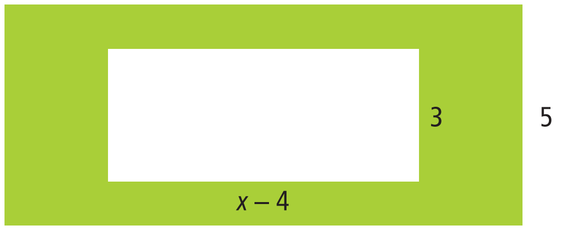
Mathematics and Statistics 1.2

Externally assessed
(CAT)
4 credits

Apply algebraic procedures in solving problems

Online practice assessment task

1. a. A rectangle of length $(2x + 1)$ cm and width 5 cm has a rectangle of length $(x - 4)$ cm and width 3 cm cut from it.

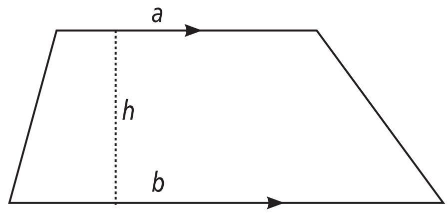


What is the area of the remaining part of the rectangle (shaded green)?

b. For what values of x is $x^2 + 3x - 18$ negative?

c. Simplify fully $\frac{6x^2 - 9x}{12x^2}$

d. The area A of a trapezium is given by $A = \frac{1}{2}(a + b)h$, where a and b are the lengths of the parallel sides and h is the perpendicular height of the trapezium.



Find a formula for the perpendicular height of a trapezium in terms of a, b, and A and use it to find the height of a trapezium with parallel sides of length 4.5 cm and 7.5 cm and area of 31.5 cm^2.

e. Zac solves an equation as shown below but he makes some errors in his working so that neither of his solutions is correct. Explain the errors in his working and find the correct solution (there is only one).

$$\frac{x^2 - 4}{x^2 - 4x + 4} = \frac{4}{5}$$

$$4(x^2 - 4x + 4) = 5(x^2 - 4)$$

$$4x^2 - 16x + 16 = 5x^2 - 20$$

$$0 = x^2 - 16x - 36$$

$$(x - 18)(x - 2) = 0$$

$$x = 18 \text{ or } x = 2$$

f. The amount of money in Dan's savings after n years is given by $D = 800 \times 2^n$. The amount of money in Ella's savings is given by $E = 100 \times 4^n$. After how many years will Dan and Ella have the same amount of money in their savings? What is this amount?

2. a. Two numbers have a product of zero. If one number is $(2x + 5)$ and the other number is $(3x - 1)$, find possible values of x.

b. If I double a number and add 3 the result is the same as tripling the number and subtracting 10. What is the number?

c. Find a quadratic equation with solutions $x = 4$ or $x = -3$. Give your answer in the form $x^2 + bx + c = 0$

d. Manu buys lunches for her office at the local café. One day she buys 5 coffees and 7 sandwiches, at a total cost of $49.10. If the sandwiches cost 50 cents more than the coffees, find the cost of a sandwich.

e. A piece of paper is 6 cm longer than it is wide.
The piece of paper is glued onto a larger piece of cardboard.
The width of the cardboard is 2 cm more than the width of the paper.
The length of the cardboard is five times the width of the paper.
The area of the shaded border of cardboard is double the area of the paper.

Find the width of the paper.

3. **a.** The side length of a cube is $3ab^2$. What is the volume of the cube?

b. One factor of $6x^2 - 5x - 4$ is $(2x + 1)$. Find the other factor.

c. One day Kim did a gardening job for a neighbour. She was paid $35 for the first two hours, then $16 per hour after that. She was paid $115 altogether. Work out the number of hours Kim was paid for.

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d. The volume V of a sphere is given by $V = \frac{4}{3}\pi r^3$, where r is the radius of the sphere. If a sphere has volume 288π cubic centimetres, find the radius of the sphere.

e. Find values of n so that $3 \times 2^{2n-3} > 100$ where n is a whole number.

f. Jack and Mia both have a bag of sweets. If Jack gave Mia 3 sweets, then both children would have the same number of sweets. If Mia gave Jack 2 sweets, then Jack would have twice as many sweets as Mia.

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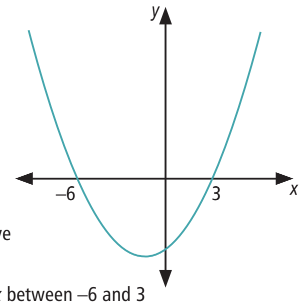
Work out how many sweets Jack and Mia have altogether.

Answers

1. a. Area $= 5(2x + 1) - 3(x - 4)$
$= 10x + 5 - 3x + 12$
$= 7x + 17$ cm^2

b. $x^2 + 3x - 18 = (x + 6)(x - 3)$
$x^2 + 3x - 18 = 0$ for $x = -6$ or $x = 3$
If $x < -6$ then $x^2 + 3x - 18$ is positive
If $-6 < x < 3$ then $x^2 + 3x - 18$ is negative
If $x > 3$ then $x^2 + 3x - 18$ is positive
So $x^2 + 3x - 18$ is negative for values of x between -6 and 3



c. $\frac{6x^2 - 9x}{12x^2} = \frac{3x(2x - 3)}{12x^2}$
$= \frac{2x - 3}{4x}$

d. $2A = (a + b)h$ so $h = \frac{2A}{a + b}$
Substituting $a = 4.5$, $b = 7.5$ and $A = 31.5$ gives height $= 5.25$ cm

e. From Line 3 the working should be:
$x^2 + 16x - 36 = 0$
$(x + 18)(x - 2) = 0$
$x = -18$ or $x = 2$
Substituting $x = 2$ into the original equation gives $\frac{0}{0} = \frac{4}{5}$ so $x = 2$ is not a solution.
This invalid solution has arisen since $\frac{x^2 - 4}{x^2 - 4x + 4} = \frac{(x + 2)(x - 2)}{(x - 2)(x - 2)}$ which is undefined when $x - 2 = 0$, i.e. when $x = 2$.
If $x \neq 2$ then, by cancelling the common factor $(x - 2)$, the equation simplifies to $\frac{x + 2}{x - 2} = \frac{4}{5}$
Cross-multiplying gives $5x + 10 = 4x - 8$, so $x = -18$
So the solution $x = -18$ is the only correct one.

f. Solve $800 \times 2^n = 100 \times 4^n$
Dividing both sides by 100 and by 2^n gives
$8 = \frac{4^n}{2^n}$ which gives $8 = \left(\frac{4}{2}\right)^n$
Solving $8 = 2^n$ gives $n = 3$
When $n = 3$, savings $= 800 \times 2^3 = \$6\,400$ (or $100 \times 4^3 = 6\,400$)
So after 3 years, both have savings of \$6 400

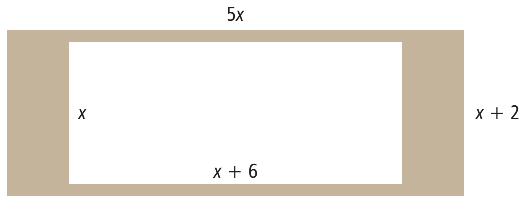
2. a. $(2x + 5)(3x - 1) = 0$
$2x + 5 = 0$ or $3x - 1 = 0$
$x = -2\frac{1}{2}$ or $x = \frac{1}{3}$

b. $3x - 10 = 2x + 3$, so $x = 13$

c. $x = 4$ is solution so $(x - 4)$ is a factor
$x = -3$ is a solution so $(x + 3)$ is a factor
Equation is $(x - 4)(x + 3) = 0$
$x^2 - x - 12 = 0$

d. $5x + 7y = 49.1$ and $y = x + 0.5$; a sandwich costs \$4.30

e. $5x(x + 2) - x(x + 6) = 2x(x + 6)$; width $= 4$ cm



3. a. Volume $= (3ab^2)^3$
$= 3^3a^3(b^2)^3$
$= 27a^3b^6$

b. Factorising
$6x^2 - 5x - 4 = (2x + 1)(3x - 4)$
So, the other factor is $(3x - 4)$

c. Let x = total number of hours Kim works
So Kim works 2 hours for \$35 and $(x - 2)$ hours at \$16 per hour
$35 + 16(x - 2) = 115$
Solving gives $x = 7$ so Kim was paid for seven hours of work

d. $\frac{4}{3}\pi r^3 = 288\pi$
Dividing by π gives $\frac{4}{3}r^3 = 288$, so $r^3 = 216$ and $r = 6$
So the radius is 6 cm

e. $2^{2n-3} > 33.33$
Since $2^6 = 32$ and $2^7 = 64$
$2n - 3 > 6$
$2n > 9$
$n > 4.5$
So $n \geq 5$, where n is a whole number

f. Equations will vary.
$j - 3 = m + 3$ and $j + 2 = 2(m - 2)$
Solve to get $m = 12$, $j = 18$, so 30 sweets altogether