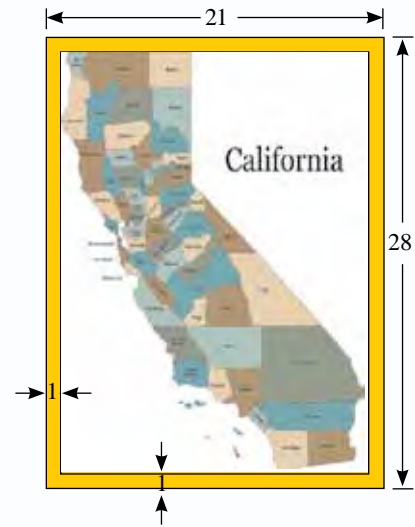




16. California, the third largest state in the United States, has an east-west dimension of 480 km.
- (a) A map of California, drawn with a scale of 1 : 1,500,000, fits exactly from north to south inside a rectangle of length 84 cm. What is the north-south dimension of California?
 - (b) If you sketch a map of California on a piece of paper 21 cm by 28 cm, what is an appropriate scale you would choose such that the map fits inside a rectangle with a uniform border of 1 cm around the sides of the paper?



14.3 Direct Proportion

When two quantities are related to each other, their changes may follow a certain pattern. For instance, if a sandwich costs \$1 at your school cafeteria, you will have to pay \$2 for two sandwiches, \$3 for three sandwiches, and so on. Thus, the amount you pay is related to the number of sandwiches ordered.

We shall explore this special relationship further in the following class activity.



CLASS ACTIVITY 1

Objective: To understand the idea of direct proportion.

Questions

The following table shows the relationship between the number of books bought (x) and the total cost of the books (\$ y).

| | | | | | | |
|-------------------------|---|----|----|----|----|----|
| Number of books (x) | 1 | 2 | 3 | 4 | 5 | 6 |
| Total cost (\$ y) | 5 | 10 | 15 | 20 | 25 | 30 |

(a) Copy and complete the following table.

| | | | | | | |
|---------------|---|----|----|----|----|----|
| x | 1 | 2 | 3 | 4 | 5 | 6 |
| y | 5 | 10 | 15 | 20 | 25 | 30 |
| $\frac{y}{x}$ | | | | | | |

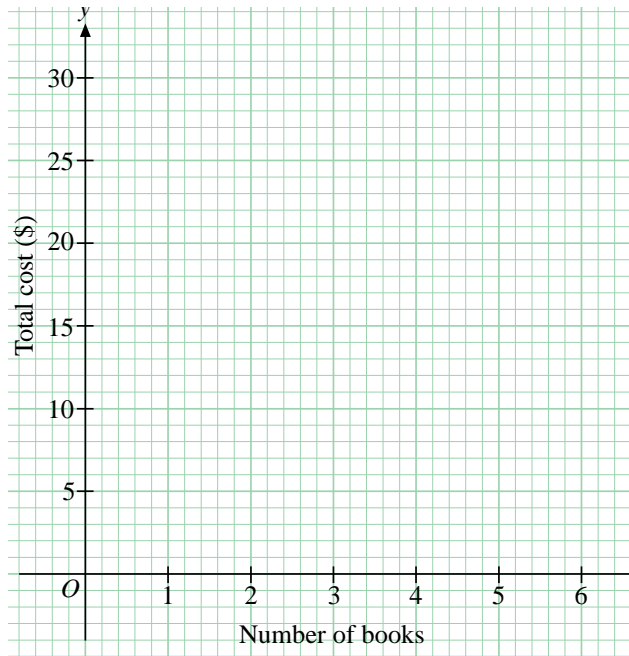
(b) On a sheet of graph paper, plot the corresponding points (x, y) in (a) using the scale for both axes as shown.

(c) What can you say about the points you have plotted in (b)?

(d) Write down an equation connecting x and y .

(e) What is the value of y when $x = 8$?

(f) Does the graph of the equation in (d) pass through the origin $(0, 0)$?



When two quantities, x and y , vary in such a way that $\frac{y}{x}$ is a constant, they are said to be in **direct proportion**, or y is said to be **directly proportional** to x .

REMARKS



Do you know that a distance on a map and its actual distance are in direct proportion?

In Class Activity 1, we have

$$\frac{y}{x} = \frac{5}{1} = \frac{10}{2} = \dots = \frac{30}{6} = 5,$$

which is a constant.

Hence, the number of books bought (x), and the total cost (y) are in direct proportion. Also, we see that the points (x, y) lie on a straight line passing through the origin. The equation of the straight line is $y = 5x$.

When two quantities, x and y , are in direct proportion, we have

$$\frac{y}{x} = k, \text{ where } k \text{ is a constant,}$$

that is $y = kx$

In Class Activity 1, the constant $k = 5$. Thus, $y = 5x$.

In general, we have the following properties:

When two quantities, x and y , are in direct proportion:

- $y = kx$, where k is a constant
- the graph of $y = kx$ is a straight line passing through the origin

DISCUSS



In the graph in Class Activity 1,
 (a) what is the gradient of the straight line?
 (b) what does the gradient represent?

Example 7

The following table shows the traveling time (t hr) and the distance covered (y mi) by a car.

| | | | | | |
|-------------------------------|-----|-----|-----|-----|-----|
| Traveling time (t hr) | 0.5 | 1.0 | 1.5 | 2.0 | 2.5 |
| Distance covered (y mi) | 30 | 60 | 90 | 120 | 150 |

- Show that t and y are in direct proportion.
- Draw the graph of y against t .
- Find the equation connecting t and y .
- Find the value of y when $t = 1.25$.
- Find the value of t when $y = 72$.

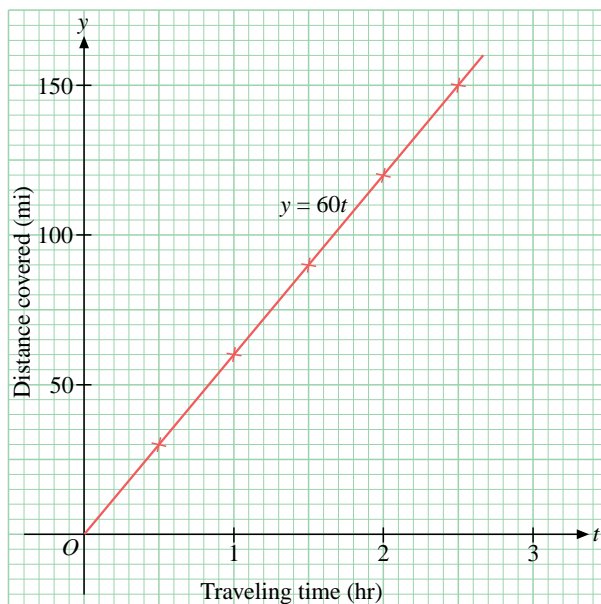
Solution

(a)

| | | | | | |
|---------------|-----|-----|-----|-----|-----|
| t | 0.5 | 1.0 | 1.5 | 2.0 | 2.5 |
| y | 30 | 60 | 90 | 120 | 150 |
| $\frac{y}{t}$ | 60 | 60 | 60 | 60 | 60 |

Since $\frac{y}{t}$ is a constant, t and y are in direct proportion.

(b) The graph of y against t is shown below.



(c) Since $\frac{y}{t} = 60$, the equation connecting t and y is $y = 60t$.

(d) When $t = 1.25$, $y = 60 \times 1.25 = 75$. Substitute $t = 1.25$ into the equation in (c).

(e) When $y = 72$, $72 = 60t$ Substitute $y = 72$ into the equation in (c).

$$t = \frac{72}{60} = 1.2$$

Try It! 7

The following table shows the number of hours (t) worked and the corresponding wages ($\$w$) of a worker.

| | | | | | |
|--------------------------------|-----|-----|-----|-----|-----|
| Number of hours worked (t) | 10 | 20 | 30 | 40 | 50 |
| Wages ($\$w$) | 150 | 300 | 450 | 600 | 750 |

- Show that t and w are in direct proportion.
- Draw the graph of w against t .
- Find the equation connecting t and w .
- If the worker worked 35 hours, find his wages.
- If the wages of a worker were $\$675$, find the number of hours he worked.

When two quantities, x and y , are in direct proportion,
 $y = kx$, where k is a constant.

Suppose (x_1, y_1) and (x_2, y_2) are any two pairs of values of x and y , where $x_2 \neq 0$ and $y_2 \neq 0$,

then $y_1 = kx_1$ and $y_2 = kx_2$.

Hence, $\frac{y_1}{y_2} = \frac{kx_1}{kx_2}$

that is $\frac{y_1}{y_2} = \frac{x_1}{x_2}$.

When two quantities, x and y , are in direct proportion, given any two pairs of values of x and y , (x_1, y_1) and

(x_2, y_2) , we have $\frac{y_1}{y_2} = \frac{x_1}{x_2}$, where $x_2 \neq 0$ and $y_2 \neq 0$.

The equality of two ratios is called a **proportion**. We say that y_1, y_2, x_1 , and x_2 are **in proportion**.

Example 8

The height, H cm, of a pile of books is directly proportional to the number of books, n , in the pile. The height of a pile of 10 books is 15 cm. What is the height of a pile of 24 books?

Solution

Method 1

Since n and H are in direct proportion, $H = kn$, where k is a constant.

When $n = 10$, $H = 15$.

$\therefore 15 = k \times 10$

$$k = \frac{15}{10} \\ = 1.5$$

Hence, $H = 1.5n$

When $n = 24$, $H = 1.5 \times 24$
 $= 36$

The height of a pile of 24 books is 36 cm.

REMARKS

The value of k is actually the thickness of each book in centimeters.

Estimate: Is the answer reasonable?

24 books are slightly more than twice of 10 books. Is the answer '36 cm' slightly more than twice of '15 cm'?

Method 2

Since n and H are in direct proportion,

$$\frac{H_1}{H_2} = \frac{n_1}{n_2}$$

Let $n_1 = 10$, $H_1 = 15$, and $n_2 = 24$.

Substituting these values into the proportion, we have:

$$\begin{aligned}\frac{15}{H_2} &= \frac{10}{24} \\ H_2 &= 15 \times \frac{24}{10} \\ &= 36\end{aligned}$$

Hence, the height of a pile of 24 books is 36 cm.

Try It! 8

The number of pieces, n , of chocolate and their total mass, M grams, are in direct proportion. The mass of 15 pieces of chocolate is 180 g. Find the mass in grams of 20 pieces of chocolate.

In real life, two quantities, x and y , may not be in direct proportion but x^n and y^m , where n and m are rational numbers, may be in direct proportion. The following example illustrates such a situation.

Example 9

The weight of a solid metal sphere and the cube of its radius are in direct proportion. When the radius of the sphere is 2 in., its weight is 192 oz. Find the weight in ounces of a sphere of radius of 1.5 in.

Solution

Let the weight of a sphere of radius r in. be m oz. Then, we have $m = kr^3$, where k is a constant.

When $r = 2$, $m = 192$.

$$\therefore 192 = k \times 2^3$$

$$k = \frac{192}{8}$$

$$= 24$$

$$\therefore m = 24r^3$$

When $r = 1.5$, $m = 24 \times 1.5^3$
 $= 81$

The weight of a sphere of radius 1.5 in. is 81 oz.

Try It! 9

The mass of a square piece of glass panel is directly proportional to the square of the length of its side. When its side is 20 cm, the mass is 1,000 g. Find the mass in grams of a glass panel of side 30 cm.

EXERCISE 14.3



BASIC PRACTICE

1. In each of the following tables, determine whether x and y are in direct proportion.

(a)

| | | | | |
|-----|---|---|---|----|
| x | 1 | 2 | 3 | 4 |
| y | 3 | 6 | 9 | 12 |

(b)

| | | | | |
|-----|----|----|----|----|
| x | 2 | 4 | 6 | 8 |
| y | 10 | 20 | 25 | 40 |

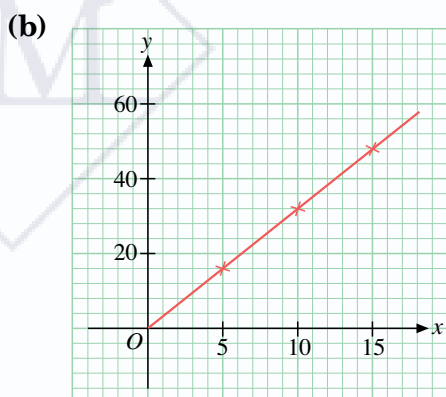
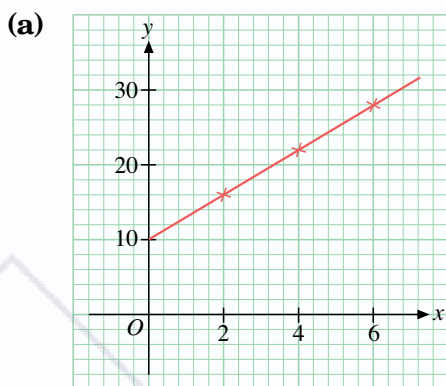
(c)

| | | | | |
|-----|----|----|----|----|
| x | 3 | 6 | 7 | 10 |
| y | 21 | 42 | 49 | 70 |

(d)

| | | | | |
|-----|----|----|----|----|
| x | 8 | 12 | 15 | 20 |
| y | 30 | 48 | 60 | 80 |

2. In each of the following graphs, determine whether x and y are in direct proportion.



3. In each of the following equations, determine whether x and y are in direct proportion.

(a) $y = 4x$ (b) $y = x + 2$

(c) $y = x^2$ (d) $y = \frac{1}{2}x$

4. If two quantities, x and y , are in direct proportion, find the values of p and q in the following table.

| | | | |
|-----|----|-----|-----|
| x | 12 | 18 | q |
| y | 8 | p | 24 |

5. It is given that w is directly proportional to t . When $t = 4$, $w = 20$. Find

- (a) the value of w when $t = 6$,
 (b) the value of t when $w = 45$.

6. It is given that A is directly proportional to r^2 and $r > 0$. When $r = 5$, $A = 75$. Find

- (a) the value of A when $r = 4$,
 (b) the value of r when $A = 147$.



FURTHER PRACTICE

7. The following table shows the total price ($\$P$) for x copies of books.

| | | | | | |
|-------------------------|----|----|----|----|----|
| Copies of books (x) | 1 | 2 | 3 | 4 | 5 |
| Total price ($\$P$) | 15 | 30 | 45 | 60 | 75 |

- (a) Show that x and P are in direct proportion.
 (b) Draw the graph of P against x .
 (c) Describe the graph in (b).
 (d) Find the equation connecting x and P .
 (e) Hence, find the total price for 8 copies of books.

8. The following table shows the mass (m g) of a pinewood cube of side x cm.

| | | | | | |
|----------------------------|-----|------|-------|-------|-----|
| Length of a side (x cm) | 2 | 4 | 5 | 8 | 10 |
| Mass (m g) | 5.2 | 41.6 | 81.25 | 332.8 | 650 |

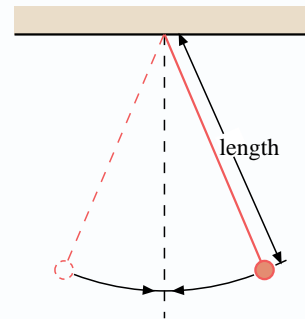
- (a) Is m proportional to x ?
 (b) Is m proportional to x^3 ?
 (c) Find an equation connecting m and x .
 (d) Hence, find the mass in grams of a pinewood cube of side 9 cm.



MATH@WORK

9. The cost of renting a car is directly proportional to the number of days the car is being rented for. The cost of renting a car for 4 days is \$240. Find the cost of renting a car for 7 days.
10. The mass of a metal plate is directly proportional to its volume. When its volume is 20 cm^3 , its mass is 210 g. If the volume of the metal plate is 50 cm^3 , what is its mass?
11. When a car is traveling steadily along a highway, its consumption of gasoline is directly proportional to the distance traveled. A car travels 100 mi on 2.7 gal of gasoline. Find, giving your answer correct to 1 decimal place,
- (a) the gasoline consumption of the car for a distance of 74 mi,
 (b) the maximum distance that the car can travel with 1 gal of gasoline.

12. The period (the time taken for one complete oscillation) of a simple pendulum is directly proportional to the square root of its length. When its length is 1.02 m, its period is 2.01 seconds. Find
- (a) the period of the pendulum when its length is 0.8 m,
 (b) the length of the pendulum when its period is 1.0 second.
- Give your answers correct to 2 decimal places.



13. The vertical falling distance of a ball is directly proportional to the square of the time of falling. The ball falls 80 m in 4 s.
- (a) Find the vertical falling distance of the ball when the time taken is 6 s.
 (b) If the ball is dropped from a height of 245 m, find the time it takes to hit the ground.



BRAINWORKS

14. (a) In our daily life, we often encounter a wide variety of quantities involving direct proportion. Describe two such quantities.
 (b) Draw a graph to show their relationship.
 (c) Find an equation connecting the quantities.
15. Jordan's height and weight increase as he grows bigger. Do you think his height and weight are in direct proportion? Give reasons for your answer.