Mathematics and Statistics 1.12

Externally assessed
4 credits

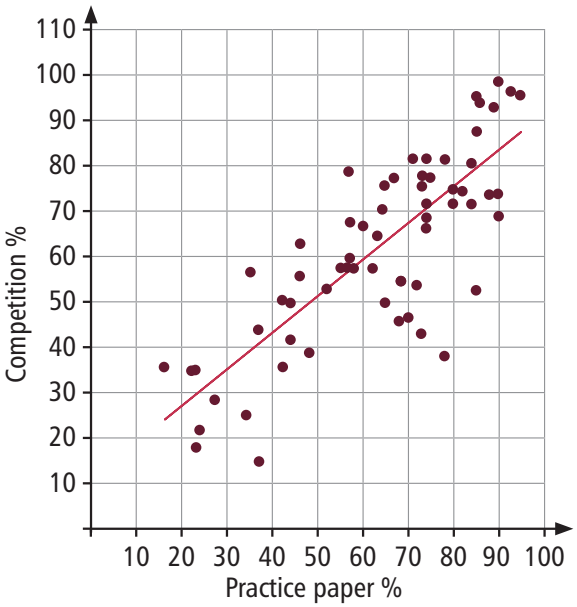
Demonstrate understanding of chance and data
Online practice assessment task

1. Each year at a secondary school, Year 9 students participate in a mathematics competition. To prepare them for the competition, the teachers give them a practice paper to sit in school before they sit the actual competition paper.

After a few years, the mathematics teachers wondered if the marks in the external competition were similar to the marks in the practice paper, so they analysed some of the data they had collected.

A scatter graph with a trend line (line of best fit) is shown below, along with a table of statistics.

Year 9 Mathematics marks



Year 9 Mathematics marks		
	Practice paper %	Competition paper %
Minimum	16	15
Lower quartile	47	48.5
Median	69	67
Upper quartile	80	77
Maximum	95	99
Range	79	84
Interquartile range	33	28.5

a. i. What was the highest mark a student scored in the practice paper?

ii. What was the practice paper mark of the person who had the lowest competition mark?

iii. How many students got a competition mark between 20% and 40%?

iv. How many students got a practice mark over 60% but below 70%?

b. True or false: the person with the lowest practice mark did worst in the competition?

c. Why is a scatter graph an appropriate graph for this data?

d. The teachers thought some of the competition marks were unexpectedly high or low, compared to the practice marks.

i. Give one unexpectedly high mark. Explain your choice.

ii. Give one unexpectedly low mark. Explain your choice.

e. **i.** On the scatter graph, draw in the line of points where the practice and competition marks are equal.

ii. How many students got equal marks in the practice and competition papers?

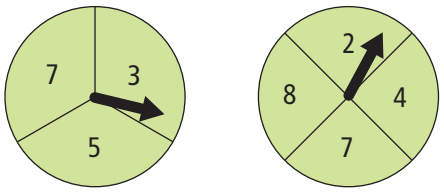
f. A student says that your practice mark will probably be more than your competition mark.

Comment on this claim, using the graphs and statistics to explain whether you agree or disagree.

g. A teacher comments that, overall, the marks that Year 9 students get in the competition are similar to the marks they get in practice tests.

Comment on this claim, using the graphs and statistics to explain whether you agree or disagree.

2. At a school gala day, Ming is running a game of chance involving two spinners, as shown below.



a. If the first spinner is spun, what is the probability of getting a number greater than 4?

b. If the second spinner is spun, what is the probability of getting a 7?

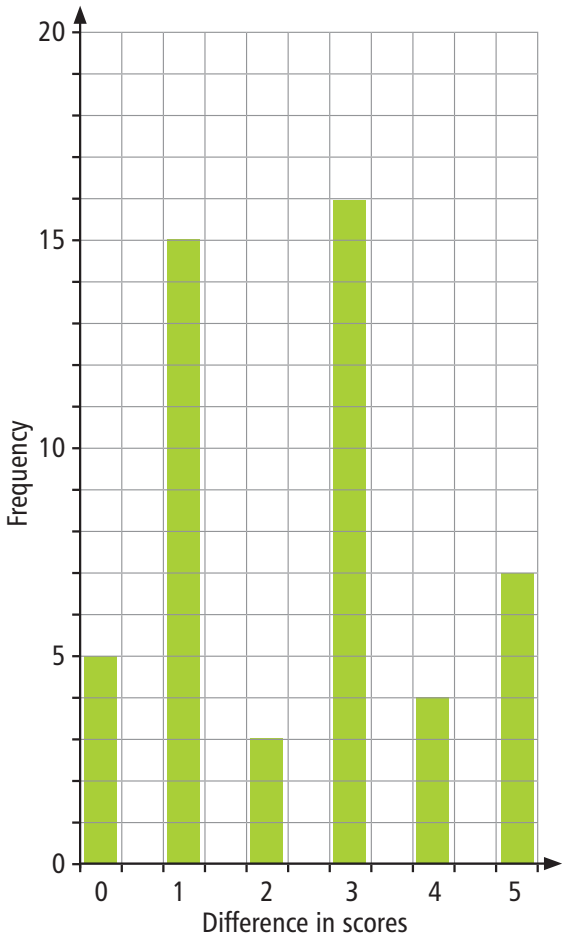
In Ming's game, the two spinners are spun together:

- If the two numbers are the same you win $10.
- If the numbers are exactly 1 apart you get your money back.
- Otherwise you lose.

The cost of playing the game is $2.

To test the game, Ming spun the two spinners together 50 times. In each trial she subtracted the larger number from the smaller to find the difference. If the numbers were the same she recorded a difference of zero. Her results are shown in the graph below.

Ming's differences from 50 spins



c. From Ming's trial, if the two spinners are spun together, how likely is it that:

i. the two numbers are 1 apart?

ii. the two numbers are the same?

d. From Ming's trial, if 100 customers come to play her game:

i. How much will they pay in total to play?

ii. How many $10 prizes would Ming expect to have to pay out?

iii. How many $2 prizes would Ming expect to have to pay out?

iv. How much profit does Ming think she would make?

v. How confident do you think Ming will be about these predictions?

e. Ming decided to draw up a table showing all the different results she could get when she works out the difference between the two numbers from her spinners.

i. Complete the table.

	2	**4**	**7**	**8**
3	1	1	4	5
5				
7				

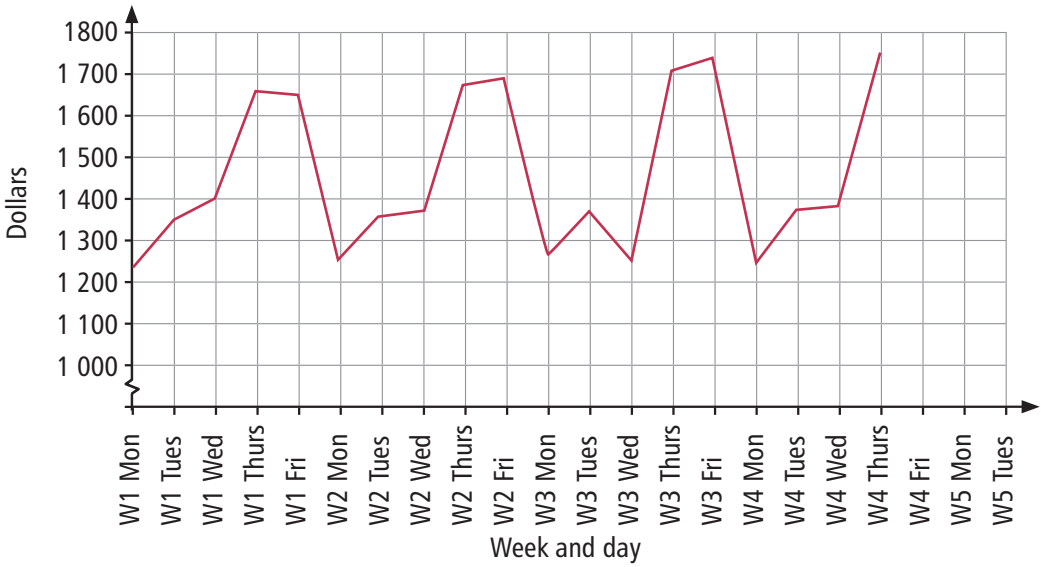
Using the results in the table:

ii. what is the probability of a win of $10?

iii. what is the probability of getting your money back?

iv. Compare these results with those of Ming's 50 trials. Discuss any differences you see.

3. A shop records the daily sales of products in its grocery section over several weeks. A graph is drawn for the data.



a. Describe any overall trends that the graph shows over this time.

b. Describe any repeating patterns that you can see in the graph.

c. Describe any unusual values of sales that you can see in the graph.

d. i. On the graph, draw your prediction for the daily sales for the next three days (Week 4 Friday, Week 5 Monday, Week 5 Tuesday).

ii. How confident are you in your predictions for these days?

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Answer

1. a. i. 95%

ii. The point (37,15) has the lowest competition mark of 15%. The practice mark was 37%.

iii. Between the horizontal lines at 20 and 40 there are 9 points. These represent the 9 students who scored between 20% and 40% in the competition.

iv. Between the vertical lines at 60 and 70 there are 7 points. These represent the 7 students who scored between 60% and 70% in the practice paper.

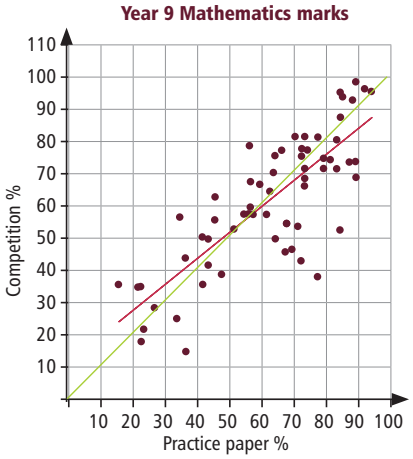
b. False. (The person with the lowest practice paper mark of 16% scored 36% in the competition paper as shown by the point (16,36); this was higher than the lowest competition mark of 15%.)

c. A scatter graph is used for pairs of related data (bivariate data), so that the relationship between the variables can be seen.

d. i. The point (55,79) represents a student who scored 55% in the practice paper and 79% in the competition paper. This was an improvement of 24%. (This point is the greatest distance above the trend line.)

ii. The point (78,38) represents a student who scored 78% in the practice paper and 38% in the competition paper. This was a reduction of 40%. (This point is the greatest distance below the trend line.)

e. i



ii. 1 student got 58% in both papers (represented by the point (58,58).

f. There are 34 points above the line of equal scores (drawn in part e.)
These points represent students whose practice mark was lower than their competition mark. And there are 29 points below the line, which represent the students whose practice mark was higher than their competition mark.

So the student is not correct to say that your practice mark will probably be higher than your competition mark, as less than half of these students had a practice mark higher than their competition mark.

Since both groups are close to half of the total group of 64 students, there may be very little difference in the overall proportions who do better in one test than the other.

g. Agree. Looking at the statistics, the practice paper median mark was 69% which was only 2% higher than the median of the competition marks, which was 67%, which is a very small shift. The middle 50% of practice paper marks were between 47% and 80%, and the middle 50% of competition paper marks were between 48.5% and 77%, so each median lies within the other's middle 50%. So the median of practice paper marks are unlikely to be different from the median of competition marks back in the populations.

2. a. $\frac{2}{3}$ b. $\frac{1}{4}$

c. i. $\frac{15}{50}$ or $\frac{3}{10}$ or 0.3

ii. $\frac{5}{50}$ or $\frac{1}{10}$ or 0.1

d. i. $100 \times \$2 = \$200$

ii. Doubling the number of players in the trial (which is 50) and doubling the number of players winning \$10 (which is 5) means that out of 100 players, 10 would be expected to win \$10.

Expected payout $= 10 \times \$10 = \$100$

iii. 15 out of 50 won their money back (which was \$2), so 30 out of 100 would be expected to win back \$2.

Expected payout $= 30 \times \$2 = \$60$

iv. Ming thinks she will make
$\$200 - \$100 - \$60 = \$40$ profit after 100 customers.

v. Ming did 50 trials, which is a large enough number of trials to make her results reasonably reliable. But another experiment could give different results, which may give her more or less profit, so Ming cannot be very sure about these predictions. More trials would be better (as this reduces variability of results) so that she can be more confident.

e. i

	2	4	7	8
3	1	1	4	5
5	3	1	2	3
7	5	3	0	1

ii. $\frac{1}{12}$ or 0.0833 (4 d.p.)

iii. $\frac{4}{12}$ or $\frac{1}{3}$ or 0.3333 (4 d.p.)

iv. The results in the trials were similar to those of the table.

In the trials, the probability of a win of \$10 was 0.1, which is 0.0167 higher than 0.0833 which was the probability of a win of \$10 from the table.

In the trials, the probability of getting your money back was 0.3, which is 0.0333 lower than 0.3333 which was the probability of winning your money back from the table.

The table gives theoretical probabilities which are probably more reliable than the experimental probabilities from Ming's trials, unless the spinners are not perfectly balanced and fair.

According to the theoretical probability table, this would mean Ming should expect $\frac{1}{12}$ of the 100 people to win \$10, which means paying out $\$(100 \times \frac{1}{12}) \times 10 = \$83.33$, and $\frac{1}{3}$ of the 100 people to get their money back, which means paying out $\$(100 \times \frac{1}{3}) \times 2 = \$66.67$

So Ming would expect to make

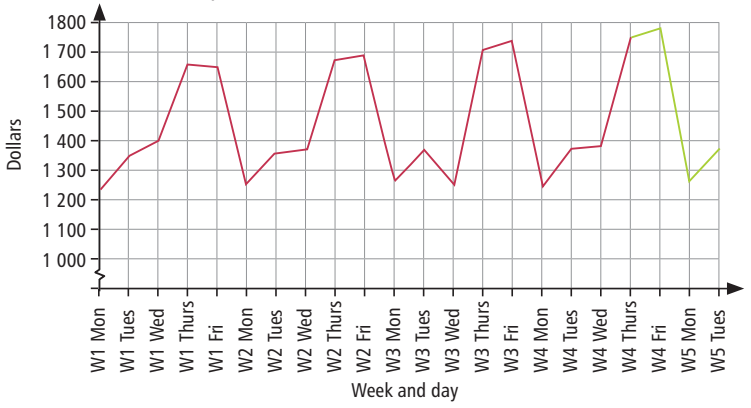
$\$200 - \$83.33 - \$66.67 = \$50$ which is \$10 higher than the \$40 profit that Ming expected to make using the results of her trials.

3. a. The long term trend is a gradually increasing one.

b. Sales are generally lowest on Mondays and highest on Fridays, with increasing sales figures from Tuesdays to Thursdays.

c. Wednesday sales' figures are usually higher than Tuesday sales' figures, but in week 3, the Wednesday sales' figures were lower than the Tuesday sales' figures. In fact, in week 3 the Wednesday sales figures were the lowest for the week which was unusual, as Monday sales' figures are usually the lowest in any week. Without more information, this unusual Wednesday week 3 sales figure cannot be explained.

d. i. *Answers will vary.*



ii. *Answers will vary; an example is shown.*

Friday sales' figures for the first three weeks shown on the graph were approximately \$1 650, \$1 690, \$1 740, which shows a steady increase of about \$40 per week, so I predict \$1 780 for Friday week 4. I am not very confident about this because there are only three weeks of data to base this prediction on. It would have been better to have had data available for a full four weeks to see if there are any monthly patterns emerging.

Monday sales for the first three weeks were approximately \$1 230, \$1 250, \$1 260, \$1 250, which shows little change over time, so I predict \$1 260 for Monday week 5. I am fairly confident about this as the pattern of little change is steady.

Tuesday sales for the first three weeks were \$1 350, \$1 350, \$1 370, \$1 370, which shows a small steady increase (of about \$7 per week on average), so I predict \$1 380 for Tuesday week 5. I am reasonably confident about this prediction as Tuesday sales' figures have consistent values over the four weeks.