$

The total price for 8 copies of books is \$120.

8. The following table shows the mass (m g) of a pinewood cube of side x cm.

Length of a side (x cm)	2	4	5	8	10
Mass (m g)	5.2	41.6	81.25	332.8	650

(a) Is m proportional to x?
(b) Is m proportional to x^3?
(c) Find an equation connecting m and x.
(d) Hence, find the mass in grams of a pinewood cube of side 9 cm.

Solution

(a) $\frac{2}{5.2} \neq \frac{4}{41.6}$

$\therefore$ m is NOT proportional to x.

(b) $\frac{5.2}{2^3} = \frac{41.6}{4^3} = \frac{81.25}{5^3} = \frac{332.8}{8^3} = \frac{650}{10^3} = 0.65$

$\therefore$ m is proportional to x^3.

(c) $\frac{P_1}{n_1} = 0.65$

$\therefore$ $m = 0.65x^3$

(d) When $x = 9$,

$m = 0.65 \times 9^3$

$= 473.85$

The required mass is 473.85 g.

Math@Work

9. The cost of renting a car is directly proportional to the number of days the car is being rented for. The cost of renting a car for 4 days is \$240. Find the cost of renting a car for 7 days.

Solution

Let the price of renting a car for n days be \$$P$.

Then $\frac{P_1}{n_1} = \frac{p_2}{n_2}$.

When $n_1 = 4$, $P_1 = 240$, $n_2 = 7$,

$\frac{240}{4} = \frac{P_2}{7}$

$P_2 = 7 \times \frac{240}{4}$

$= 420$

The cost of renting a car for 7 days is \$420.

10. The mass of a metal plate is directly proportional to its volume. When its volume is 20 cm^3, its mass is 210 g. If the volume of the metal plate is 50 cm^3, what is its mass?

Solution

Let the mass of a metal plate of V cm^3 be m g.

Then $\frac{V_1}{m_1} = \frac{V_2}{m_2}$.

When $V_1 = 20$, $m_1 = 210$, $V_2 = 50$,

$m_2 = 50 \times \frac{210}{20}$

$= 525$

The mass of the metal plate of volume 50 cm^3 is 525 g.

11. When a car is traveling steadily along a highway, its consumption of gasoline is directly proportional to the distance traveled. A car travels 100 mi on 2.7 gal of gasoline. Find, giving your answer correct to 1 decimal place,
(a) the gasoline consumption of the car for a distance of 74 mi,
(b) the maximum distance that the car can travel with 1 gal of gasoline.

Solution

(a) Let the consumption of gasoline be y gal when the distance traveled is d mi.

Then $y = kd$, where k is a constant.

When $d = 100$, $y = 2.7$.

$2.7 = k(100)$

$k = 0.027$

$\therefore$ $y = 0.027d$

When $d = 74$,

$y = 0.027 \times 74$

$= 2.0$ (correct to 1 d.p.)

The required gasoline consumption is 2.0 gal.

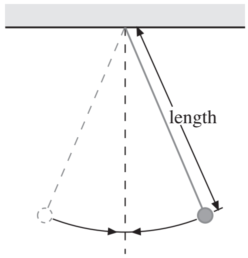
(b) When $y = 1$,

$1 = 0.027d$

$d = 37.0$ (correct to 1 d.p.)

The required maximum distance traveled is 37.0 mi.

12. The period (the time taken for one complete oscillation) of a simple pendulum is directly proportional to the square root of its length. When its length is 1.02 m, its period is 2.01 seconds. Find
(a) the period of the pendulum when its length is 0.8 m,
(b) the length of the pendulum when its period is 1.0 second.
Give your answers correct to 2 decimal places.



Solution

(a) Let the period of a simple pendulum of length L cm be T seconds.

Then $T = k\sqrt{L}$, where k is a constant.

When $L = 1.02$, $T = 2.01$.

$$2.01 = k\sqrt{1.02}$$
$$k = \frac{2.01}{\sqrt{1.02}}$$
i.e., $T = \frac{2.01}{\sqrt{1.02}}\sqrt{L}$

When $L = 0.8$,

$$T = \frac{2.01}{\sqrt{1.02}} \times \sqrt{0.8}$$
$$= 1.78 \quad \text{(correct to 2 d.p.)}$$

The required period of the pendulum is 1.78 seconds.

(b) When $T = 1$,

$$1 = \frac{2.01}{\sqrt{1.02}}\sqrt{L}$$
$$\sqrt{L} = \frac{\sqrt{1.02}}{2.01}$$
$$L = \frac{1.01}{2.01^2}$$
$$= 0.25 \quad \text{(correct to 2 d.p.)}$$

The required length of the pendulum is 0.25 m.

13. The vertical falling distance of a ball is directly proportional to the square of the time of falling. The ball falls 80 m in 4 s.
(a) Find the vertical falling distance of the ball when the time taken is 6 s.
(b) If the ball is dropped from a height of 245 m, find the time it takes to hit the ground.

Solution

(a) Let the vertical falling distance be y m in t seconds.

Then $y = kt^2$, where k is a constant.

When $t = 4$, $y = 80$.

$$80 = k \times 4^2$$
$$k = 5$$
$$\therefore\ y = 5t^2$$

When $t = 6$,

$$y = 5 \times 6^2$$
$$= 180$$

The required vertical falling distance is 180 m.

(b) When $y = 245$,

$$245 = 5t^2$$
$$t^2 = 49$$
$$t = \sqrt{49}$$
$$= 7$$

The required time taken is 7 seconds.

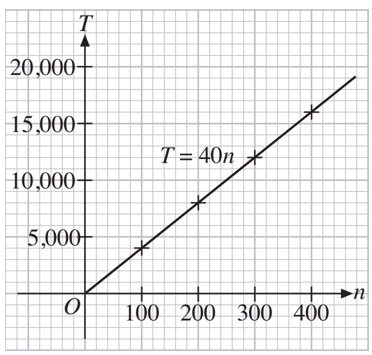
Brainworks

14. (a) In our daily life, we often encounter a wide variety of quantities involving direct proportion. Describe two such quantities.
(b) Draw a graph to show their relationship.
(c) Find an equation connecting the quantities.

Solution

(a) If the admission fee per person for a concert is \$40, then the total admission fee, \$$T$, is directly proportional to the number of people, n, attending the concert.

(b) The graph of T against n is shown below.



(c) The equation connecting T and n is $T = 40n$.
Note: Students may provide other relevant cases.